PRACTICAL ELECTRICITY

PART II

BY

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SECTION 29

PRINCIPLES OF ELECTRIC GENERATORS

546. Electric generators or dynamos are machines whereby by utilizing the principle of magnetic induction (Art. 451), mechanical energy may be converted into electrical energy (Art. 200). They are sometimes referred to as electricity generators. This term is, however, incorrect Electricity can not be generated. A generator may be thought of as a device by means of which electricity which is already in existence (Art. 105) can be forced to move and thereby transmit energy and do work—light lamps, operate motors, and the like. A generator does not generate electricity any more than a hydraulic force pump generates water.

547. How a generator can convert mechanical energy into electrical energy is outlined in Art. 105. Briefly, the mechanical turning of the rotating portion of the generator causes conductors associated with the machine to cut or to be cut by a magnetic flux. This flux is usually produced by electromagnets. The cutting of the flux induces an e.m.f. in the conductors. Now if this e.m.f. be impressed on a closed, conducting circuit it will force through the circuit an electric current—which is a transference of electrons. As outlined in Art. 202, whenever a current is impelled electrical energy is generated.

548. The principle of the generator has already been briefly indicated in Art. 453. If the bar shown in Fig. 236 be pushed through the magnetic field between the poles of the magnet an e.m.f. will be induced in the bar. The intensity of the e.m.f. will be proportional to the speed with which the bar is moved—that is, proportional to the rate of cutting. This e.m.f. will force a current (of an intensity inversely proportional to the resistance of the circuit) through the circuit which consists of the bar and the external circuit. The e.m.f. will be in one direction while the bar is being moved up and in the other while the bar is being moved down. The directions of these induced e.m.fs. can be ascertained by applying the hand rule of Art. 462. Power will

be required to move the conductor between the poles (since current is forced through the circuit), and the power, P, thus required (neglecting the weight and the eddy-current loss) will be equal, in watts (Art. 186) to $E \times I$. If the circuit is open, $I = 0$. Hence, with an open circuit, $P = 0$, that is, no power is required to move the bar.

Example.—If it be assumed that there are 200,000,000 lines in the uniform field of Fig. 236 and that just 1 sec. is required to move the bar at a

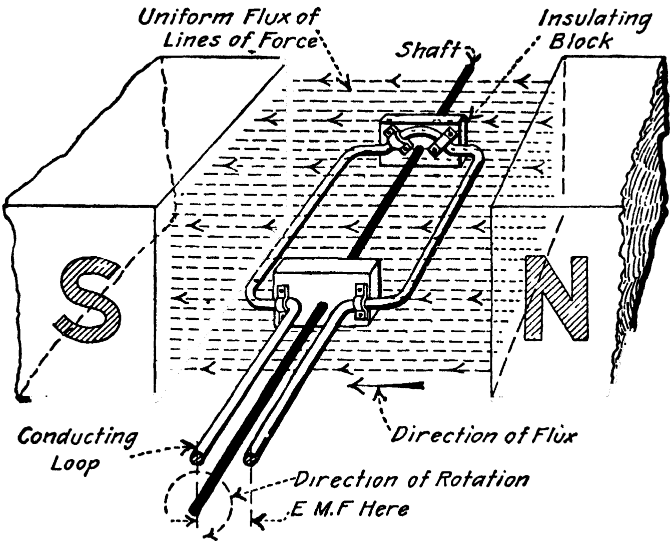


FIG. 274.—Illustrating the principle of the generator.

uniform rate through them [since cutting at the rate of 100,000,000 lines per sec. induces 1 volt, (Art. 248)], an e.m.f. of 2 volts (Art. 480) will be generated during that second. Assuming the resistance of the entire circuit, comprising bar and external circuit, to be ½ ohm, the current will (Art. 151) be $I = E \div R = 2 \div \frac{1}{2} = 4$ amp. The power expenditure—the rate of doing work—will (Art. 186) be $P = E \times I = 2 \times 4 = 8$ watts.

549. In commercial generators it is necessary to use strong electromagnets to produce the fields and it is necessary to move conductors through the fields at high speeds to generate the e.m.fs. required. These high speeds, viz., high rates of cutting lines are best attained by rotating conductors formed into loops through magnetic fields. The elements of such an arrangement are shown in Fig. 274.

550. The hand rule for determining the relative directions of motion, e.m.f., and flux as applied to a loop rotated in a field is stated graphically in Fig. 275. This is merely a specific adaptation of the rule given in Art. 462. Note that the right hand is

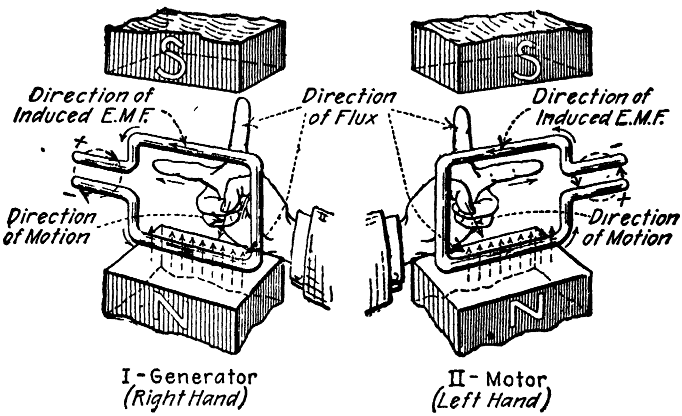


FIG. 275. —Application of hand rule to rotating coils.

always used for a generator as at I while the left hand is used for a motor as at II.

551. What occurs when a conducting loop is rotated in a magnetic field will now be discussed in connection with Figs. 276 to

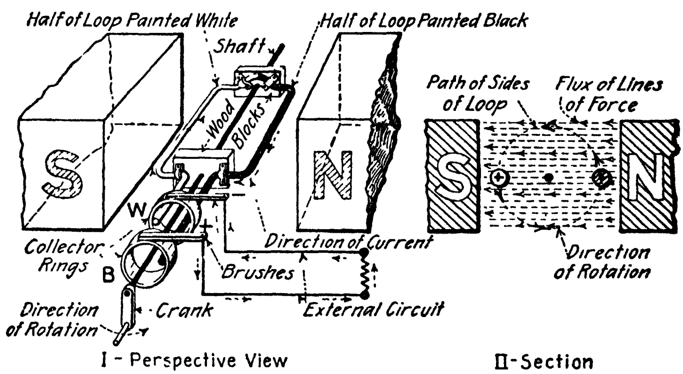


FIG. 276.—First position of conducting loop in a magnetic field.

283. Imagine the conducting loop of Fig. 274 arranged in a magnetic field, as shown in Fig. 276, with a collector ring electrically connected to each side of the loop and provided with two metallic brushes, each so mounted as to make electrical

contact with one of the collecting rings. The brushes bear down on their rings and always make electrical connection with them, but they do not interfere with the turning of the loop. The external circuit is connected to the two brushes as shown. Furthermore, assume that one-half or side of the loop is painted black, as shown, merely so that it can be readily distinguished from the other half which is not painted.

Now if this loop of Fig. 276 be rotated—in either direction—both of its sides, the painted and the unpainted one, will cut lines of force (Fig. 276,I), and an e.m.f. will be generated within the

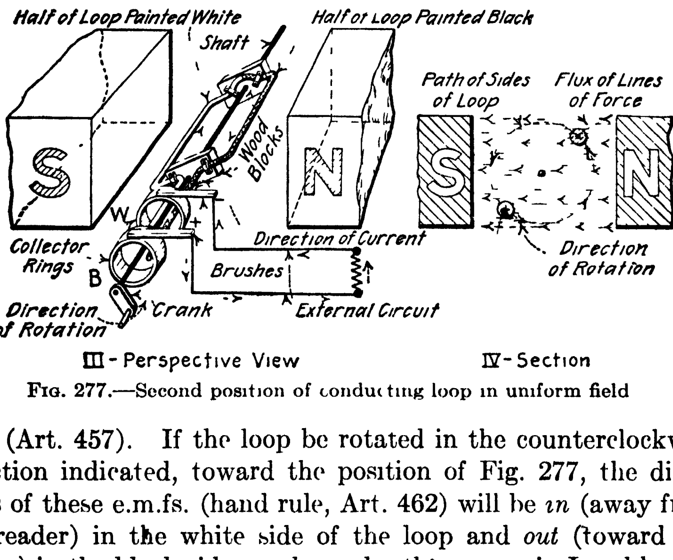


FIG. 277.—Second position of conducting loop in uniform field

loop (Art. 457). If the loop be rotated in the counterclockwise direction indicated, toward the position of Fig. 277, the directions of these e.m.fs. (hand rule, Art. 462) will be *in* (away from the reader) in the white side of the loop and *out* (toward the reader) in the black side, as shown by the arrows in I and by the cross and dot in II. Brush *B* (Fig. 277) then will be positive (+), because the direction of the induced e.m.f. at this time is out of or away from it, and the brush *W* will be negative (−).

Current will be forced in the direction shown through loop and the external circuit by this e.m.f. as outlined in the following example:

Example.—Now assume that the rotation of the above-described loop be continued at a uniform speed and in a counterclockwise direction. Its sides will cut lines of force as it moves through the position of Fig. 277 until it reaches the position indicated in Fig. 278. At the instant illustrated in Fig. 278 no lines of force will be cut, because at this instant the sides of the conductor are moving parallel to the lines and cannot, therefore, cut them.

Hence, at this instant, the e.m.f. induced will be zero, and therefore no current will be forced through the loop or external circuit.'

It can be shown (Art. 555) that, as the loop is rotated, at a uniform speed, from the position of Fig. 276 to that of Fig. 278, the e.m.f. induced in it gradually decreases from a maximum to zero. (By "a maximum e.m.f." is

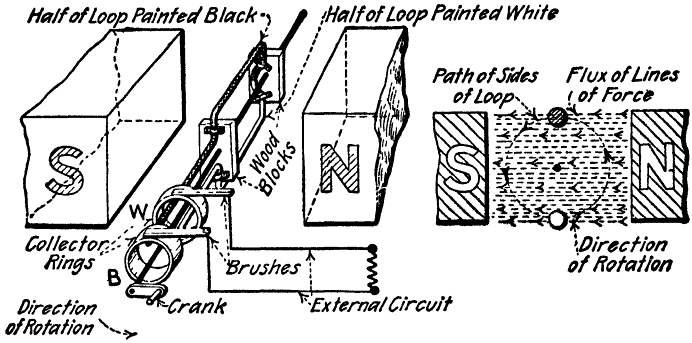


FIG. 278.—Third position of conducting loop in uniform field.

meant the greatest possible e.m.f. that can be induced in the loop with the given flux and given speed of rotation; the value of such a maximum e.m.f. can be determined in any given case by using the formula of Art. 480.) The rate of cutting gradually decreases as the loop is rotated until, in the

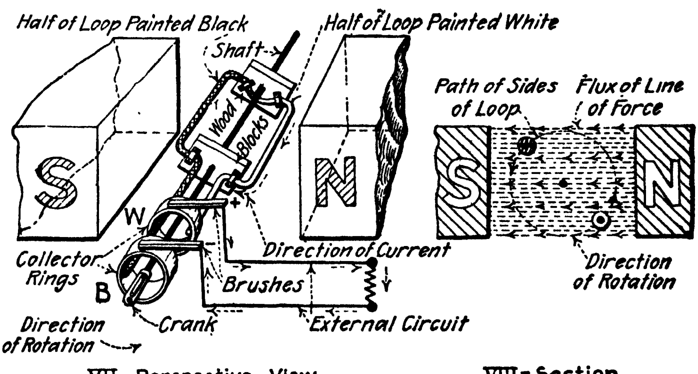


FIG. 279.—Fourth position of conducting loop in uniform field.

position of Fig. 278, no lines are being cut—the rate of cutting is then zero and the induced e.m.f. is therefore zero.

The uniform rotation of the loop is continued. Just as the loop leaves the "neutral" position (Fig. 278) its sides—both of them—will again commence to cut lines. But the e.m.f. now induced in each of the sides of the loop will be in the opposite direction from that induced during the first quarter of

the revolution. Apply the hand rule of Art. 462 and verify the directions of the arrows showing the e.m.f. directions in Fig. 279. The e.m.f. is now *in* in the black side of the loop and *out* in the white side. The brush *W* now becomes positive and *B* becomes negative. Compare this with the reverse condition of Fig. 276.

The current impelled through the loop and external circuit by this induced e.m.f. will reverse in direction as the direction of the e.m.f. reverses. As the uniform revolution of the loop is continued through the positions of Figs. 280, 281, 282 and 283, e.m.fs. will be induced during the time that the sides of the loop are cutting lines and none will be induced at the instants (Figs. 278 and 282) when they are not cutting. The e.m.f. induced at the instant of Fig. 280 will again be a maximum because at this instant the

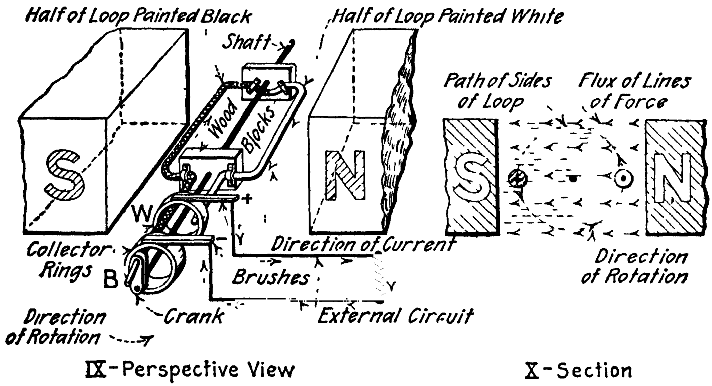


FIG. 280.—Fifth position of conducting loop in uniform field.

loop is cutting directly across lines—cutting at the maximum rate. Study the illustrations and verify the directions of the e.m.fs. and the currents indicated by the arrows. Also verify the polarity signs at the brushes.

Figure 284 shows in one picture the positions of a rotating loop at five different instants. Note that this loop is being turned in a clockwise direction and that it starts, at I, from the neutral position where, for an instant, the sides of the loop are moving in a direction parallel to that of the flux—and the induced e.m.f. at this instant is zero. The *sine curve* (Art. 554) in the lower part of the illustration indicates graphically how the e.m.f. induced varies in strength and direction as the uniform rotation continues. The vertical distance at any point between the horizontal line and the sine curve itself is proportional to the e.m.f. induced at the corresponding instant.

552. Why the e.m.f. induced in a loop rotating at a uniform speed in a uniform field is different at different instants may

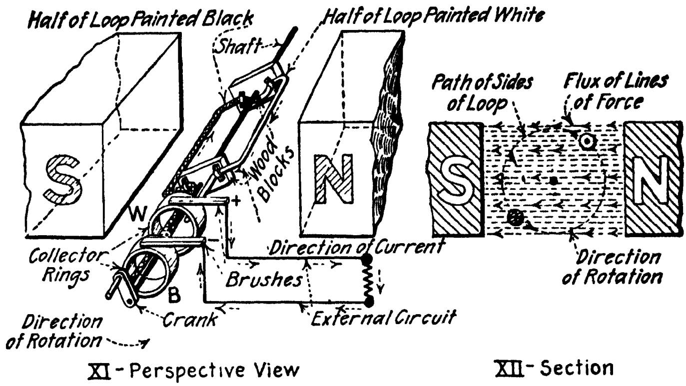


FIG. 281.—Sixth position of conducting loop in uniform field.

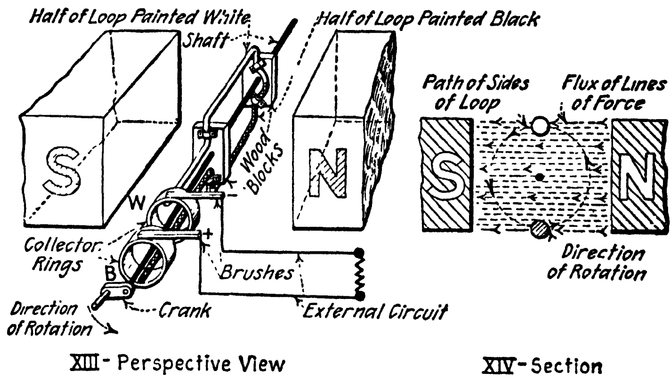


FIG. 282.—Seventh position of conducting loop in uniform field.

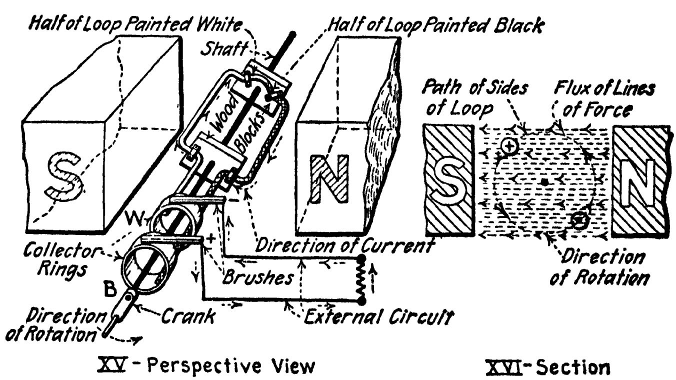


FIG. 283.—Eighth position of conducting loop in uniform field.

be better understood from a consideration of the example of Figs. 285 and 286. Imagine that the loop $A'A$ is revolved at a

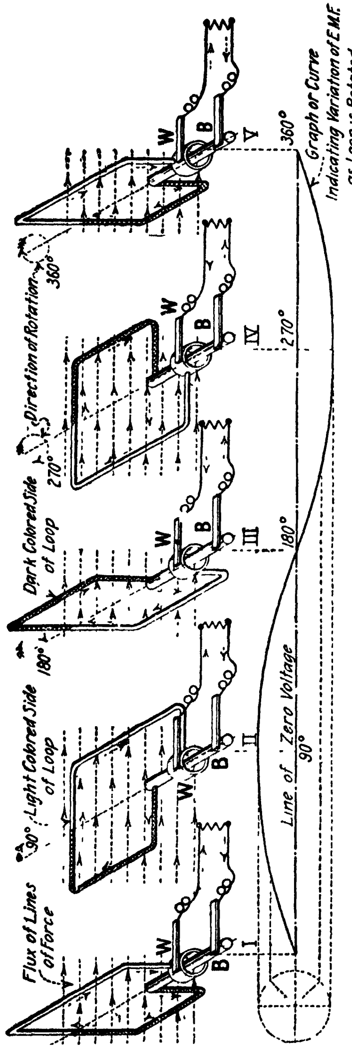


FIG. 284.—Illustrating the relation of the values of a sine curve to the e.m.fs. induced at different instants during the revolution of a conducting loop.

uniform rate (at a steady speed of 1 complete revolution in 16 sec.) in the uniform magnetic field (Art. 70) shown:

Example.—As the loop moves from position $A'A$ to $B'B$ both of its sides S' and S will cut lines. Thereby an e.m.f. will be induced in the loop as

described in a preceding article. One complete revolution, that is the circumference of a circle, is always equivalent to 360 degrees. From position $A'A$ to $B'B$ is $\frac{1}{16}$ revolution; hence it is $360 \div 16 = 22\frac{1}{2}$ degrees. Since 16 sec. is required for 1 revolution, 1 sec. will be required for $\frac{1}{16}$ revolution or $22\frac{1}{2}$ degrees. In moving from A to B side S cuts five of the lines of force shown—in 1 sec. But in moving from B to C, S cuts only three lines—in 1 sec. In moving from C to D, S cuts two lines—in 1 sec. In moving from D to E, S cuts one line—in 1 sec.

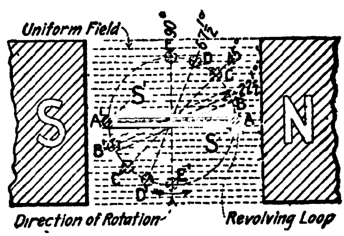


FIG. 285.—Showing how the rate of cutting lines varies as a loop is rotated in a uniform magnetic field.

Obviously the rate of cutting—the number of lines cut per second—decreases as side S approaches position E. The other side of the loop S' is, also, cutting lines at the same rate as in S. And the e.m.f. induced in side S' acts in unison around the loop with that induced in side S, as suggested in Fig. 276. The e.m.f. induced in S, while it is moved from E around to A', will obviously increase as A' is approached. It will be a maximum at the instant of position A' when S will be cutting directly across lines. Likewise, the e.m.f. induced in the sides of the loop varies as rotation is continued.

The example given above explains the situation in general terms and outlines in an illustrative way an important truth. However, the values given should be considered as qualitative

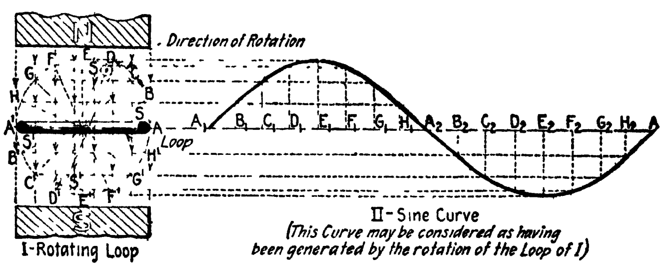


FIG. 286.—The generation of a sine curve.

rather than quantitative. The actual rate of cutting at position E is, at that instant, zero.

553. The sine of an acute angle of a right-angled triangle is that value by which the length of the hypotenuse must be multiplied to obtain the length of the side opposite the angle. Tables of sines of all angles have been computed and may be found in the Appendix.

Example.—The angle A of the triangle of I of Fig. 287 is known to be 45 deg., and the length of the hypotenuse AB is known to be 8 in. What is the length of the BC, which is opposite angle A? *Solution.*—Referring to any table of sines of trigonometric functions, it will be found that the sine of 45 deg. is 0.707. Hence, it follows from the above-given definition of sine that if the length of the hypotenuse AB be multiplied by this value 0.707, the length of BC will be the result. Thus: $8 \times 0.707 = 5.65$ in., which is the length of the side BC.

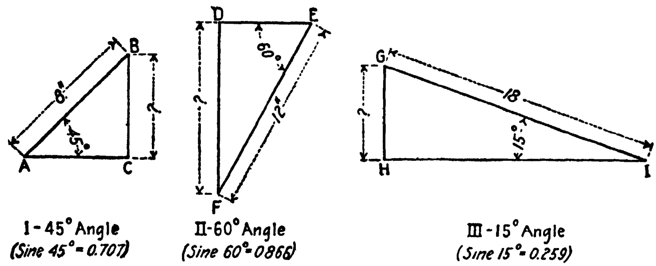


FIG. 287.—Problems illustrating the application of the sine of an angle.

Example.—What is the length of the side DF of triangle DEF? *Solution.*—From a table it is ascertained that the sine of 60 deg. is 0.866. Then $0.866 \times 12 = 10.4$ in., which is the length of DF.

Example.—The sine of 15 deg. is 0.259; therefore the length of GH of triangle GHI is $18 \times 0.259 = 4.66$ in.

554. A sine curve or sinusoid (Fig. 288) is a curve whose abscissas (horizontal distances from the origin or starting point)

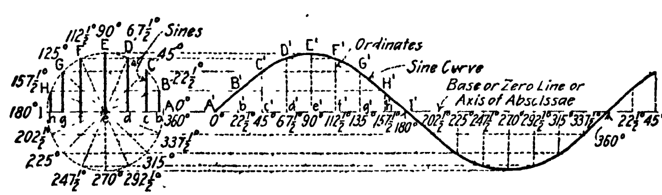


FIG. 288.—Showing a sine curve and the method of its construction.

represent the lengths of arcs and whose ordinates (vertical distances from the base line) represent the corresponding sines.

Example.—The circumference of the circle of Fig. 288,I (having a radius equal to 1) is divided into equal arcs AB, BC, etc. The length of a line, such as Bb, Cc, etc., drawn from the left termination of each section (AB, AC, AD, etc.) to the horizontal diameter IA, is the sine of the opposite or included angle. For example: length Bb is equal to the sine of 22½ degrees, length Cc is equal to sine of 45 deg. (For proof see note which follows.) Now by laying off, as shown in II, equal divisions such as $A'b'$, $b'c'$, $c'd'$, etc.,

on a horizontal line, erecting lines ($B'b'$, $C'c'$, etc.) equal in length to the corresponding sines in order and joining their extremities, a sine curve or sinusoid is formed.

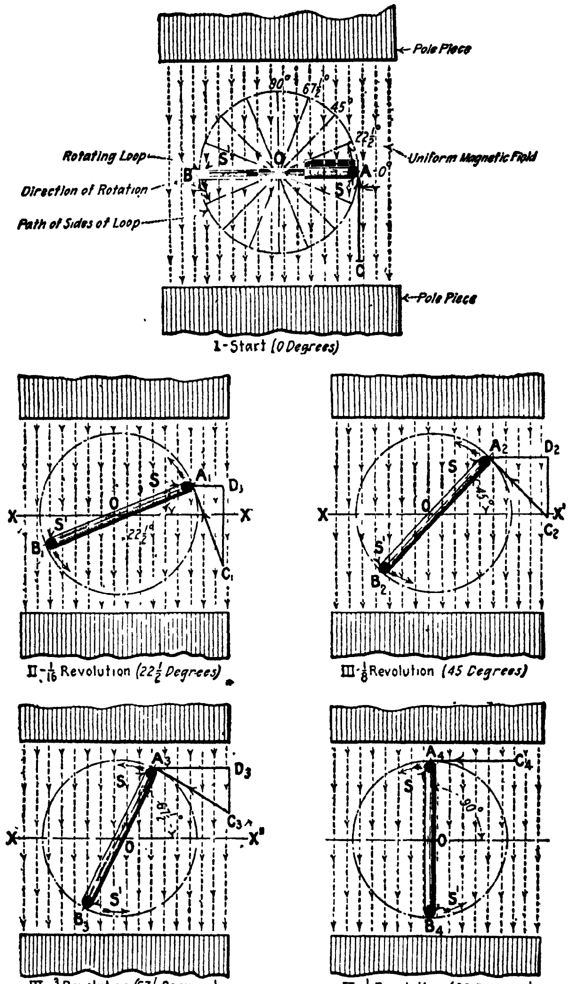


FIG. 289.—Showing why the e.m.f. induced in a loop varies as the sine of the angle through which the loop is turned.

NOTE.—Why the lengths of the lines Bb, Cc, Dd, etc., are equal to the sines of the included angles opposite them may be explained in this way: It follows from the definition of a sine of Art. 553. The hypotenuses eB, eC, eD, etc., of the different triangles in the circle are all the same length, being in each case the radius of the circle. The length of this radius may

be taken as being equal to 1. Then if length eB be multiplied by sine of $22\frac{1}{2}$ deg., the result should be length Bb—from the definition of sine of Art. 553. Hence length of Bb = sin $22\frac{1}{2}$ deg. The same sort of proof can be written for the other sines, Cc, Dd, etc.

555. The e.m.f. induced at any instant in a loop, rotated at a uniform speed in a uniform field, varies as the sine of the angle through which the loop has been turned from the neutral plane which lies at right angles to the field. Art. 552 and Fig. 285 indicate why the e.m.f. induced in a rotating loop varies at different instants as the loop is passing through different positions in the field—but they do not indicate how much the e.m.f. varies. By carefully considering the diagrams of Fig. 289 in combination with the following example it will be evident that the variation, from instant to instant, in the intensity of the induced e.m.f., as the loop is rotated, will be as stated in the opening paragraph of this article.

Example.—The loop BA (Fig. 289,I) is being rotated in a counterclockwise direction in the uniform field shown. It is a fact that a line, AC, drawn at right angles to BA (that is, tangent to the circle outlining the path of the loop) will, when drawn to some—any—scale, represent the direction and velocity of the movement of the side S of the loop at the instant shown. The line AC should be drawn to scale proportional in length to the velocity of the side S. The direction of this line shows the direction of S at that instant.

Diagrams I to V inclusive of Fig. 289 show different positions of the loop as it is being rotated in the field. In each of the five positions, the length of line AC (or A_1C_1, A_2C_2, etc.) indicates the actual velocity or speed of the loop at the instant pictured. Since the speed is uniform, AC, A_1C_1, A_2C_2, etc., are all the same length in each of the five diagrams shown. The direction of the line AC, etc., in each of the five diagrams shows the actual direction of the side S of the loop at each of the five instants pictured.

Now at the instant of I, the side S of the loop is, as represented by line AC, in the neutral plane and is moving directly parallel to the direction of the flux. Side S is, therefore, cutting no flux at this instant, and the e.m.f. in it is zero. But consider the instant of II: A_1C_1 represents the direction and speed of side S at this instant. Now *any* line, for instance A_1C_1, which represents graphically the movement, may be resolved into two components or parts at right angles to each other.

Thus, A_1D_1 and D_1C_1 are components of A_1C_1. It is a fact that A_1D_1 actually represents the instantaneous movement of side S in a direction at right angles to the lines of force. The direction of A_1D_1 represents the direction of movement at this instant. The length of A_1D_1 represents or is proportional to the speed of S in the horizontal direction at this instant. The actual rate at which S cuts lines is, obviously, proportional to the speed with

which it cuts horizontally across the lines that is, at right angles to the lines. This speed is represented at the instant of II by the length A_1D_1.

Now A_1D_1 is proportional to the sine of angle C_1 (Art. 553). But, angle C_1 = angle O, as can be shown by geometry. And angle O is the angle through which the loop has turned from the neutral plane. Since A_1D_1 represents the speed of S at right angles across lines at the instant shown, it represents the actual rate of cutting lines at this instant. Hence, it is apparent that the rate of cutting lines is proportional to the sine of the angle through which the loop has been turned from the neutral plane. Since O in picture II is 22½ deg. the rate of cutting lines—or the length of A_1D_1 —is at this instant, proportional to the sine of 22½ deg.

Similar reasoning is followed for the three other positions of the loop diagramed: At the instant III, the actual rate of cutting lines is represented by line A_2D_2 which is proportional to the sine of 45 deg. At the instant of IV, the actual rate of cutting lines is represented by A_3D_3 which is proportional to the sine of 67½ deg. And, at the instant of V, the actual rate of cutting lines—now a maximum—is represented by A_4C_4, the actual speed of rotation, because the sine of 90 deg. is 1. It has, therefore, been shown that the induced e.m.f. varies as the sine of the angle through which the loop has turned.

556. The elementary alternating-current generator is, obviously, shown in Figs. 276 to 283. The e.m.f. induced in the loop of this arrangement and plotted in the curves of Figs. 284 and 290,I is, since it alternates regularly in direction, by the definition of Art. 127 an alternating e.m.f. If the revolving coil be connected to an external circuit, since the resistance of the external circuit of Fig. 276 remains constant, the current in amperes forced through this circuit by the e.m.f. induced in the revolving loop must obviously be an alternating current. Now the e.m.f. induced in a coil or loop being rotated in a uniform field varies (Art. 555) as the sine of the angle through which the loop has turned from the neutral plane. It follows, therefore, that the curve of e.m.f. of this loop (Fig. 290) is a sine curve (Art. 554).

557. An alternating e.m.f. is generated in any loop rotated in a magnetic field even though the e.m.f. (rectified with a commutator, Art. 566) impressed on the external circuit be a direct e.m.f. and the current in the external circuit a direct current (Art. 121). Figure 290,I and II, illustrates this principle. Then, the current in the armature (Art. 571) of any direct-current machine is an alternating current—though of course the current in the external circuit is a direct current.

558. Meaning of Positive and Negative Direction of Rotation.—The counterclockwise direction of rotation is usually

called, or assumed to be, the positive direction of rotation. Counterclockwise means opposite in direction to the direction of rotation of the hands of a clock. Likewise, the clockwise direction of rotation is usually designated as the negative direction of rotation. The terms as above defined have been arbitrarily selected and, while the definitions given are the generally accepted ones, there is no real reason why they could not be reversed. When used to indicate direction of rotation, then, the term

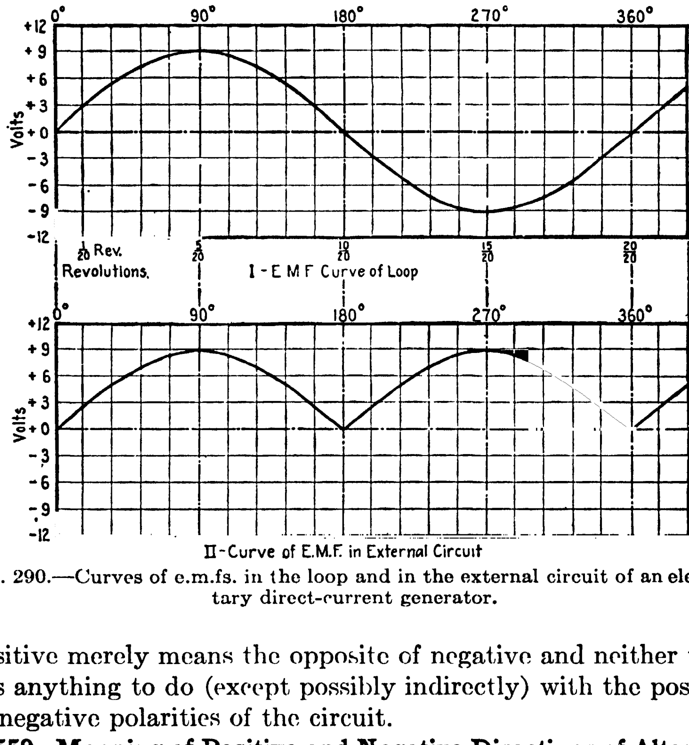


FIG. 290.—Curves of e.m.fs. in the loop and in the external circuit of an elementary direct-current generator.

positive merely means the opposite of negative and neither term has anything to do (except possibly indirectly) with the positive or negative polarities of the circuit.

559. Meaning of Positive and Negative Directions of Alternating E.m.fs. and Voltages (review Art. 126).—In Fig. 276,I the current is flowing away from brush *B* and into brush *W*. But in Fig. 280,IX, current flows into brush *B* and out of brush *W*. Obviously, the direction of the current in Fig. 280 is just opposite to that of the current in Fig. 276. To distinguish between these two possible directions of e.m.f. or of current flow in any circuit, one (either one) direction is called the positive direction and then the other is then designated as the negative direction.

If the loop of Fig. 285 be rotated in a counterclockwise direction from the position AA', the direction of the induced e.m.f. will be out S and into S'. Now if the loop from position AA' is rotated in a clockwise direction (Art. 558) the induced e.m.f. direction will again be out of S and into S'. (If the rotation of the loop is started from any position other than one parallel with the flux, AA', Fig. 285,A, the e.m.f. will be in one direction if the rotation is commenced counterclockwise and in the other direction if it is commenced clockwise.) Verify these statements by applying the hand rule of Art. 462. If the direction of the field is reversed, the direction of the induced e.m.f. will be correspondingly reversed. If in Fig. 285 the direction into S' and out of S is taken as the positive direction, then the direction out of S' and into S will be the negative direction. On this basis, considering S alone, the positive direction is out of S, toward the reader; the negative direction is into S, away from the reader.

NOTE.—It is well to designate the positive direction of e.m.f. in each case, as the direction of the e.m.f. or current in the loop while the loop is being rotated in a counterclockwise direction through the first 180 deg. from the neutral axis. Then the direction of e.m.f. during the next 180 deg. will be negative. The reason why it is well to so designate the directions is this: the values of a sine curve that are plotted above the neutral or zero line (as $A'C'D'$ of Fig. 288) are usually most conveniently designated as positive. That is, they usually indicate values of e.m.fs. or currents that are in what is arbitrarily designated a positive direction. Values plotted below the zero line are then regarded as negative. If the positive direction of e.m.f. is taken as that just suggested, this convention will be preserved.

This matter of positive and negative directions is, then, merely one of convention, and either direction of e.m.f. or current may be regarded as positive as the loop rotates through the first 180 deg. from the neutral plane, provided the direction of e.m.f. and current as the loop rotates through the remaining 180 deg. is regarded as negative.

NOTE.—Positive and negative directions of currents or e.m.fs. must not be confused with positive or negative polarities of electric circuits. A positive or negative direction of current or e.m.f. bears no particular relation (except perhaps, indirectly) to the positive and negative polarities of a circuit. However, the polarities of a circuit do change as the directions of the e.m.f. and current change. There are, then, at least three possible usages of the terms positive and negative: (1) To indicate polarity of circuits (Art. 109). (2) To indicate direction of rotation (Art. 558). (3) To indicate direction of current or e.m.f. as described in the article just preceding.

560. A graphic statement of the variation of the e.m.f. values induced in the loop of Fig. 276 at the different instants of its

revolution can be plotted into a curve like that of Fig. 290,I. Assuming that the loop starts from the neutral position of Fig. 278, this curve of Fig. 290,I shows how the e.m.f. induced in the loop increases from zero and attains a maximum value in one direction (Fig. 280), which can be designated arbitrarily (Art. 559) as the positive direction, at 5/20 or 1/4 revolution.

NOTE.—The maximum e.m.f. in the illustration plotted in Fig. 290,I is shown to be 9 volts. This means that the uniform speed at which the loop is being rotated and the number of lines in the flux are such that at the instants when the sides of the loop are passing through the positions of Fig. 280,IX and X and Fig. 276 the e.m.f. induced is just 9 volts—4 1/2 volts in each side of the loop.

Figure 290,I, also shows how, as rotation is continued, the e.m.f. decreases to zero at 10/20 (or 1/2) revolution (Fig. 282), how it reaches its greatest value in the other direction, which we can call the negative direction (Art. 559), at 15/20 or 3/4 revolution (Fig. 276), and how it then again becomes zero at 20/20 or 1 revolution (Fig. 278). Note that the maximum negative and positive e.m.fs. are equal in value but opposite in direction. Any corresponding negative and positive (Art. 559) locations on the curve are of equal value but of opposite directions. The e.m.f. generated by the loop will continue to vary as shown by the curve (Fig. 290,I), as long as the loop is rotated uniformly. This curve is a sine curve (Art. 554).

Example.—In Table 562, in the column headed E, are shown numerical values indicating how a loop being rotated in a field and at a speed such that the maximum e.m.f. induced is 10 volts, induces different e.m.fs. at different instants. The sine curve (graphic statement) of these values is given in Fig. 291. The voltage value for each instant in this column E was obtained by multiplying the maximum value, 10 volts, by the sine of angle through which the loop had been rotated (from the neutral position shown at Fig. 291,I) up to that instant.

561. The e.m.fs. and currents developed by alternating-current generators have sine-wave forms or wave forms that closely approximate that of a true sine wave (see Arts. 554 and 556). The current in any circuit varies directly as the applied e.m.f. It follows from this that, if the e.m.f. wave has a sine-wave form, any current due to this e.m.f. must also have a sine-wave form. This does not mean that the curve representing current (see Fig. 291) will be the same size as the curve representing voltage;

it only means that both will have the same general shape or form (see Fig. 291). The derivations of the formulas for many alternating-current calculations are based on the assumption of perfect sine-wave forms for alternating e.m.fs. and currents. The reason for this is that it is only on the basis of true sine curves, that is, true sine-wave forms, that computations can be readily made or equations derived.

562. Table Indicating E.m.fs. Induced at Given Instants (by a Loop Revolved in a Field) and the Instantaneous Currents Impelled Thereby.—Resistance of circuit = 2 ohms.

Reading Numbers	Time at which reading is taken			Direction of e m.f. and current	E, volts. E.m.f. induced at the given instant	I, amperes. Current forced through the circuit at the given instant, $I = E \div R =$
	Clock time	Degrees	Revolutions			
1	2 : 00 P.M.	0°	0	....	0.00	0 ÷ 2 = 0.0 amp.
2	1/16 sec. after 2	22½	1/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
3	2/16 sec. after 2	45	1/8	+	7.07	7.07 ÷ 2 = 3.5 amp.
4	3/16 sec. after 2	67½	3/16	+	9 24	9 24 ÷ 2 = 4 6 amp.
5	4/16 sec. after 2	90	1/4	+	10 00	10.00 ÷ 2 = 5 0 amp.
6	5/16 sec. after 2	112½	5/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
7	6/16 sec. after 2	135	3/8	+	7.07	7 07 ÷ 2 = 3.5 amp.
8	7/16 sec. after 2	157½	7/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
9	8/16 sec. after 2	180	1/2	+	0.00	0.00 ÷ 2 = 0.0 amp.
10	9/16 sec. after 2	202½	9/16	−	3.83	3.83 ÷ 2 = 1.9 amp.
11	10/16 sec. after 2	225	5/8	−	7.07	7.07 ÷ 2 = 3.5 amp.
12	11/16 sec. after 2	247½	11/16	−	9 24	9.24 ÷ 2 = 4.6 amp.
13	12/16 sec. after 2	270	3/4	−	10 00	10.00 ÷ 2 = 5.0 amp.
14	13/16 sec. after 2	292½	13/16	−	9.24	9.24 ÷ 2 = 4 6 amp.
15	14/16 sec. after 2	315	7/8	−	7.07	7.07 ÷ 2 = 3.5 amp.
16	15/16 sec. after 2	337½	15/16	−	3.83	3.83 ÷ 2 = 1.9 amp.
17	1 sec. after 2	360	1	−	0 00	0.00 ÷ 2 = 0.0 amp.
18	1 1/16 sec. after 2	22½	1 1/16	+	3.83	3.83 ÷ 2 = 1.9 amp.
19	1 2/16 sec. after 2	45	1 1/8	+	7.07	7.07 ÷ 2 = 3.5 amp.
20	1 3/16 sec. after 2	67½	1 3/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
21	1 4/16 sec. after 2	90	1 1/4	+	10.00	10.00 ÷ 2 = 5.0 amp.
22	1 5/16 sec. after 2	112½	1 5/16	+	9.24	9.24 ÷ 2 = 4.6 amp.
23	1 6/16 sec. after 2	135	1 3/8	+	7.07	7.07 ÷ 2 = 3.5 amp.

563. A Sine Curve May Represent the Variation of an Alternating E.m.f. or of an Alternating Current with the Time (see Art. 758).—Sine curves may be drawn to any convenient scale.

Example.—The heavy line lying radially in the circle to the left of Fig. 292, which may be considered as revolving (counterclockwise) in the

direction of the arrow, is called a vector; see also Art 758. Its length is proportional to the maximum value (Art 729) of the alternating e m f or the current which it represents The vertical distance from the arrowhead point, *A*, of the vector to the horizontal zero line, *CD*, is, at any given instant, as the vector revolves proportional to the instantaneous value of the e m f or the current at that instant The lengths of the lines *AB* represent the instantaneous values at the 45-deg. instant.

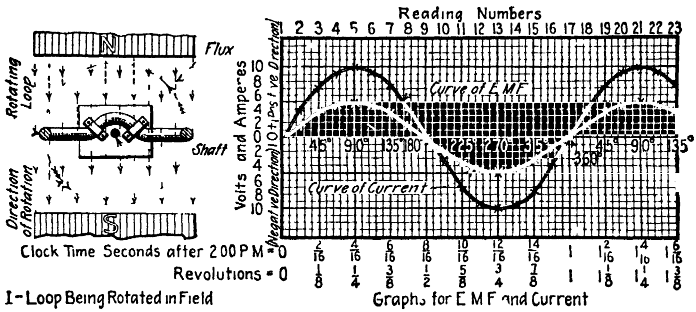


FIG 291 —Loop which is being rotated in a magnetic field and curves of e m f and current induced thereby

The length of any such line representing an instantaneous value is proportional to the trigonometric sine (Art 553) of the angle lying between the vector and the horizontal line It is because of this fact that a curve like that to the right of Fig 292 is called a sine curve.

564. The alternating current (amperes) which is forced through the circuit of Fig 276 by the alternating e m f (Art 720) induced in the loop, will also vary as the loop is turned, that is, it varies as the time elapses The rate of variation will, as with the e.m f ,

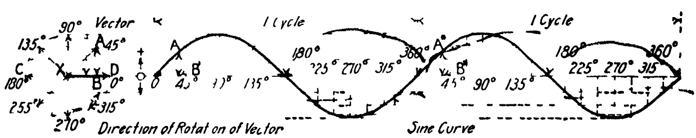


FIG 292 —Showing a rotating vector and the sine curve generated thereby

be proportional to the sine of the angle through which the loop has been turned from the neutral position. The curve indicating this rate of variation will be a sine curve. For example, again starting at the neutral position of Fig. 278,V, the current through the circuits at this instant would be zero. As the loop is rotated (Fig. 279,VII), current is forced through the circuit in a direction from collector ring and brush *W* toward and into *B*.

The current (amperes) gradually increases in accordance with the sine law from position Fig. 278,I, to that of Fig. 280,IX. The loop has now been turned through 90 deg. and the direction of motion of its sides is at right angles to the direction of the flux—the e.m.f. and current are now a maximum. As the rotation of the loop is continued (Fig. 281), the current decreases until, it having turned through 180 deg., the position of Fig. 282,XIII, is reached—and the e.m.f. and current are again zero. Rotation continuing (Fig. 283,XV), current is again forced through the circuit, but it now flows away from *B* and into *W*—it has reversed in direction.

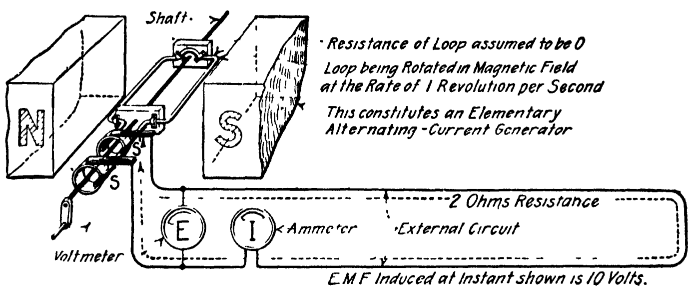


FIG. 293.—Elementary alternating-current generator forcing current through an external circuit. (The sine curves of e.m.f. and current for this arrangement are shown in Fig. 291.)

The same sequence of increases and decreases and changes in direction occur periodically and repeatedly so long as rotation is continued. Since the current reverses periodically, that is with the time, in direction, obviously it is, by the definition of Art. 127, an alternating current. Note that the current attains its maximum value when the loop has been turned through 180 degrees from the neutral or starting position.

Example.—The last column of Art. 562 shows the values of current at different instants as the loop of Fig. 293 is rotated in its field. The maximum e.m.f. induced is 10 volts. The resistance of the external circuit is 2 ohms and the resistance of the loop itself is assumed to be zero. Figure 291,II shows the curve of this current.

565. A graphic or sine-curve portraiture of an alternating e.m.f. and the current it produces is shown in Fig. 291 for the circuit of Fig. 293. The current in any circuit varies as the e.m.f. which is forcing it through the circuit varies. This must be true in

order that the requirements of Ohm's law (Art. 151) be satisfied. Since an alternating e.m.f. can be represented by a sine curve (Art. 563), it follows that its alternating current may also be represented by a sine curve. However, the sine curve of a current will be numerically equal to (the same size as) the curve of the e.m.f. that produces it only when the resistance is numerically equal to one. This follows, because $I = E \div R$; now if $R = 1$ then, $I = E \div 1$, that is then, $I = E$. If R is greater than 1 and the curves of both e.m.f. and current are plotted to the same numerical scale, the current curve will lie within the e.m.f. curve as shown in Fig. 291. But if R is less than 1, the current curve will lie without the e.m.f. curve.

Example.—Consider the elementary generator and circuit of Fig. 293. It will be assumed that the loop there shown is being rotated at the uniform rate of 1 turn (or 1 revolution) per sec. The loop is assumed to have no resistance but the external circuit has 2 ohms resistance. The flux in the field is assumed to be such that the maximum instantaneous e.m.f. induced in the loop is 10 volts. That is, at the instants when the sides of the loop are cutting lines at right angles—the maximum rate—the e.m.f. then induced is 10 volts. The e.m.fs. induced at other instants can be ascertained as suggested in Art. 555 by multiplying this maximum e.m.f. by the sine of the angle between the neutral position of the loop and its position at the given instant.

The values of the instantaneous e.m.fs. induced at successive instants are shown in column E of the table in Art. 562. These values were computed as described above. Now, consider what occurs as the loop, being started from the neutral position shown, is rotated at this uniform rate of 1 turn or revolution per sec.

Assume that rotation is commenced at just 2:00 P.M.—it could be commenced at any other time just as well but 2:00 P.M. will be taken so that there will be a definite starting time. As it is started, at this instant—at 2:00 P.M.—the loop does not cut any lines because, at this instant, its sides are moving parallel to the direction of the flux. Hence, at this instant, the induced e.m.f. is zero as shown in Art. 562. Furthermore, no current is forced through the external circuit at this instant because the e.m.f. is then zero. As the rotation is continued, the sides of the loop begin to shear through lines, and thereby e.m.fs. are induced in the loop.

However, the e.m.fs. induced at successive instants increase as the rotation is continued (through the first 90 deg.) because as the sides of the loop move farther away from the neutral position their rate of cutting becomes greater—they cut more lines per second.

Now, consider the instant when the loop has been rotated just 22½ deg. away from the neutral position. The e.m.f. induced at this instant is shown as 3.83 volts. Why 3.83 volts? This is the reason: The maximum e.m.f. has been assumed to be 10 volts. If this value of 10 volts is multiplied

by the sine of $22\frac{1}{2}$ deg., which is 0.383, the instantaneous e.m.f. at the $22\frac{1}{2}$-deg. position or instant should be the result, as suggested in Art. 555. Thus 10 volts $\times$ 0.383 = 3.83 volts. Note that this $22\frac{1}{2}$-deg. position corresponds in every instance to $\frac{1}{16}$ revolution because $22\frac{1}{2}$ deg. is $\frac{1}{16}$ of 360 deg., and 360 deg. always represents a complete revolution.

Note also that since the loop started at 2:00 P.M. and is being rotated at a uniform rate of 1 turn or revolution per sec., the loop will have reached the $22\frac{1}{2}$-deg. position just $\frac{1}{16}$ sec. after 2:00 P.M. It is evident that the position of the loop can be designated by any one of these methods: (1) *by degrees,* (2) *by revolutions* or parts thereof, and (3) *by time* or by fractions or multiples of a second. It is so designated for a number of different positions in the table of Art. 562.

What current is being forced through the circuit of Fig. 293 at this $22\frac{1}{2}$-deg. instant? In the preceding article it was ascertained that the e.m.f. developed at this instant is 3.83 volts. The entire circuit on which this e.m.f. is impressed comprises the loop itself and the external circuit. But it has been assumed that the loop has zero resistance. Hence the total resistance of the entire circuit is 2 ohms, the resistance of the external circuit.

By Ohm's law $I = E \div R$. Then, the current at this $22\frac{1}{2}$-deg. instant is $I = E \div R = 3.83 \div 2 = 1.9$ amp. The current curve in Fig. 291 is plotted accordingly.

By proceeding as suggested above, the e.m.f. and current values at any other instants during a revolution of the loop can be ascertained. They have, accordingly, been computed and shown in Art. 562 for successive instants, $22\frac{1}{2}$ deg.—or $\frac{1}{16}$ sec. or $\frac{1}{16}$ revolution—apart. These values have then been plotted in the curve of Fig. 291. This curve shows pictorially the relation of an alternating e.m.f. to the current it forces through a circuit under the conditions specified. Since it requires 1 sec. for the e.m.f. of this elementary generator (Fig. 293) to complete 1 cycle, it has a frequency (Art. 722) of 1 cycle per sec.

After the loop has rotated through 180 deg. the direction of the e.m.f. induced in it will be reversed as described in Art. 551. The direction of the current will then be reversed correspondingly. This is shown by the change from the + to the − sign in the table. These signs indicate the directions and not the polarities of the e.m.fs. and currents as described in Art. 559.

As long as the rotation of the loop is continued, the e.m.f. and current will continue to vary and reverse regularly. The reversals will occur at each successive $\frac{1}{2}$ sec. The current and e.m.f. will vary in accordance with the sine law as indicated in the curve of Fig. 291.

566. A commutator (Fig. 294) may be defined as a ~~device or~~ rectifier for changing in one portion of a circuit the directions of the e.m.f. or current produced in another portion. Ordinarily, a commutator is used for changing an alternating e.m.f. or current to a direct e.m.f. or current. The process involved in this changing may be referred to as commutation or rectification. Large commutators are shown in Figs. 336 and 337.

Example.—The commutator on the loop of the elementary generator of Fig. 295 rectifies the alternating e.m.f. induced in the loop (the curve of which is shown in the lower portion of the figure) in Fig. 290,I so that a direct e.m.f. (Fig. 290,II) is impressed on the external circuit. This process of rectification is described in another article.

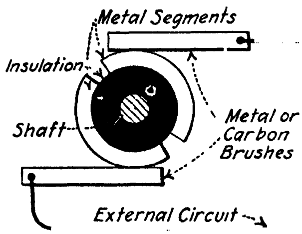


FIG. 294.—Sectional view of two-segment commutator.

567. To Produce a Direct E.m.f. with a Generator, the Alternating E.m.f. Induced in Its Armature Is Rectified with a Commutator.—A simple commutator is shown in Fig. 295 connected to a loop which may be rotated in a magnetic field. One-half of the loop and its commutator segment are painted black and the other side and segment are white, merely for identification. A consideration of the following example will make it clear as to how a commutator rectifies the alternating e.m.f. which is always induced (Art. 557), in a loop which is rotated in a field.

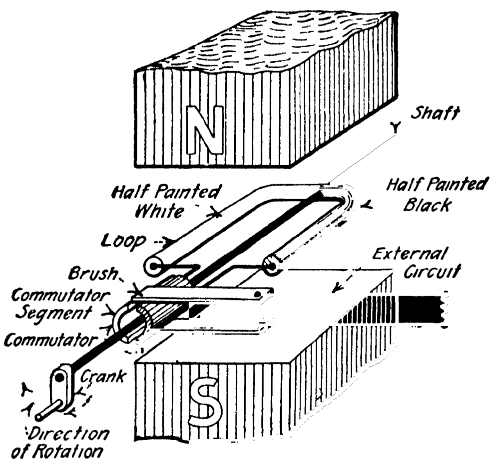


FIG. 295.—Commutator arranged on loop in field.

Example.—Refer to Fig. 295. The commutator segments are electrically connected to their respective sides of the loop, but they are insulated from each other and from the shaft by an air gap. The shaft is insulated from the loop. Assume that the loop is rotated at a uniform rate in a clockwise direction by turning the crank. Figure 296,I, which is a duplicate of 295 except that it is a simplified sectional view, shows conditions at the starting instant.

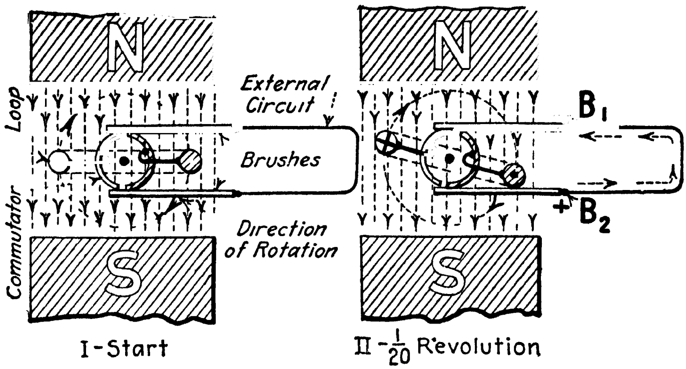


FIG. 296 – Conducting loop with commutator rotating in a field.

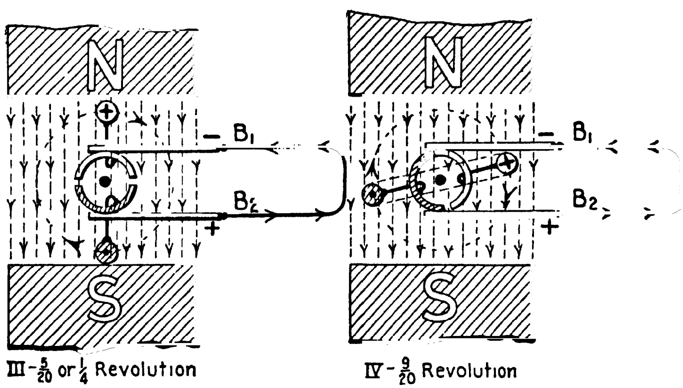


FIG. 297.—First positions as loop is started in rotation.

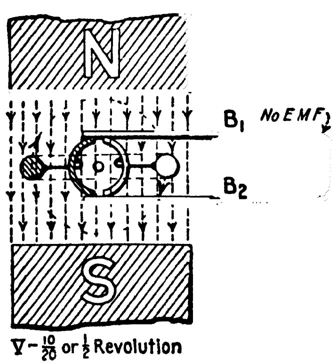
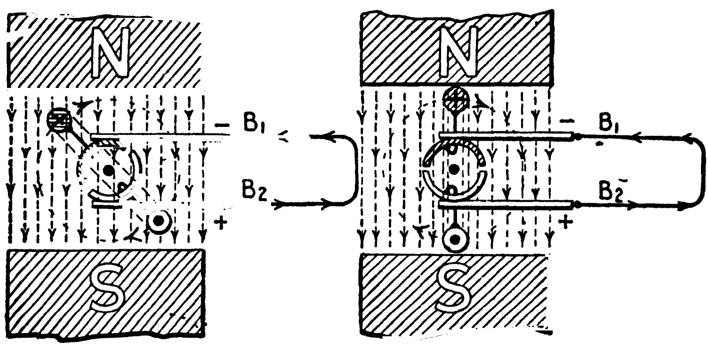


FIG. 298.—Neutral position of loop.

No e.m.f. is induced in the loop at this instant because its sides are then moving in a direction parallel to that of the flux.

Rotation is continued. After the loop has been rotated through $\frac{1}{20}$ revolution the conditions are as indicated at II. At this instant the sides of the loop are cutting lines and an e.m.f. is being induced which forces current through the loop and external circuit. By applying the hand rule of Art. 550 it will be found that the direction of the e.m.f. and current is *in* in the white side of the loop (shown by the cross) and *out* of the black side (as shown by the dot). Hence, the current is forced out through the brush marked + and enters the brush marked −. B_2 is then the positive polarity (+) brush because the e.m.f. is directed away from it and because the current flows out of it. B_1 is the negative polarity (−) brush because the direction of the e.m.f. is into it and because the current flows into it.



VI - $\frac{11}{20}$ Revolution VII - $\frac{15}{20}$ or $\frac{3}{4}$ Revolution

FIG. 299.—Positions of conducting loop as its rotation continues.

If this elementary generator is to impress a direct e.m.f. on, and force a direct current through, its external circuit as long as its rotation is continued, the current must continue to flow out of B_2 and to flow into B_1. It will, by virtue of the commutator, do this, as will be shown.

As the rotation of the loop is continued it passes through the positions of Fig. 297,III and IV. The direction of the e.m.f. and current in the loop remains (hand rule, Art. 462) *out* of the black side and *in* the white side of the loop as indicated by the dot and the cross respectively in the illustration. B_1 retains its − polarity and B_2 its + polarity.

Now, as the rotation is continued, at the instant, pictured in Fig. 298,V, when the sides of the loop are moving parallel to the direction of the lines of force, no e.m.f. is induced in either the white or the black side of the loop. Hence, at this instant, there can be no current—the brushes are neither positive nor negative. Note that, at this instant, the brushes bridge both commutator bars.

As the turning of the loop is continued, the instant depicted in Fig. 299,VI, is reached. The sides of the loop are again cutting lines and an e.m.f. and current are again induced. But now the current and e.m.f. direction is *in in the black side* and *out of the white side*. Note that the direction of

e.m.f. and current has reversed in the loop. But at the instant the direction within the loop started to reverse, the white commutator segment slid out from contact with brush B_1 and the black segment slid into contact with it. A similar change occurred with the segments at brush B_2. This change of brush contact from one segment to the other at the proper instant—that is, when the direction of e.m.f. in the loop reverses—maintains the e.m.f. impressed on the external circuit always in the same direction in spite of the fact that the direction of e.m.f. in the loop changes in direction once during each revolution. This illustrates the function and action of the commutator. Figure 300 shows the situation in one illustration. Compare this with Fig. 284.

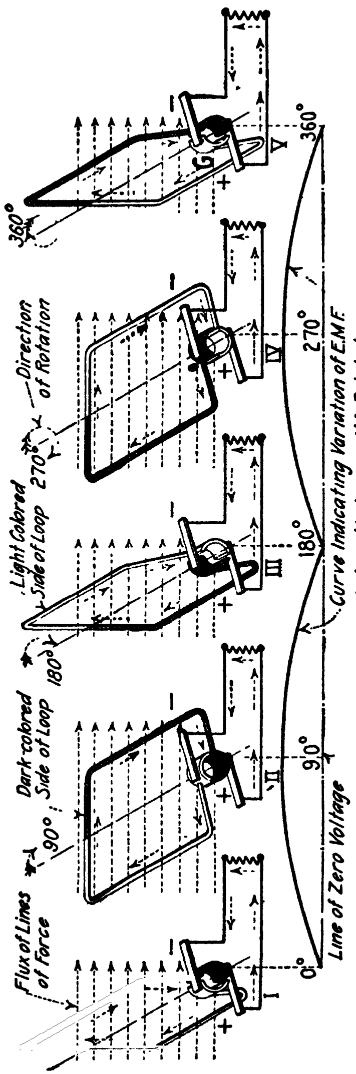


FIG. 300.—Illustrating the rectifying action of a commutator.

As the rotation of the loop is continued it passes successively through the positions shown in Figs. 299,VII and 301,VIII and IX, the direction of e.m.f. and current in the external circuit remaining out of B_2 and in B_1 during the instants when the sides of the loop are cutting lines. As the loop is rotated the e.m.f. induced in it varies at different instants as described in the discussion of the elementary alternating-current generator.

568. A loop or a coil of a few concentrated turns when provided with a commutator and rotated in a field produces a pulsating e.m.f. and current if the external circuit is closed (see Art. 124 for definition of pulsating e.m.f. or current). Thus the curve of Fig. 290,II, for the arrangement of Fig. 295, is that of a pulsating current. All pulsating currents are direct currents, but the reverse is not true. To produce a continuous (Art. 122) direct current the curve of which would be about like

that of Fig. 320, it is necessary to interconnect several groups of coils to commutator bars in a manner indicated in Art. 602.

569. The fundamental difference between an alternating-current and a direct-current generator is that the alternating-current machine impresses on the external circuit connected to it an e.m.f. which regularly varies in magnitude and in direction in accordance with the sine law (Art. 561) as shown in the curve of Fig. 290,I. A direct-current generator impresses on its external circuit an e.m.f. which is always in the same direction (for a given arrangement of connections) and which remains practically

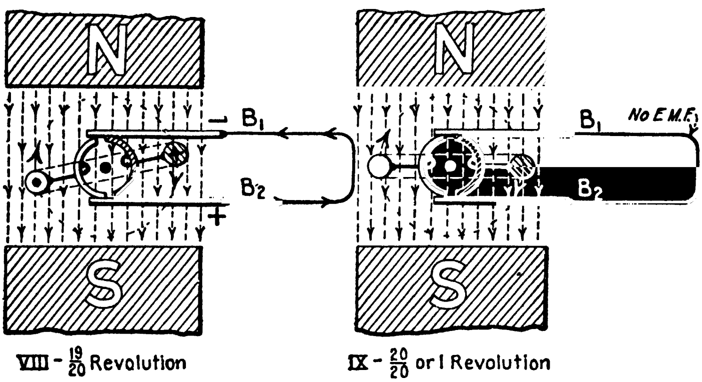


FIG. 301.—Final positions of conducting loop.

constant in magnitude. These distinctions logically follow from the definitions of a direct current (Art. 121) and of an alternating current (Art. 127). The mechanical differences between these two types of machines are discussed in following articles.

570. A graphic comparison of the e.m.fs. of elementary alternating- and direct-current generators is given in Fig. 290, wherein I shows the curve of e.m.f. induced in the loop and impressed on the line by a loop like that of Fig. 276,I when it is rotated. It is assumed that at the instant when the sides of this loop are cutting lines at right angles—at the maximum rate—the e.m.f. is then 9 volts. The curve is plotted on this basis. Now if a commutator is added to the loop, as in Fig. 295, the e.m.f. induced in the loop when it is rotated in the field can still be represented by the curve of Fig. 290,I. But the e.m.f. impressed on the external circuit would then be represented by the curve of II. A comparison of Figs. 300 and 284 will further illustrate this idea.

571. An armature of an electric generator comprises: (1) the conducting loops in which the e.m.f. is induced, when the loops cut or are cut by the magnetic flux, and (2) the structure immediately associated with them. The armature of an actual machine is usually thought of as comprising the armature winding or conducting loops, the iron core or structure on which the loops are wound, and the necessary insulation which prevents the turns of the loops from making electrical contact with each other or with the armature iron.

Examples.—The loops of the elementary generators shown in Figs. 276, 295, and 302 may be considered as primitive revolving armatures. The revolving armature of a magneto generator is shown in Fig. 303. The revolving armature of an actual direct-current generator is shown in Fig. 334. The stationary armature of an alternating-current generator is illustrated in Fig. 417.

572. Every generator must have an armature and a field, that is, a magnetic field (Art. 70). This statement is perfectly general and applies to all dynamos or electric generators both alternating and direct current. The distinctive features of the field structures and armatures of the machines of these two general types are described in the specific articles relating to them.

573. Generators May Have Stationary Armatures and Revolving Fields or the Reverse.—Commercial direct-current generators always have stationary field structures and rotating armatures. Theoretically, direct-current generators could be made which would have stationary armatures and rotating fields. Modern alternating-current generators—except possibly the very smallest ones—usually have rotating fields and stationary armatures.

Examples.—Figures 307 and 308 delineate a direct-current generator and its rotating armature. Figure 407 shows a small alternating-current generator having a stationary field structure and a revolving armature; Fig. 412 illustrates a modern alternating-current generator having a stationary armature and a rotating field structure.

574. The factors that determine the voltage developed by any generator are: (1) *the flux* or ϕ, that is, the number of lines of force which are cut by or cut the armature conductors; (2) the *number of cutting conductors* on the armature which cut the flux; (3) *the speed* at which the conductors move through the flux. Obviously these three factors determine the rate of cutting, which, as outlined in Art. 474, always determines the intensity of

an induced e.m.f. If the flux, the number of cutting conductors, or the speed of the cutting conductors is increased, the e.m.f. induced is increased proportionately. If any one (or two or all) of these factors is decreased, the e.m.f. will be decreased accordingly. The above noted factors are combined into an equation in Art. 579.

575. The amount of flux which is cut by the armature conductors of a generator is determined largely by the size and design of the machine. Obviously it is desirable to have the flux in each case as great as is consistent with economical design. To secure the greatest flux compatible with economy the portion of the generator which carries the flux should have low reluctance (Art. 249) that is, high permeance (Art. 260). Consequently the air gaps which the flux must cross should be as short

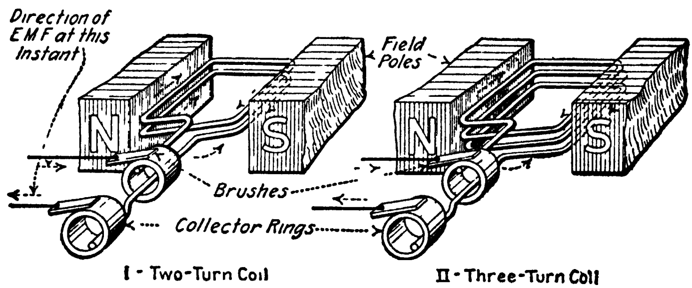


FIG. 302.—Inductor coils of two and three turns.

as possible. The conductors which cut or are cut by the flux are wound on iron cores to insure low reluctance.

576. Increasing the number of cutting conductors can be effected by increasing the number of turns in the armature coil—or by increasing the number of coils as described under "Direct-current Generators." In alternating-current generators which have stationary armatures (Art. 747) the number of coils or the number of turns per coil may be increased in a somewhat similar way. For example, a rotating armature coil may comprise a number of turns as in Fig. 302. The e.m.f. induced in any one turn of the coil will be equal, approximately, to that induced in each of the other turns. These e.m.fs. act in conjunction or in series, and the greater the number of turns the greater the e.m.f. If 1 volt is induced in each turn, a coil of 5 turns would induce an e.m.f. of 5 volts. That the e.m.fs. in the different

turns act conjunctively can be verified by applying the hand rule of Art. 462.

While e.m.f. can, theoretically, be increased to any extent desired by increasing the number of turns, there are other considerations that tend to limit the number of turns per coil feasible in a practical machine. If a coil of many turns is to occupy the same space as one of a few turns, smaller wire must be used for the many-turn coil. But in every case the wire comprising a coil must be sufficiently large that it will not become excessively hot when the generator operates continuously at its full load and the coil is carrying its full-load current. Furthermore, although the e.m.f. may be increased by increasing the number

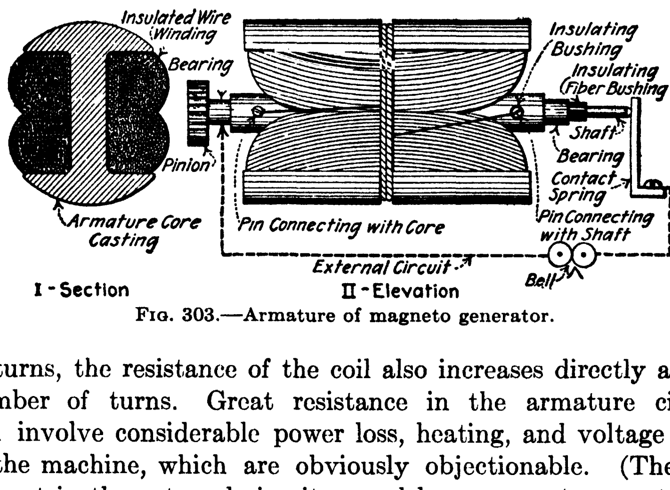


FIG. 303.—Armature of magneto generator.

of turns, the resistance of the coil also increases directly as the number of turns. Great resistance in the armature circuit will involve considerable power loss, heating, and voltage drop in the machine, which are obviously objectionable. (The line current in the external circuit served by a generator, or at least a certain definite proportion of it, flows in every armature coil and this current is always relatively large.)

The necessary insulation on the armature or coil conductors further increases the space required by them and since, because of certain design considerations, the space allowed for the conductors is limited (usually it is a slot in the armature core), the number of turns in the average coil does not ordinarily exceed 6 or 8. However, a number of coils can be arranged on the same armature to increase the e.m.f. as described in Art. 576.

Example.—The armature of the magneto generator described in Art. 580 is an example of an armature having a winding of many turns.

577. The speed at which the armature conductors move, that is, the revolutions per minute of the armature, is limited by considerations of safety and economy. (In modern alternating-current generators the armature conductors are almost invariably stationary, and the field structure rotates; in direct-current generators, the armature rotates, and the field structure is stationary.) Where a generator is to be direct-connected to an engine or other prime mover, the prime-mover speed determines the generator speed. In any case the probable speed of the prime mover must be considered. Excessively high speeds are not permissible because of the great centrifugal stresses they impose.

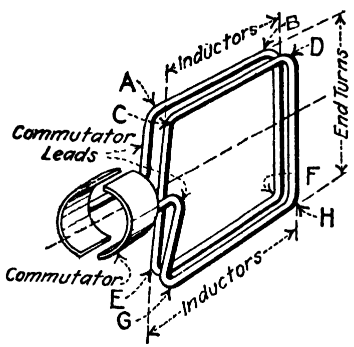


FIG. 304.—Illustrating inductors and end turns.

A peripheral speed of about a mile a minute is probably the upper limit for armatures of ordinary machines, but higher speeds can be and have been used. Greater speeds are used in small generators than in large ones.

578. The inductors (Fig. 304) of an armature are the conductors in which the e.m.f. is induced. Usually, each of the two sides of every turn in an armature coil is an inductor. However, with coils wound in certain ways (Art. 609) only one of the sides of a turn may be an inductor. Thus, as a rule, every turn of an armature coil comprises two inductors. The end turns of armature coils do not ordinarily cut flux but move parallel to the lines of the flux when the coil rotates and are not, therefore, effective in inducing e.m.f. and hence are not inductors.

Examples.—The armature coil of Fig. 304 has 2 turns and 4 inductors. The loop of Fig. 276 has 1 turn and 2 inductors. The coil of Fig. 302,I also has 2 turns and 4 inductors, while that of II has 3 turns and 6 inductors.

579. The e.m.f. induced in any coil which is rotated in a magnetic field may be computed on the following basis: If the number of lines cut per second by a single inductor of the coil be divided by 10^8 (Art. 480), the result will be the average e.m.f., in volts, induced in that inductor during that second. Then the total average e.m.f. induced in the coil will be the average e.m.f. per inductor multiplied by the number of inductors in series. Pro-

ceeding on the basis of the principle just outlined, the following formula (144) for the e.m.f. induced in a coil which is rotated in a magnetic field may be derived:

Let the symbol ϕ_T stand for the total number of lines cut by *each* inductor or side of each turn of the coil during one revolution of the coil. Then the total number of lines cut by all of the inductors of the coil per revolution will be equal to the number of inductors in the coil multiplied by the total number of lines that are cut by each inductor per revolution. Let C = the number of inductors in the coil. Then, $\phi_T \times C$ = the total number of lines cut by all of the inductors of the coil per revolution.

Now if r.p.m. = the revolutions per minute of the coil, it follows that r.p.m. $\div$ 60 = the number of its revolutions per second. If the number of lines cut per revolution be multiplied by the revolutions per second, the result will be the total number of lines cut per second. Hence $(\phi_T \times C) \times (\text{r.p.m.} \div 60)$ = total number of lines cut per second by all of the turns of the coil. As stated above, if this total number of lines cut per second be divided by 10^8 (100,000,000), the result will be the average e.m.f. in volts, the quantity sought. Expressing the above operations in an equation,

$$E_A = \frac{\phi_T \times C \times \text{r.p.m.}}{100{,}000{,}000 \times 60} \text{ (volts)} \tag{144}$$

Wherein E_A = average e.m.f. in volts induced in the rotating coil.
ϕ_I = flux or total number of lines of force cut by each inductor during one revolution.
C = number of inductors (Art. 578) which are in series and form one circuit between brushes or collector rings of opposite polarity.
r.p.m. = revolutions per minute.

NOTE.—The above formula will not give results which are strictly accurate for alternating-current armatures where the inductors are widely distributed over the surface of the armature (Arts. 576 and 600). Where the inductors are concentrated, that is where they are grouped very closely together as in Fig. 303, the result given will be quite accurate for alternating current machines. The result given will be strictly accurate for all direct-current generators. Note also that the term average e.m.f. has a specific meaning (Art. 730) when applied to an alternating e.m.f.

Example.—What average e.m.f. would be induced in the loop of Fig. 276 if it be rotated at the rate of 1,800 r.p.m. and the flux is 400 kilolines, that is 400,000 lines? *Solution.*—Each inductor or side of the loop cuts the

flux twice per revolution; hence $\phi_T = 2 \times 400{,}000 = 800{,}000$. The number of inductors in series between collector rings is two. Substituting in formula (144),

$$E_A = \frac{\phi_T \times C \times \text{r.p.m.}}{100{,}000{,}000 \times 60} = \frac{800{,}000 \times 2 \times 1{,}800}{100{,}000{,}000 \times 60} = 0.48 \text{ volt}$$

This would be an alternating e.m.f.

Example.—See example under Art. 580, the magneto generator, for another illustration of the application of this equation.

Example.—What average e.m.f. would be induced in the loop of the elementary direct-current generator (Fig. 295) if it were rotated at a speed of 1,200 r.p.m., assuming that the flux is 800,000 lines? *Solution.*—Each inductor or side of the loop cuts the flux twice per revolution, therefore $\phi_T = 2 \times 800{,}000 = 1{,}600{,}000$ lines. The number of inductors in series between brushes is two. Now substituting in formula (144),

$$E_A = \frac{\phi_T \times C \times \text{r p m}}{100{,}000{,}000 \times 60} = \frac{1{,}600{,}000 \times 2 \times 1{,}200}{100{,}000{,}000 \times 60} = 0.64 \text{ volt}$$

This is a direct e.m.f.

580. The magneto generator (Figs. 303, 305, and 306) such as is used in local-battery telephone instruments and in magneto testing sets offers an example of a small generator illustrating the essential principles. Its construction also illustrates the fact that by connecting a number of inductors or coil turns in series, a relatively high total e.m.f. may be induced. The armature winding consists of a single length of wire, wound into a coil of many turns, around the armature-core casting; hence all of the inductors are in series. The flux is produced by several steel permanent magnets. Usually these magnetos are provided with collector rings or their equivalent and hence impress alternating e.m.fs. on the circuit connected to them. However, they can be—and are for certain special purposes—equipped with a commutator and then they impress direct, pulsating e.m.fs. on the external circuit. The effective e.m.f. (Art. 731) of the average magneto generator is about 60 to 75 volts.

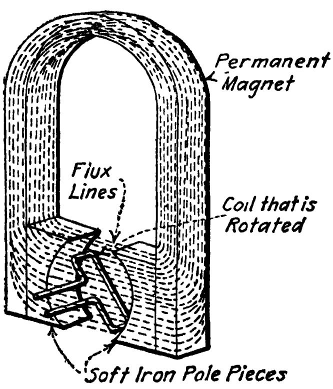


FIG. 305.—Elements of a magneto generator.

Example.—When a certain magneto generator is turned at a uniform speed of 100 r.p.m., the alternating e.m.f. it produces at its terminals is 70 volts, as indicated by a voltmeter there connected. The armature is wound with 3,000 turns of wire. What is the flux produced by the permanent magnets? *Solution.*—Each turn of the armature winding obviously has two sides or inductors; hence the total number of inductors is $2 \times 3{,}000 = 6{,}000$ inductors. A voltmeter connected to an alternating-current circuit always indicates the effective e.m.f. (Art. 735) of the circuit. But the symbol E_a in formula (144) stands for the average e.m.f. (Art. 730) or E_{av}.

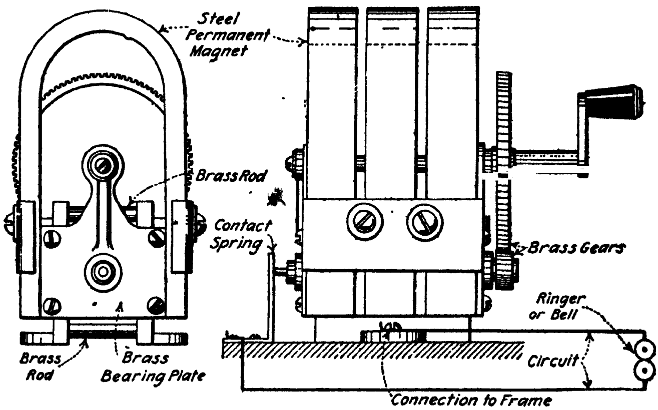


FIG. 306.—Magneto generator assembled.

Now (Art. 737) $E_a = 0.901 \times E_e = 0.901 \times 70 = 63.5$ volts = average e.m.f. Now substitute in the formula

$$\phi_T = \frac{10^8 \times 60 \times E_A}{C \times \text{r.p.m.}} = \frac{100{,}000{,}000 \times 60 \times 63.5}{6{,}000 \times 100} = 635{,}000 \text{ lines}$$

This means that 635,000 lines is the total flux cut. Half of this, or $635{,}000 \div 2 = 317{,}500$ lines of force, spans the gap wherein the armature is rotated. (In the above example it is assumed that a magneto generator produces an e.m.f. of true sine-wave form (Art. 554). In practice this is not strictly true, since the wave form is usually rather peaked. The actual e.m.f. induced will be somewhat greater than that given by the above formula.)

581. Constant-potential and Constant-current Generators.—All commercial generators may be divided into the two classes or groups just specified.

A constant-potential or constant-e.m.f. generator is one which will, under normal conditions, impress a practically constant e.m.f. or voltage on the external circuit connected to it. (The term "constant difference of potential" is a better one than "constant potential" because it better describes the machine.) Nearly all modern commercial generators, both

alternating-current and direct-current, except the relatively few used directly for series street lighting and other series circuits, are constant-e.m.f. generators. In practice, the term constant e.m.f. is applied when the e.m.f. referred to never varies more than, say, 10 per cent from a constant value. Constant-e.m.f. generators are designed to have their armatures rotated at practically constant speeds.

The current in the external circuit of a constant-e.m.f. generator—and in the generator—will, in accordance with Ohm's law (Art. 151), vary inversely as the resistance of the external circuit. The circuits connected to a constant-potential machine are parallel circuits (Art. 212)—the receiving devices such as lamps and motors are connected in parallel to the circuits.

NOTE.—By adjusting the field rheostat (Art. 599) of a constant-e.m.f. generator, the voltage which it impresses on the external circuit may be varied within a considerable range. Also, by adjustment of the field rheostat, the impressed voltage may be held practically constant, even if the prime-mover speed decreases or increases somewhat.

Example.—Constant-e.m.f. generators are used for all indoor incandescent lighting, railway traction, and electrical power transmission in North America. The transmission circuits in all of these cases are parallel or multiple circuits.

A constant-current generator (see Art. 123 for definition of "constant current") is one which maintains a constant current in the external series (Art. 208) circuit connected to it, the terminal e.m.f. of the machine varying as the resistance of the external circuit changes. Their principal application is for series street lighting. See author's "American Electricians' Handbook" for method of varying impressed e.m.f. so as to maintain current constant.

Example.—In series street lighting circuits the current is maintained at about 10 amp. The lighting devices are connected in series so this constant current flows through them in tandem. The e.m.f. required to force a current through a lighting device having a resistance of 5 ohms would be ($E = I \times R = 10 \times 5$) 50 volts. Then if 100 such lighting devices were connected on one circuit the generator would have to impress 100×50 volts = 5,000 volts on the circuit to circulate this 10-amp. current. A small additional e.m.f. would also be required to overcome the IR drop in the line wires.

The constant-current series system of distribution is economical of copper where the circuits are very long, as in street lighting. No. 6 copper is usually employed for these circuits. Each unit on the series circuit must receive its proper proportion of energy, since the current is maintained constant. Its disadvantages are its high potential and the fact that motors and similar devices to operate on constant current can not be designed readily. Lamps and other devices for use on constant-current circuits are provided with automatic cutouts which automatically provide a short-circuit path around the device if it accidentally opens or develops other troubles.

582. Direct-current Generators.—It has hereinbefore been shown (Art. 557) that an alternating e.m.f. is induced in the

armatures of all generators but that, when a suitable commutator is provided, this alternating e m f may be so rectified that an e.m.f. which is always in the same direction—a direct e.m.f.—

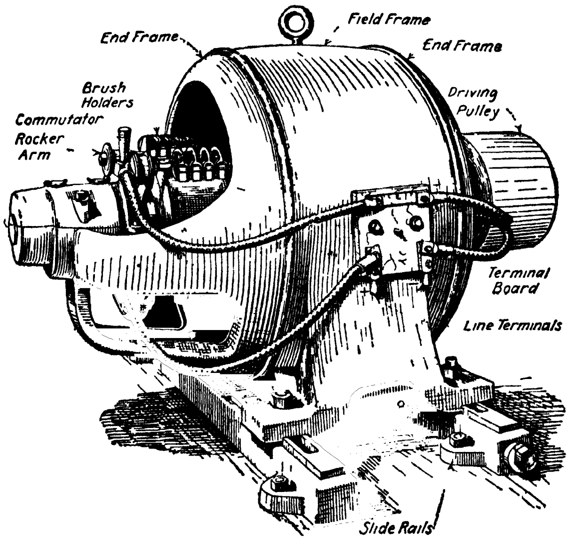


FIG 307 — Assembly view of a small direct-current, compound-wound generator A disassembled view of the same machine is shown in Fig 308 (*Allis-Chalmers Manufacturing Company*)

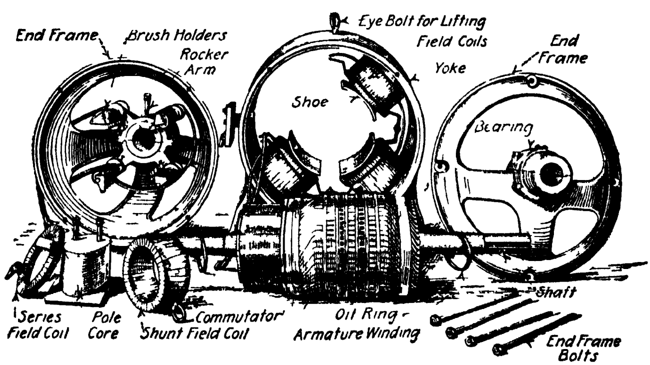


FIG. 308.—Disassembled view of the small direct-current generator shown in FIG 307. (*Allis-Chalmers Manufacturing Company*)

may be impressed on the external circuit. Generators designed thus to produce direct e.m.fs. are direct-current generators. The fundamental principles of these machines have already been

briefly indicated. In following articles their principal characteristics will be considered.

583. The necessary components of a direct-current generator are:—(1) *A field structure*, (2) *an armature*, (3) *a commutator*, and (4) *brushes*. Each of these essential parts is shown in an assembled machine in Fig. 307, and in a disassembled one in Fig. 308. The functions, construction, and general arrangement of these different components are treated in articles which follow.

SECTION 30

DIRECT-CURRENT GENERATOR FIELDS

584. A magnetic field is necessary in every generator, as indicated in Art. 572, so that the inductors can cut or be cut by this field and thereby have induced in them an e.m.f.

585. Methods of Producing the Field. Field Magnets.—The magnetic field of a generator is produced by field magnets which may be either permanent magnets or electromagnets. However, very powerful fields are essential in most commercial generators and these can be obtained only with electromagnets. Another disadvantage of permanent magnets is that with them there is no convenient method of controlling or varying their strengths. Sometimes the fields of very small generators, such as magnetos (Art. 580) used for telephone signaling and internal-combustion engine ignition, are produced by permanent magnets. In all generators used for electric lighting, energy transmission, and industrial applications the fields are produced by electromagnets. The field magnets of every generator are modified horseshoe-shaped magnets. It will be found that in every case each magnet unit consists of a yoke and two legs the ends of which form the poles. The legs are so disposed that the armature revolves between them.

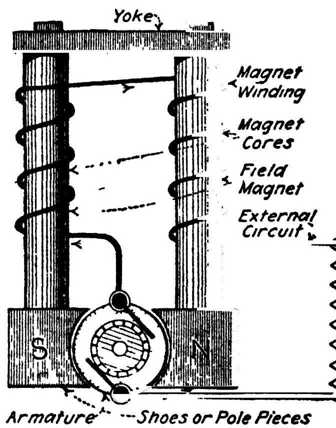


FIG. 309.—Diagrammatic drawing of a series generator.

Example.—Figure 309 shows (diagrammatically) a generator. Note that the field magnet is essentially of horseshoe form. Note also the same condition in the magneto generator of Fig. 306. Considering the multipolar generator of Fig. 310, each pair of magnet coils and the portion of the magnetic circuit associated with them in reality comprise a horseshoe magnet.

586. The field structure (Fig. 310) is that portion of a generator comprising and immediately associated with its field magnets. The field coils or magnet coils (Fig. 311) are the insulated copper wire or strap coils through which electrons are forced to produce

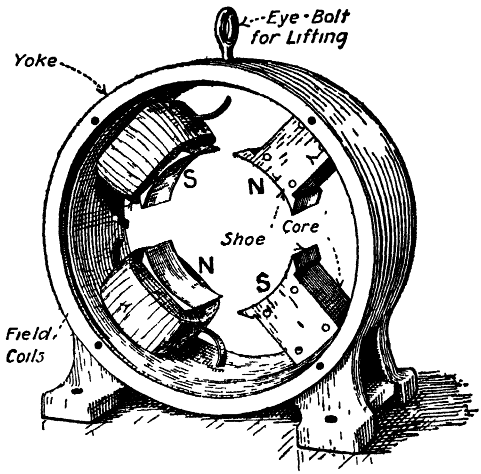


FIG. 310.—Field structure of a multipolar (four-pole) direct-current generator.

the magnetic field. Frequently, particularly in the large machines, these coils are (Fig. 311,I) provided with air spaces between layers or turns to facilitate ventilation. The magnet cores (Fig. 312) are of soft iron or steel to minimize hysteresis

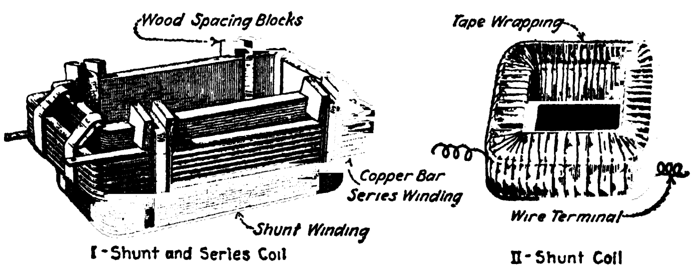


FIG. 311.—Field coils. (Note how the series coil of I is composed of bar copper with spaces between its turns to assure effective ventilation and cooling. Also note the space between the series and the shunt coil for the same purpose.)

losses and are frequently, particularly in the larger machines, laminated to minimize eddy-current losses. Figure 313 shows the magnet core of a large machine thus laminated. The ends of the cores which are nearest the armature are flared out into

pole shoes, sometimes called pole pieces or, merely, shoes. The shoes are occasionally separate blocks of metal as in Fig. 309, but more often they comprise enlarged portions of the material of the core. Shoes are provided for three reasons: (1) They decrease the distance between the end of the core and the armature. (2) They spread the flux along the armature and produce

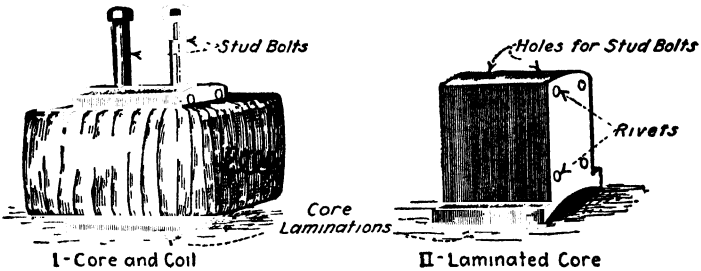


FIG. 312.—Laminated magnet-core for a direct-current generator.

the most effective distribution thereof. (3) They serve to hold the magnet coils in place.

587. Direct-current generators may be classified or grouped in accordance with: (1) *The number of poles* (Art. 588); (2) *the method of field excitation* (Art. 590); (3) *the type of field winding* (Art. 594). All three classifications are applicable to the same generator. That is, to describe comprehensively a direct-current generator one should indicate: (*a*) *how many poles it has;* (*b*) *how its fields are excited or magnetized;* (*c*) *the type of winding used for its fields.* These different classifications and their subclassifications will be described.

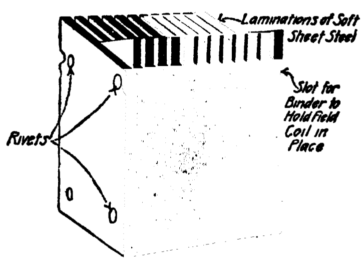


FIG. 313.—Showing the laminated core for a field magnet of a large direct-current generator.

588. In classifying direct-current generators on the basis of number of poles, they may be grouped into: (1) *bipolar* and (2) *multipolar* machines. Bipolar machines are those which have only two magnet cores or poles. Multipolar machines have more than two magnet cores or poles. The bipolar design, though formerly used exclusively, is now applied for only the smallest machines. See Art. 631 for an outline of the advantages of the multipolar design.

Examples.—Figure 309 shows the general characteristics of the bipolar design while Figs. 307 and 310 show multipolar machines.

589. The arrangement of poles is always such that they alternate in polarity around the frame. This is true of the poles of all generators, both alternating-current and direct-current.

Example.—In Fig. 310, the shoes alternate in polarity thus, *NSNS* around the frame. This same principle is also shown in following illustrations.

590. Two methods of field excitation are used for direct-current generators. A machine may be either (1) *separately excited* (Fig. 314) or (2) *self-excited* (Figs. 315, 316, 317, 318).

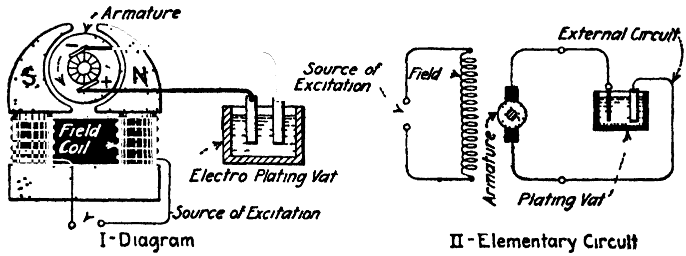


FIG 314 —Diagrams of a separately excited, direct-current generator serving an electroplating vat

591. Separately Excited generators are those in which the current which flows in the field magnet coils is impelled by some source other than the generator itself. Such machines are used in special cases (for testing in electrical machine factories) where very close regulation of the field strength is desirable and for electrolytic or electroplating work where it is of importance that the polarity of the machine be not reversed. They are also used in many of the special motor applications where field control must be independent of the load that the motor is carrying. Another generator or a storage battery (Fig. 314) may be used for impelling the excitation current. The field magnets may be wound for any desirable voltage which is available, inasmuch as they have no electrical connection with the armature of the separately excited machine. Practically all alternating-current generators are separately excited.

592. Self-excited Generators.—Practically all direct-current generators for lighting and power purposes are self-excited. That is, the current which excites or magnetizes their field magnets is

impelled by the armature of the generator itself. The field current of a self-excited generator is determined almost entirely by the brush voltage of the machine.

Example.—Figure 315 shows typical diagrams of self-excited generators. Note that in each case the field windings are so connected that the e.m.f. induced in the armature of the generator forces the current, which magnetizes the field magnets, to flow around the field coils.

593. The Excitation of Self-excited Generator Field Magnets. It is not readily apparent why a machine can be self-exciting, because the current which magnetizes the fields must be impelled by the e.m.f. induced in the armature inductors when they cut flux, and it seems reasonable that there should be no flux to cut until a current is circulating in the field coils. The explanation is that practically all commercial iron and steel has some *residual*

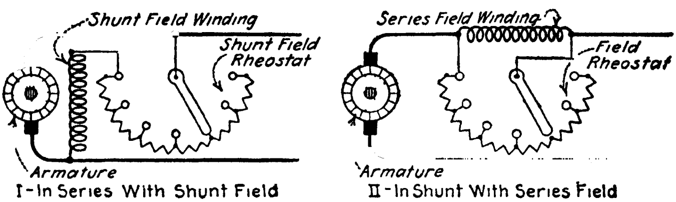


FIG. 315.—Diagrams of a series-wound generator.

magnetism—magnetism retained in the material—which causes the generator to build up. Hence, when a machine is not in operation, there is practically always a weak though appreciable flux emanating from the poles. Then, when the armature is caused to rotate, its inductors (Art. 578) cut this flux and a low voltage is induced in the armature. This voltage impels a current through the field coils, which increases the flux. This, in turn, increases the current in the field coils, which still further increases the flux. The increases continue until the normal voltage of the machine is attained. All self-exciting machines "build up" in this same manner.

NOTE.—Building up may require 20 to 30 sec. Occasionally machines appear to lose all of their residual magnetism and will not build up. Then, while the machine is being started, the fields must be excited weakly by using some source of low e.m.f. (for example, several dry-cells in series, or a low-voltage direct-current lighting circuit) to force current through the field coils and produce an initial magnetization. Usually after this treatment

the machine will build up satisfactorily. Sometimes the earth's field (Art. 60) can be made to induce the initial magnetization. See the author's "Electrical Machinery" for detailed directions as to how to make machines build up.

594. The Three Types of Field Windings.—The field-magnet windings of direct-current, self-excited generators may be arranged or connected so as to produce the required number of ampere turns to develop the necessary flux, in accordance with one of three different methods. Generators may, therefore, be classed as regards their method of field winding arrangement into: (1) *series-wound generators* (Fig. 316 and Art. 595); (2) *shunt-wound generators* (Fig. 317 and Art. 596); (3) *compound-wound generators* (Fig. 318 and Art. 597). Each of these types has distinctive characteristics and is inherently fitted for certain services as will be shown.

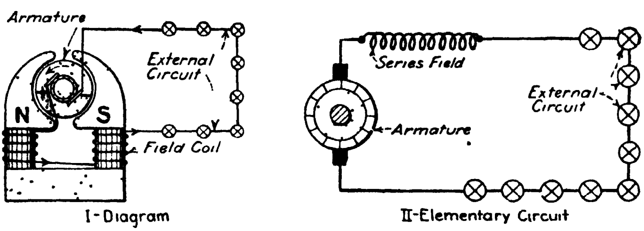


FIG. 316.—Diagrams of a series-wound generator.

595. Series generators (Fig. 316) have their armatures, field coils, and external circuits, all in series. The same current flows in the external circuit, the field coils, and the armature—obviously this must be true since these components are in series (see Art. 208, "Series Circuit"). The applications and characteristics of series generators are discussed in Art. 655. Since the current in the series field coil is relatively large, a comparatively small number of turns produces the required ampere turns for magnetization.

596. Shunt-wound generators (Fig. 317) have their field windings connected in shunt or parallel with the armature. Hence the current in the field coils is only a portion of that in the armature. Since the armature e.m.f. is impressed across the shunt-field winding, it follows that the current in the field winding will, in accordance with Ohm's law, be determined by its resistance Usually shunt-wound machines are so proportioned

that from 5 per cent (small machines) to 1 per cent (large machines) of the total armature current circulates in the field coils. Inasmuch as the shunt-field current is relatively small, a much larger number of field turns is required than in series machine coils to produce the necessary ampere turns for magnetization.

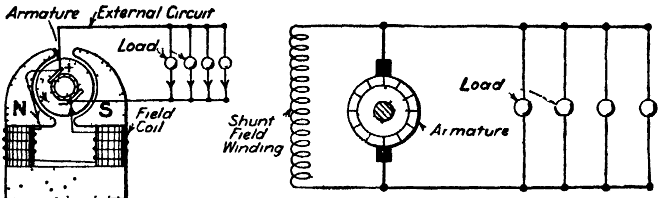


FIG. 317.—Diagrams of a shunt-wound generator.

NOTE.—By Ohm's law, the current circulating through the shunt-field winding is $I = E \div R$ = (the e m f across brushes of opposite polarity) $\div$ (the resistance of the field winding + the resistance of the field rheostat) The current circulating through the armature of a shunt generator equals (the current in the field winding) + (the current in the external circuit).

597. Compound-wound generators (Fig. 318) are hybrids, partaking of the characteristics of series and of shunt machines.

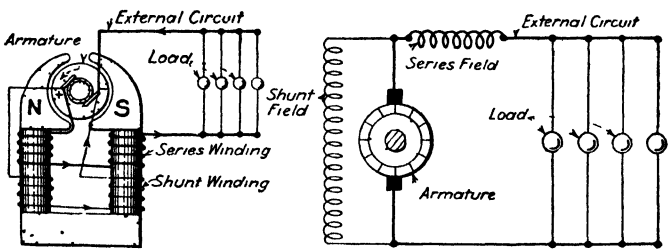


FIG 318.—Diagrams of a compound-wound generator.

The main-line current (or a certain definite proportion of it when a series shunt is provided) flows in the series coil. Hence the series-coil current varies with the load. The shunt coil is connected directly across the armature and the current forced through the coil by the armature e.m.f. is always the same (practically) and is inversely proportional to the shunt-field circuit resistance

598. Long-shunt and short-shunt connections for compound-wound generators are illustrated in Fig. 319. A short-shunt field connection is one wherein the shunt-field winding is connected directly across the brushes or armature. A long-shunt field connection is one where the shunt-field winding is connected across the armature and the series-field winding. Any compound-wound generator can be readily changed from a short-shunt to a long-shunt machine or vice versa by merely altering the field connection accordingly. The effect of each of these connections on the performance of the generator is discussed in

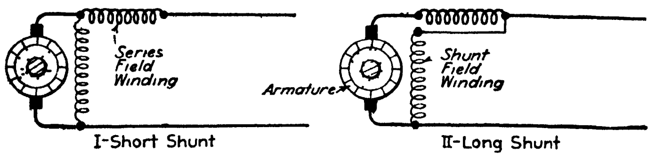


FIG. 319.—Long and short shunt connections of compound-wound generators.

the author's "American Electricians' Handbook" and in his "Electrical Machinery."

599. The methods of controlling field strength are indicated diagrammatically in Fig. 315. The rotational speeds (r.p.m.) of generators are constant (that is, they do not vary more than a few per cent); hence the only feasible way of controlling or varying the rate of cutting flux, the e.m.f. that the machine produces, is by varying the flux or the field strength. Where a generator has a shunt-field winding, the field strength may be varied by altering the shunt-field current by incorporating a rheostat in series with it. Increasing or decreasing the resistance in the field circuit by moving the rheostat handle decreases or increases the field current and consequently the flux.

With a generator having a series winding, field control may be effected by arranging a rheostat in shunt (Fig. 315,II) with the winding. With the rheostat circuit open, line current flows in the series coil and the flux is a maximum. But if current is permitted to flow in the rheostat circuit, the current in the series coil is decreased accordingly. By varying the resistance in circuit in the field rheostat, the flux is correspondingly varied.

SECTION 31

DIRECT-CURRENT GENERATOR ARMATURES

600. The function of a direct-current armature is, for generators for most commercial purposes, to produce an unvarying,

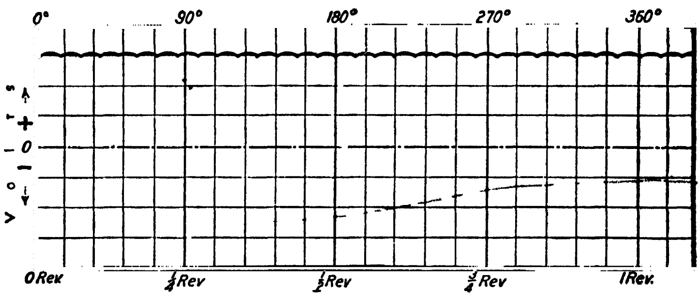


Fig. 320.—Curve of a practically constant e.m.f.

continuous e.m.f. The definition of an armature is given in Art. 571. In Art. 567 the method whereby the alternating e.m.f. induced in a rotating loop may be rectified into a direct e.m.f. in the external circuit is explained. However, the e.m.f. thus produced by the loop, as shown in Fig. 290,II, is a pulsating e.m.f. because it is always in the same direction. Obviously, this pulsating e.m.f. would not be applicable for a majority of applications because it consists merely of a series of impulses. How armatures may be arranged so as to produce the smooth continuous e.m.fs., such as that shown graphically in Fig. 320, will be explained.

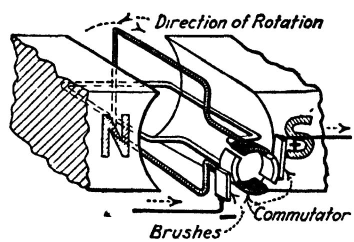


Fig 321.—Showing an elementary direct-current generator with a two-coil armature.

601. Production of a Constant or Unvarying E.m.f. by Increasing the Number of Coils.—Imagine the primitive direct-current generator of Fig. 295 modified by the addition of another coil and pair of commutator segments as suggested in Fig. 321, and

compare the two arrangements. With the single-coil arrangement it is obvious that there are certain instants (Figs. 298 and 301) at which no e.m.f. is being induced. Such instants occur twice during each revolution. Also, for a considerable proportion of each revolution the e.m.f. induced is small, as the curve of Fig. 290 shows. But when the two-coil armature of Fig. 321 is rotated in the field there is no instant when the e.m.f. impressed across the brushes is zero. The brushes are so disposed that they

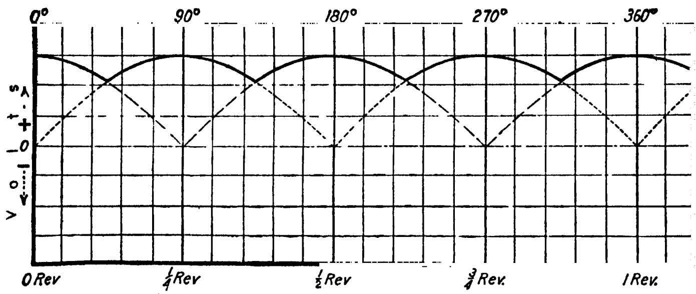


FIG. 322.—Curve of e.m.f. of the two-coil generator of Fig. 321.

always connect with the coil in which the greater e.m.f. is being induced.

Example.—Consider the elementary generator of Fig. 321. (One coil has been shown dark-colored and the other white, merely for distinction; otherwise they are the same.) The light-colored coil is cutting flux at a maximum rate at the instant depicted and the dark coil is not cutting flux. At this instant it is the white coil which is impelling the current in the external circuit. As the coil is rotated, the e.m.f. induced in the white coil will decrease and that in the dark coil will increase. The e.m.fs. in both coils will be equal when the coils have been turned ⅛ revolution or 45 deg. from the starting position, which is shown in Fig. 321. When the e.m.fs. thus become equal, the brushes bridge the commutator segments connected to the coils. An instant later, the brushes contact only with the segments connected with the dark coil. As rotation of the coil is continued the cycle of occurrences just described will be repeated.

If the e.m.fs. induced by this two-coil armature be plotted as a curve, it will be of the form indicated in Fig. 322. The heavy line indicates the e.m.f. impressed on the brushes. The current circulated in the external circuit will vary in accordance with a similar curve. The dotted line shows the e.m.f. induced in the white coil and the dashed line that in the dark coil—or vice versa. Every ¼ revolution or 90 deg. the e.m.f. is a maxi-

mum. Furthermore, every $\frac{1}{4}$ revolution—midway between the maximum instants—the e.m.fs. are equal and less than the maximum. Note that by the addition of one coil the brush e.m.f. has been prevented from decreasing to zero at any instant. It is apparent that by adding more coils the variation of brush e.m.f. during a revolution can be still further decreased.

602. To generate a constant, direct e.m.f., many inductors are properly arranged on an iron core so as to form an armature (Art. 571). The conductors are suitably connected together, in ways which will be described, and to commutator bars as shown in Fig. 323. Then to generate an e.m.f. this armature is rotated at uniform speed in a magnetic field. Some of the inductors—not the same ones constantly but different ones at different instants of a revolution—are always cutting flux at the maximum rate, that is, cutting at right angles across the flux. Thereby a direct, constant e.m.f. is impressed on the brushes. The curve of such an e.m.f. is given in Fig. 320. Obviously, the current impelled in an external circuit of constant resistance by this e.m.f. will be constant (practically), also and in every case will be inversely proportional (see Ohm's law equation) to the resistance of the entire circuit (Art. 153).

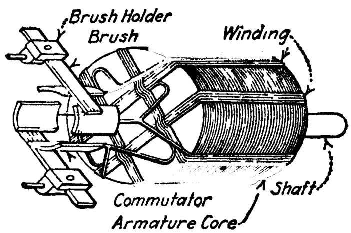


FIG. 323.—Showing a four-coil armature. (This is a closed circuit, drum winding.)

603. Open-coil and closed-coil armature windings are diagramed in Fig. 324. In open-coil windings the coils as a whole do not form a closed circuit, but each coil is in circuit only when the commutator bars to which it is attached contact with the brushes. In closed-coil windings the coils as a whole, together with the commutator segments to which they are connected, form a closed circuit upon themselves, and each coil always comprises part of the circuit. Practically all modern armatures are of the closed-coil type. Open-coil armatures have been used to some extent in series arc-lighting generators.

604. Effect of Open- and Closed-coil Windings on Brush E.m.f.—With an open-coil winding (Fig. 324,I) the e.m.f. of each coil decreases to zero every $\frac{1}{2}$ revolution, or 180 deg., as shown by the dotted and the dashed curves of Fig. 325. The dotted

curve shows the variation with the time of the e.m.f. of the white coil and the dashed curve that of the dark-colored coil. With a closed-coil winding (Fig. 324,II), since the coils are connected in series, the e.m.f. impressed on the brushes is at any instant the

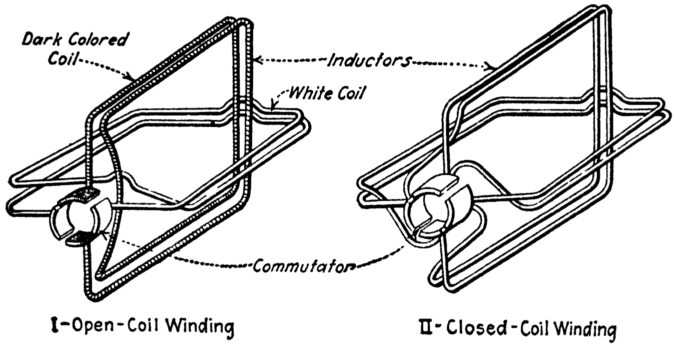


FIG. 324.—Diagrammatic illustration of open-coil and closed-coil armature windings.

sum of the e.m.fs. induced in the coils at that instant. Thus the full-line curve of Fig. 325 may be taken as the e.m.f. curve of the closed-coil winding of Fig. 324,II. This full-line curve was obtained by adding together the values of the dotted and the

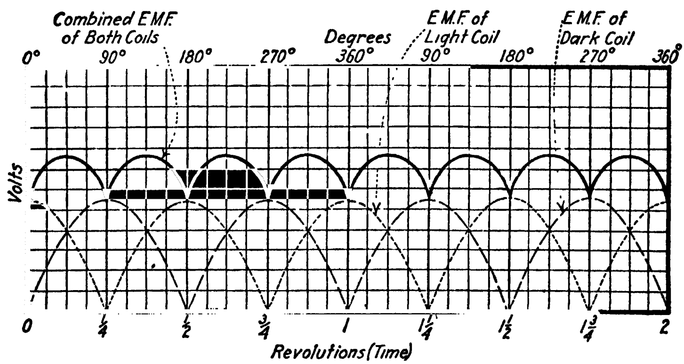


FIG. 325.—Showing e.m.fs. impressed on brushes by open- and closed-coil windings illustrated above.

dashed curves. Note, then, that a winding connected closed-coil has the property of producing a higher e.m.f. than the same winding connected open-coil.

605. The functions of the armature core are two: (1) It constitutes a strong mechanical support for the inductors. (2)

It decreases the reluctance of the magnetic circuit and induces the flux to follow paths where it will be most effective.

Examples.—If the inductors of Fig. 321 were wound on a cylindrical iron core instead of on an air core arranged for rotation between the *N* and the *S* pole, the reluctance of the magnetic circuit would be very greatly

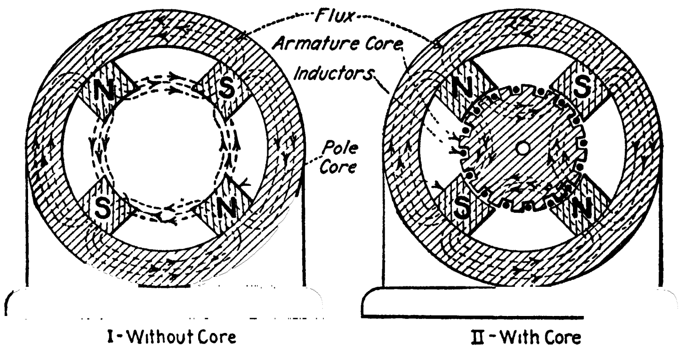


FIG. 326.—Multipolar-generator magnetic circuit showing effect of armature core on flux distribution.

decreased and a correspondingly fewer number of ampere turns would produce the flux necessary for the induction of the required voltage. In a multipolar machine, if there were no armature core, the flux would tend to bridge directly between the poles, as at Fig. 326,I, where the inductors would not cut across it. With an iron core, incorporated as shown at II, practically all of the flux traverses the coil and crosses the air gap where the inductors will cut through it.

606. Construction and Material of Armature Core.—So that the core will decrease the reluctance of the magnetic circuit as much as possible, it should be of highly permeable material. Hence, in practice, soft iron or electrical steel is used. Since the core is, when the generator is in operation, being magnetized by induction in one direction (*N*) and then demagnetized and magnetized in the other direction (*S*), as portions of its surface pass under the poles of opposite polarity in succession, it is subject to hysteresis (Art. 325). High-grade iron should therefore be used to maintain the hysteresis loss (Art. 327) at a minimum. Eddy currents (Art. 541) are also induced in the iron when it is being rotated in the magnetic field. Hence, cores should not be solid but should be built up or laminated. The cores are composed of sheet-iron laminations called armature-core punchings. To maintain the core temperature at a minimum, ventilating

spaces or ducts (Fig. 327) are provided at certain locations between laminations. Such ducts are always provided in the cores of large generators and frequently in those of small ones.

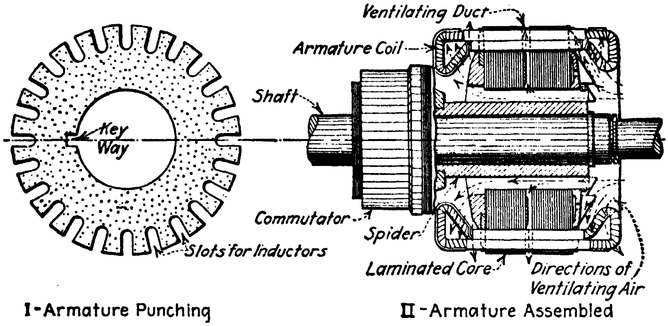


FIG. 327.—Illustrating armature punching in armature construction.

The arms of the armature spider, when it is rotated, act like blades of a ventilating fan and force cooling air currents through the ducts and around the conductors.

607. Armature-core punchings are illustrated in Figs. 327 and 328. These punchings comprise the laminations of which the core is assembled. For generators of the smaller capacities each punching is a suitably perforated, toothed disk, as shown in Fig. 327,I. Cores for the larger machines are assembled from segmental punchings as shown in Fig. 328. Slots of suitable size are provided in the punchings for the accommodation of the armature winding or inductors.

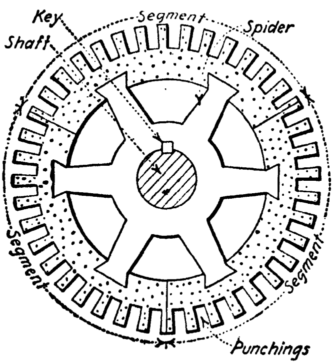


FIG. 328.—Armature core built up of segments.

608. Drum-wound and ring-wound armatures are illustrated diagrammatically in Fig. 329. These comprise the two classes into which armatures may be divided as regards disposition of the winding. A drum-wound armature is one the inductors of which lie wholly on, or in slots, in its cylindrical surface; the inductors do not pass through the interior of the rings. A ring-wound armature is one the coils of which are wound around the rim of a ring-shaped iron core.

NOTE that armatures may be classified into open-coil and closed-coil as regards the interconnection of the coils, as indicated in Art 604, and also into drum-wound and ring-wound as above as regards the disposition of the winding on the armature core

In effect, a ring winding constitutes a continuous helix wound around the armature core, as shown in Fig 330 Ring windings are seldom if ever used for the armatures of modern generators.

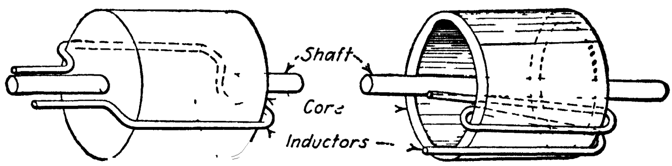


FIG 329 —Diagrammatic illustrations of drum-wound and ring-wound armatures

609. The principle of operation of both drum- and ring-wound armatures is the same, but since the ring-wound type is now seldom used most of the treatment herein will relate specifically to drum-wound armatures Note that, whereas with drum-wound armatures both sides of each loop or coil cut flux, with ring-wound armatures only one side cuts flux; the other side of each coil lies within the ring where there is practically no flux.

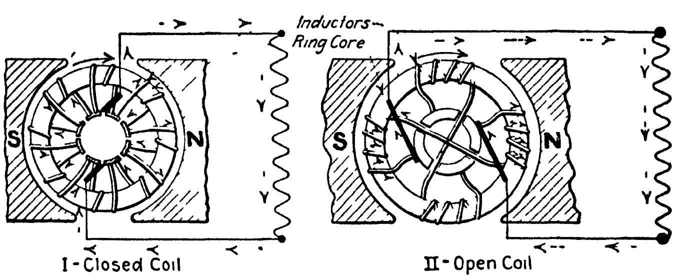


FIG 330.—Diagrams of ring-wound armatures, closed-coil and open-coil

610. The disadvantages of the ring winding are: (1) It is necessary in making a ring winding to thread the winding conductor through the space within the hollow-cylinder core which necessitates bending the conductor back and forth. Thus the coils must be wound on the core by hand. (2) Because of this bending it is very difficult to form large conductors into a ring winding. (3) The insulation on the conductors is likely to be injured by the bending process; hence insulation must, in such

cases, be placed on the coils while they are being wound. (4) It is difficult to secure ring-wound coils securely in their positions.

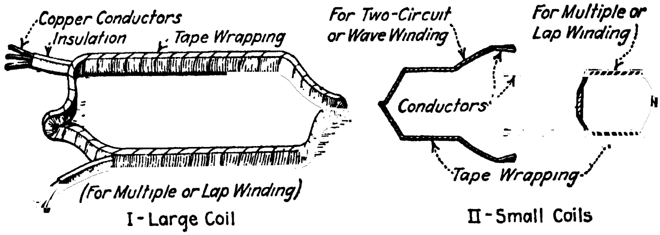


FIG. 331.—Form-wound armature coils for large and small generators.

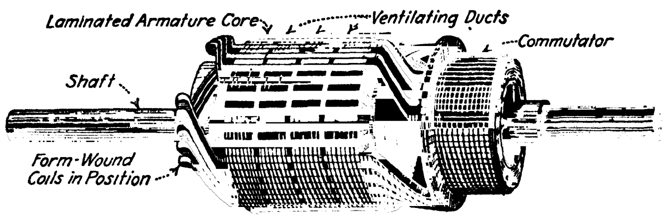


FIG. 332.—A drum-wound armature with a portion of its form-wound coils in position.

(5) Only half, approximately, of the length of wire wound on the armature is effective for cutting flux; thus the armature resistance is greater than for an equivalent drum-wound armature.

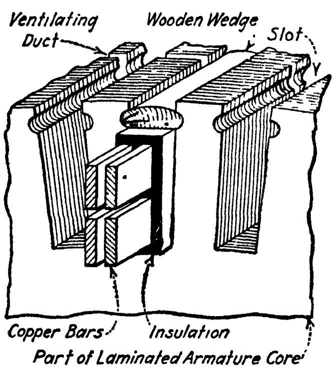


FIG. 333.—Portion of a drum-wound armature for a large generator showing inductors in position in slot.

611. The advantages of drum-windings are: (1) The coils may be form-wound (Fig. 331), that is, since all of the coils for a given armature are of the same shape and size they may be wound on forms. In many cases these form-wound coils may be made with automatic machinery. The resulting economy in cost is obvious. (2) Conductors of any reasonable size may be made up into form-wound coils. (3) The form-wound coils can be readily placed and firmly secured in the armature-core slots prepared for their reception.

612. The Process of Making a Form-wound Coil Is as Follows. The conductors, which are wires or bars covered with a thin insulation, such as a winding of cotton, are first wound on a form into a coil of the contour and size required. They are then wrapped with insulating tapes, heated to eliminate moisture, and impregnated with some waterproof insulating compound.

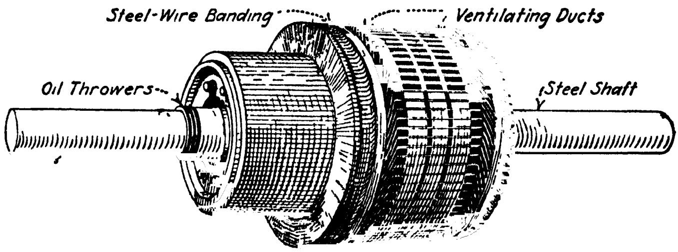


FIG. 334.—A completed armature for a direct-current generator of medium capacity.

613. In winding an armature with form-wound coils, the coils are all placed in their proper position in the slots (Fig. 332) and are held in place therein with wedges of fiber, wood, or similar tough insulating material, as shown in Fig. 333. Servings of steel wire wound into banding (Fig. 334) are usually also placed around the armatures further to secure the winding in place.

SECTION 32

ARMATURE REACTION, COMMUTATORS, AND COMMUTATION

614. Commutators in generators always have a number, sometimes a great number, of bars. It is almost obvious from a consideration of Figs. 321 and 335 that as the number of coils in an armature is increased the number of commutator segments must be proportionately increased.

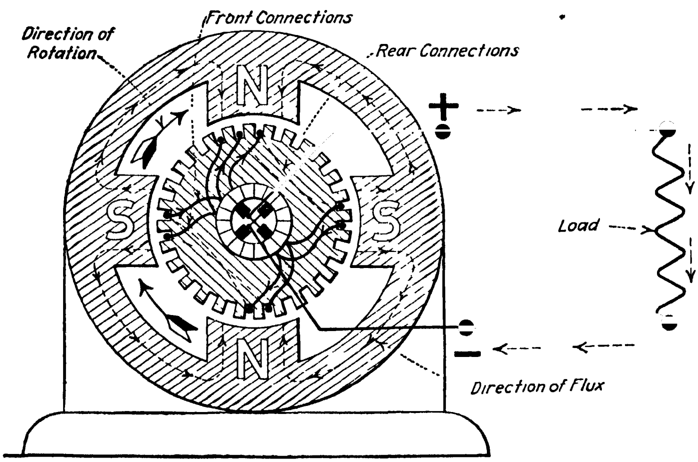


FIG. 335.—Diagram showing part of a direct-current armature winding in position on an armature. Verify the e.m.f. directions shown by applying the right-hand rule.

615. Commutator Construction.—The function of a commutator and its elementary construction are described in Art. 566. Actual commutators, though simple in principle, are very difficult to construct properly. The commutator is the weakest member both mechanically and electrically of any direct-current generator. The great majority of difficulties encountered with direct-current machines are commutation troubles. The forged copper segments (Fig. 336,I) are assembled into a cylinder on a sleeve or spider (Fig. 337), each copper segment being insulated from its

neighbor by a thin segment of mica The segments are also insulated from the iron or steel member which carries them with mica sleeves and rings. The wedge rings (Fig. 337), when

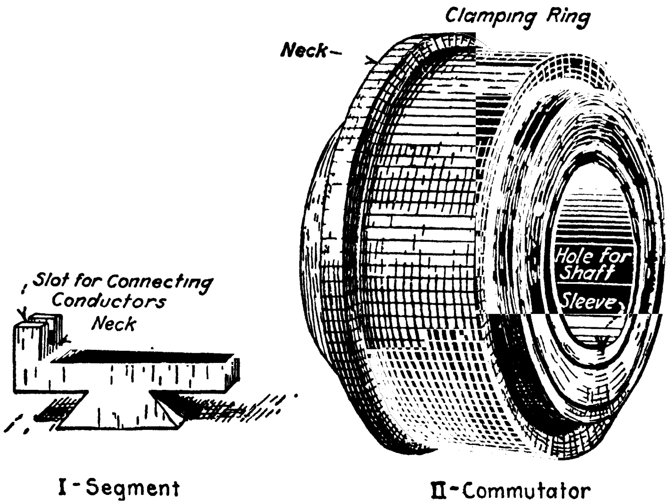


FIG 336 —Commutator segment and assembled commutator.

tightened by the turning of a clamping ring or the drawing up of clamping bolts, bind the segments securely in position. The commutator having been thus assembled, it is mounted in a lathe and its surface machined into a perfect cylinder

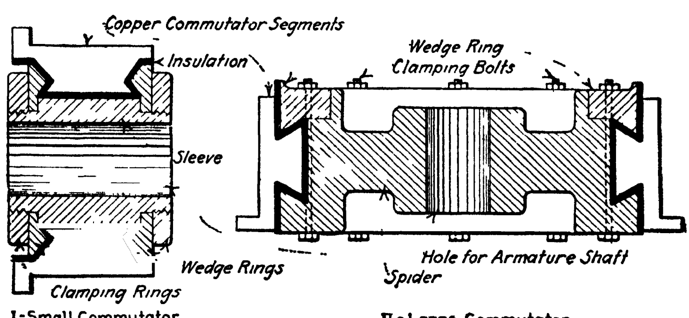


FIG 337 —Sectional views illustrating typical construction of a small and a large commutator

616. Brushes for most generators are blocks of carbon or graphite. Brushes made of packs of copper gauze or copper wire are sometimes used for certain low-voltage, high-current elec-

trolytic generators. Graphite is a suitable material because it is in a measure self-lubricating and has a relatively high resistance which, if the brushes are suitably proportioned, tends to minimize sparking between the brushes and the commutator (Art. 625). Brush holders (Fig. 338) are used to hold the brushes in correct position in relation to the commutator. Adjustable tension springs are provided whereby the brush pressure against the commutator may be regulated. The brush holders are mounted on brush-holder studs which, in turn, are mounted on a rocker frame. The rocker frame is so arranged that it may be rotated through a small arc to permit rotating of the brush-holder studs, brush holders, and brushes as a unit to different positions (within the limited range) around the commutator. Where the brushes must carry a large current, a number are arranged side by side on one stud. It is seldom, except for the very smallest generator, that any one brush-holder stud carries less than two brushes. The grade of graphite and the proportions for the brushes for a given machine are in design a matter of considerable importance.

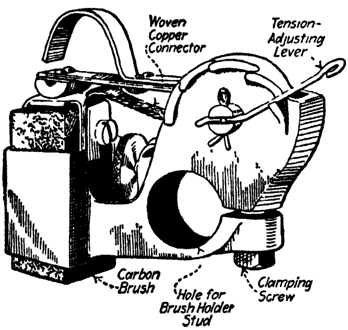


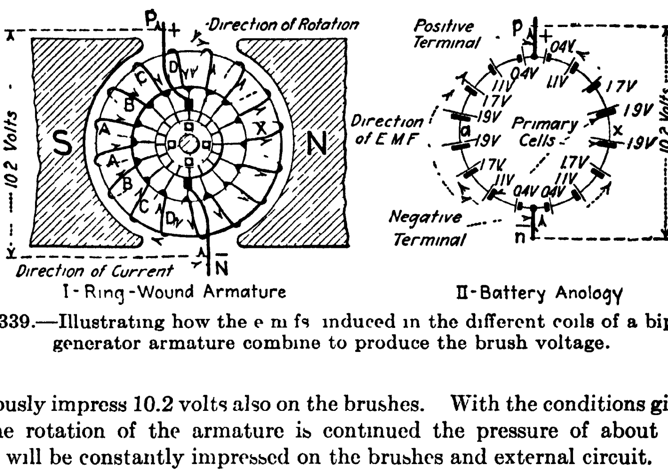
FIG. 338.—Brush and brush holder of modern generator.

617. How the voltages induced in the different coils of an armature combine to produce the brush voltage is indicated in Fig. 339. Although this illustration shows the armature of a ring-wound machine, the principle involved is the same in the armatures of all machines. In every case it will be found that an armature winding consists essentially of a number of turns or loops of flux-cutting conductors connecting between the brushes of the machine. (The conductors of a wave winding (Art. 635) do not form closed loops, but the electrical effects are practically the same as if the loops were closed.) Also, it will be found that the e.m.fs. induced in the different coils or loops combine in every case in about the manner to be described:

Example.—Assume that the armature of Fig. 339,I is rotated at a uniform speed in the clockwise direction shown. The outside parts of the coils will cut flux and induce e.m.fs. in the directions indicated by the dotted

arrows. (Check the e.m.f. directions by applying the hand rule of Art. 550.) It is assumed that the field is uniform. Then the e.m.f. induced in any inductor at a given instant will be proportional to the sine of the angle through which that inductor has been rotated from the neutral position (Art. 555).

Assume that the e.m.f. induced in the coils A at the instant shown is 1.9 volts. Then the e.m.fs. induced at this instant in the other coils will be about: $B = 1.7$ volts. $C = 1.1$ volts. $D = 0.4$ volt. The e.m.f. impressed on the brushes will be the sum of the e.m.fs. induced in the separate coils, that is $1.9 + 1.9 + 1.7 + 1.7 + 1.1 + 1.1 + 0.4 + 0.4 = 10.2$ volts. The opposite group of coils around the right-hand half of the armature will



I - Ring-Wound Armature II-Battery Anology

FIG 339.—Illustrating how the e m fs induced in the different coils of a bipolar generator armature combine to produce the brush voltage.

obviously impress 10.2 volts also on the brushes. With the conditions given, as the rotation of the armature is continued the pressure of about 10.2 volts will be constantly impressed on the brushes and external circuit.

Primary cells, 16 in number (Fig. 339,II), of the different voltages shown (if it were feasible to obtain cells of these different voltages) might be so connected as to impress 10.2 volts on an external circuit. This diagram may assist the reader to understand what is occurring in the armature.

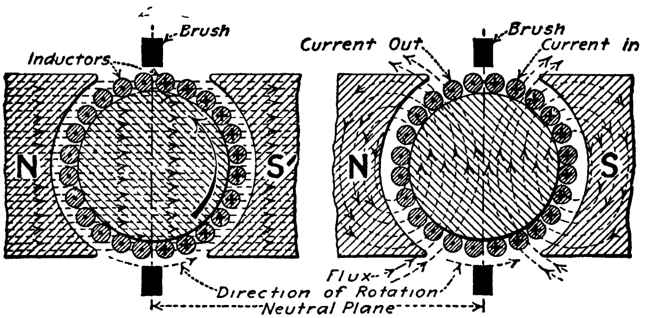
Since the two groups of coils (that to the left and that to the right of the brushes) are in parallel and each has the same resistance, half of the current that the generator impels in any external circuit connected to it will flow in each of the groups. Half will take the route NAP and the other route NXP. If the current in the external circuit is 12 amp., 6 amp. will flow in NAP and 6 amp. in NXP. A similar situation would obtain if the battery at II impelled current in an external circuit. Half would pass in the route nap and half in the route nxp.

In Fig. 350 are illustrated the conditions described above as they occur in the armature of a multipolar generator.

NOTE.—In the diagram of Fig. 339,I there is only one turn of the winding connected between adjacent commutator segments. However, in practical generators there are several turns of a coil in series between adjacent segments.

618. The neutral plane (Fig. 340,II and Fig. 341) through an armature is that plane at which the e.m.f. induced in the inductors is zero, because at this plane the inductors move parallel to the flux and hence do not cut it. The normal neutral plane (Fig. 341) is the plane which is the neutral one when there is no current in the armature inductors; this plane lies midway between the adjacent poles of opposite polarity.

619. The commutating plane is that imaginary plane, passing longitudinally through the armature and brushes, at which com-



I-Flux Due to Field Coil Current II-Flux Due to Current in Armature

FIG. 340.—Showing the two fields which cause armature reaction and field distortion in a generator. The dots and crosses in I indicate e.m.f. directions only. No armature current is flowing in I.

mutation occurs. The commutating plane may not coincide with the neutral plane, because, as described in Art. 626, it may be necessary (in the case of a generator, as will be hereinafter shown) to shift the brushes ahead (in the direction of rotation) of the neutral plane in order to insure sparkless commutation.

620. Armature reaction is the reactive magnetic influence produced by the current in the armature of a generator or motor, which is under load, on the magnetic circuit of the machine. Armature reaction is the cause of the phenomenon called field distortion. How it is that armature reaction and field distortion occur in generators will be explained:

Explanation.—Consider a portion of the magnetic circuit of a bipolar, direct-current generator, having a drum-wound armature with 12 coils, as shown in Fig. 340. The illustration delineates a cross-sectional view of the armature and parts of the pole pieces. With the fields excited and the armature circuit open at the brushes so that no current can flow in it, the direction of the flux will be straight across from pole to pole as at I and

the field will be practically uniform. The conditions just stated will obtain whether the armature is in rotation or not.

Now assume that the armature of I is rotated in the counterclockwise direction. Its inductors will cut flux, and e.m.fs. will be induced in them in the directions indicated (hand rule, Art. 462). To the right of the neutral plane (Art. 618), the e.m.f. in each inductor will be *in* as suggested by the crosses (Fig. 139) in the ends of the inductors. To the left of the neutral plane the e.m.fs. will be *out* as shown by the dots. Since the e.m.f. induced in the inductors at the neutral plane is zero (0), the brushes should be set at this plane as shown. Note that the field of I is now (while an e.m.f. is being induced in the armature but while there is no current in the armature) uniform—there is no field distortion.

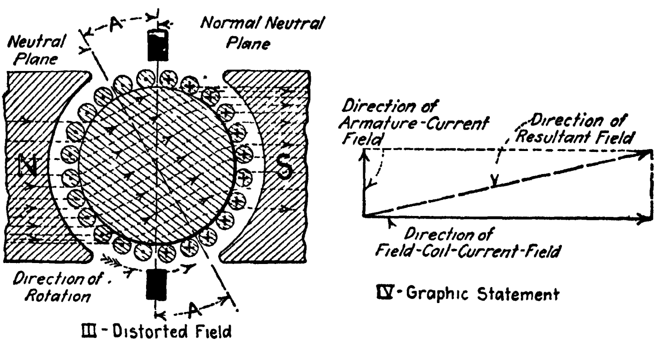


FIG. 341.—Field distortion caused by armature reaction (generator).

Now assume that the armature circuit be closed through an external circuit. Then, the e.m.f. of the armature will impel a current through the external circuit and armature inductors. This current of itself will produce a field or flux in the direction shown at II. This illustration of II shows only the flux produced by the armature current; it does not show the flux produced by the field-coil current. The condition shown at II could be reproduced in an actual machine by opening the field circuit, thereby killing the main field, and impressing some external source of e.m.f. across the brushes which would create a current, in the proper direction, in the armature, which should be held stationary. It is apparent then that, when a generator is in operation, there are two fields which tend to react on its armature: One, due to the field-coil current, has a direction between the two poles, as at Fig. 340,I. The other, due to current in the armature inductors, has a direction almost at right angles to the first, as shown at II. In an operating generator—one impelling a current—the effects of these two fields are simultaneous and superimposed. They combine to produce a distorted field as shown in Fig. 341. The flux across the air gap in the pole pieces and armature core is now no longer uniform but becomes dense toward the toes of the pole shoes in the direction of rotation. Furthermore, the neutral plane is no longer coincident with the normal neutral plane but is shifted through some angle as A, Fig. 341,III, in the direction of rotation. The

field-coil-current field is usually much stronger than the armature-current field, as shown graphically in IV.

Observe that with *no* current in the armature circuit, the neutral plane coincides with normal neutral plane, as at I. But, *with* current in the armature inductors, the neutral plane will shift in the direction of rotation, as at III. The greater the armature current, the greater will be the effect of armature magnetization (II) and the greater will be the shifting of the neutral plane. This means: The greater the load on a generator the greater is the armature reaction.

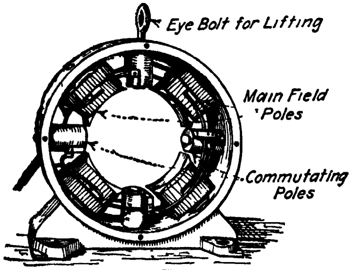


FIG. 342.—Field structure of commutating-pole machine.

621. The effects of armature reaction may be minimized by providing a magnetic circuit—air gap and pole pieces—of suitable design. Pole pieces of special forms and construction are sometimes used for the purposes. A description of the methods involved would require more space than is here available. Commutating poles (sometimes called interpoles or auxiliary poles, Figs. 342, 343 and 344) provide the most effective means of

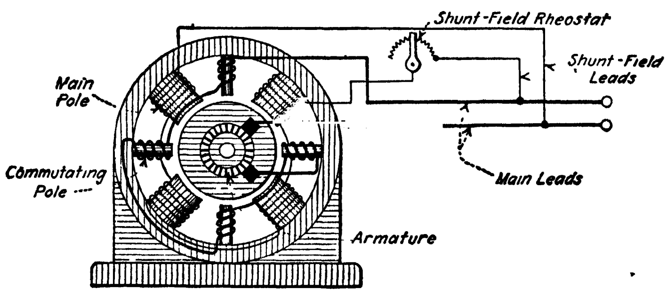


FIG. 343.—Arrangement of main and interpole windings in a commutating-pole generator.

eliminating the effects of armature reaction and are used in the best, modern direct-current machines (see following note).

NOTE.—Commutating poles are small auxiliary poles mounted between the main poles. They are so connected that the armature or line current—or a certain definite proportion of it—circulates in their windings. These windings are so proportioned that they produce a magnetic field in the air gap and in the armature core which is always of somewhat greater strength than that which the armature current (Fig. 345,V) tends to produce. As the line current and hence tendency toward armature reaction increases,

the strength of the commutating poles increases also Furthermore, the commutating-pole coils are so wound that their field opposes the armature-current field (see Fig 344) Thereby the armature-current field is neutral-

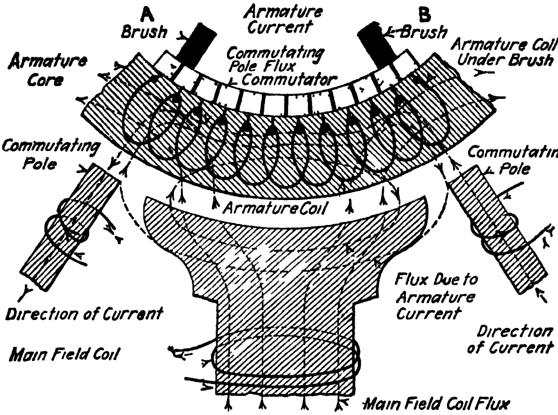


FIG 344 —Illustrating the principle of the commutating pole

ized Since it is the field due to the armature current which causes armature reaction (Art 620), this neutralization of it eliminates the possibility of armature reaction or field distortion

There is another effect of armature reaction that should be understood When the field becomes distorted, because of arma-

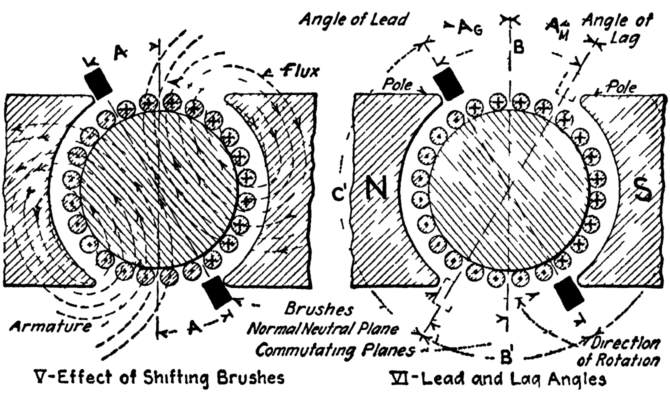


FIG 345 —(V) Effect of shifting brushes and (VI) lead and lag angles generator

ture current, as in Fig 341,III, the brushes should be shifted to some position (shown in Fig 345,V) so that they will lie in the neutral plane. The magnitude of the angle A (Fig 345), through which the brushes should be shifted, will depend on the amount

of armature reaction. But when the brush position is thus changed, the direction of the current in the inductors included within angle A will be reversed, as shown in V. This will alter the direction of the magnetization of the core due to the armature current, from that of II, Fig. 340, to that of V, Fig. 345. The result is a further distortion and weakening of the resultant field (combination of the field-current and the armature-current fields) of the machine.

622. The angle of lead and angle of lag, as these terms apply to the brush positions in direct-current machines, are illustrated in Fig. 345,VI. The angle of lead is the angle (A_G, Fig. 345,VI) through which the brushes of a generator must be shifted ahead (in the direction of rotation) of the normal neutral plane so as to lie in the commutating plane. The angle of lag is the angle (A_M) through which the brushes of a motor (Art. 677) must be shifted behind (against the direction of rotation) the normal neutral plane so as to lie in the commutating plane.

NOTE.—The terms "angle of lead" and "angle of lag," as above defined, have no connection with the same terms as used in alternating-circuit terminology. (With the armature of Fig. 345,VI operating as a motor, the direction of rotation and flux remaining as shown, the direction of current in the armature conductors would be opposite to that shown.)

623. Cross Ampere Turns and Back Ampere Turns.—The conductors included in the double angle B (Fig. 345,VI), which is the sum of the lag angle and the lead angle, are sometimes called the back turns, and their magnetizing effect is called demagnetizing ampere turns. These terms are applied because the turns within angle B may be considered as being in series with those of angle B', forming a complete helix around the armature core. Their effect is opposed to and tends to weaken that of the field coils of the machine. The turns not included in the lag and lead angles may similarly be called the cross turns, and their effect, cross-magnetizing ampere turns. They tend to set up a field at right angles to the main field.

624. The process of ideal commutation is illustrated in Figs. 346, 347, and 348. The armature conductors of any direct-current machine, in effect, constitute loops or coils between the commutator bars. Therefore, the process of commutation may be examined by considering the diagram, Fig. 346, which

shows part of a direct-current-generator armature, commutator, and field stripped of the nonessentials (see Art. 566).

Explanation.—The armature (Figs. 346 to 348) is being rotated at a uniform speed in the clockwise direction shown. Each coil of the armature is marked in a different way to facilitate ready identification. The brush is on the neutral plane and it is located inside of the commutator to simplify

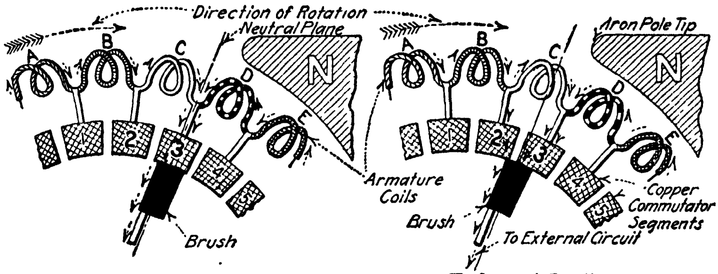


FIG. 346.—Final conditions in process of commutation.

the diagrams. What occurs when coil C—the white one—passes through the commutating plane will be considered. The current directions are as indicated by the small arrows.

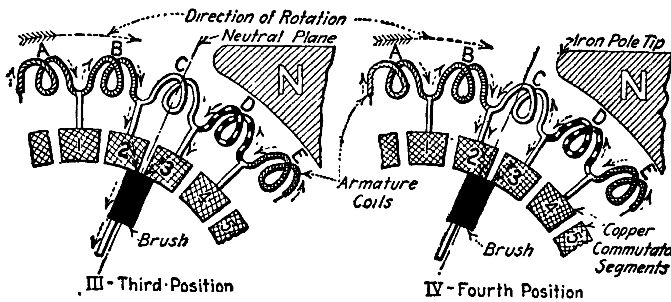


FIG. 347.—Initial conditions in the process of commutation.

At the instant of diagram I, the brush bears on segment 3. The currents in the left-hand half and that in the right-hand half of the armature meet at this segment (Art. 617), combine, and flow through the brush out into the external circuit.

At the instant of II the armature has been rotated far enough so that segment 2 just contacts with the brush. Segment 3 is also in contact with it. Now, a small part of the current from *B* flows directly into segment 2, through the brush and into the external circuit. Only a small part flows from *B* directly into 2, because the area of the brush contacting with 2 is small. Most of the current from *B* flows via coil *C*, through 3 and the brush into the external circuit, because the area of brush in contact with 3 is rela-

tively great. (The contact resistance (Art. 176) between the brush and a segment on which it bears will be inversely proportional to the area in contact.) Also, as in I, one-half of the total current through the brush flows from D. It unites with the portion from C and flows through 3 and out through the brush. As the armature is rotated further, the contact resistance between the brush and 2 decreases, and that between the brush and

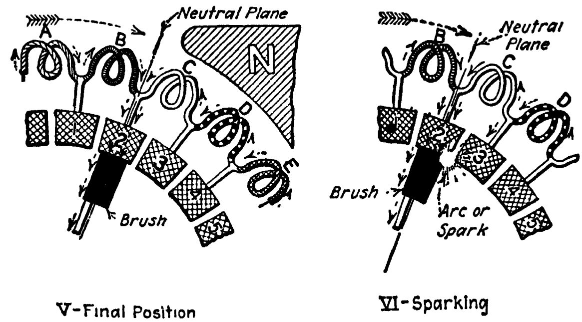


FIG. 348.—Intermediate conditions in the process of commutation.

3 increases. The currents through these two paths are, respectively, increased and decreased correspondingly.

At the instant of III (Fig. 347), the brush area contacting with 2 is the same as that bearing on 3 Hence, half of the current—that from B—flows directly out through 2 to the brush, where it unites with the other half, which flows from D through 3 to the brush. The coil C being in the neutral plane, is cutting no flux; hence the e.m.f. and current in it is zero.

At the instant of IV, the armature has been so rotated that the brush is barely in contact with 3. The brush area in contact with 2 is much greater than that in contact with 3. Now the half of the total current from B flows directly into 2 and to the brush. The other half of the total current flowing from D divides, the smaller portion of it flowing directly to 3 and into the brush, the greater portion, from D through C and 2 to the brush.

At the instant of V (Fig. 348) the brush contacts only with segment 2. Half of the current through 2 and through the external circuit flows from C and the other half flows from B.

The important fact to note from the above explanation is that the current direction in a coil reverses as it undergoes commutation. For example, in the coil C of Fig. 346,I, prior to commutation, the current direction in C is toward the right. At Fig. 348,V, subsequent to commutation, the current direction is toward the left. The ideal process as above described may not obtain exactly as described in the case of an actual generator for reasons outlined in the following article.

625. Sparking during commutation is due to the self-inductance (Art. 516) of the armature coils. If armature coils had no inductance, with the brushes set on the neutral plane, there should be no sparking. However, armature coils, since they are wound on iron cores, do have considerable inductance. Also, since armatures revolve so rapidly that frequently the current in a coil must reverse in a hundredth of a second and since they may carry very large currents, the counter e.m.f. of self-induction (Art. 501) may be considerable. How this produces arcing or sparking at the commutator will be described:

Explanation.—In an ordinary generator the current in the coil under commutation must reverse during the short interval between (1) (Fig. 346) the instant before the brush first contacts with one commutator segment of the coil (Fig. 346,I), and (2) the instant at which the brush ceases contacting with the other segment (Fig. 348,V). At the instant of Fig. 347,IV, the current should be reversed in direction in the coil *C*, as shown in the diagram, because this coil has been rotated to the right of the neutral plane.

Actually in an ordinary generator the current will not be wholly reversed at this instant—the instant after the coil has been rotated beyond the neutral plane. The self-inductance of the coil tends to maintain the current in it in its original direction. The consequence is that at the instant pictured at IV, there will be (in actual generators) a considerable current circulating through the short-circuit path constituted by the coil and the brush, in a direction from left to right through the coil *C* (in IV) and from right to left through the segment 3, the brush and segment 2. The current through the brush tip may be so great as greatly to heat or fuse the edge of the segment 3 and the adjacent edge of the brush.

This short-circuit current still flows at the instant when the armature has been turned so that the brush has ceased contacting with 3 (as shown in VI) and is wholly contacting with 2. The result is that when the brush slides from 3 to 2 this current is interrupted, with an air gap between 3 and the brush, and an arc or spark is formed, as shown at VI. Obviously, such excessive heating and arcing may, if permitted to continue, prove very injurious to both commutator and brushes.

626. Prevention of Sparking.—Obviously, any means whereby the counter e.m.f. of self-induction in the short-circuited coil (Fig. 348,VI) is neutralized will eliminate sparking at the commutator. The common method of accomplishing this is to move the brush a trifle ahead (in the direction of rotation in the case of a generator) of the neutral plane (Fig. 349) so that the coils as they reach the commutating plane can cut some flux. Then, commutation will not occur with the coils as they successively lie in the neutral plane. But it will occur with the coils as they

pass through the commutating plane—a location ahead of the neutral plane—where there will be induced in them an e.m.f. opposite in direction to the counter e.m.f. of self-induction of the coil. The e.m.f. induced in the coil is opposed to and will just neutralize the counter e.m.f.—if the brush has been shifted ahead the right distance. The brush position to ensure the least sparking is ascertained by trial. Sparking is automatically eliminated in commutating-pole generators (Art. 621) without its being necessary to shift the brushes.

627. Change of Brush Position with Change of Load.—Armature reaction and field distortion vary with change in load (Art. 620). Therefore, the neutral plane shifts as the load on the generator—the current in its armature—changes. The neutral plane shifts ahead as the load increases. It follows that the brushes of a generator (commutating-pole generators, Art. 621, excepted) should be shifted ahead as the load on the machine increases, so that the sparking will be a minimum. The brushes should always be in such a position that the coils, at the instants when they successively pass through the commutating plane, will cut enough flux to neutralize their counter e.m.fs. of self-induction.

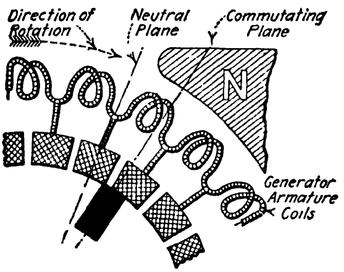


FIG. 349.—Showing how the brushes of a generator should be shifted ahead of the neutral plane to promote sparkless commutation.

NOTE.—In all well-designed modern noncommutating-pole generators little shifting of the brushes is necessary with change of load because the carbon brushes are so selected and proportioned as to have proper resistance to maintain the short-circuit currents in them at a minimum, thus minimizing sparking.

628. In Commutating-pole Generators It Is Not Necessary to Shift the Brushes with Changes in Load to Minimize Sparking at the Commutator.—As suggested in the preceding article, shifting of the brushes is necessary in ordinary generators because of changing armature reaction. But with commutating-pole machines the armature reaction effects are not present. Hence, with these machines brush shifting is not necessary or desirable. The best brush position for a commutating-pole machine is determined at the factory where it is manufactured, and usually the brushes are there permanently set at the proper position.

SECTION 33

MULTIPOLAR DIRECT-CURRENT GENERATORS

629. Multipolar Generators.—The term is defined in Art. 588. Multipolar machines have certain inherent advantages. The multipolar construction is used therefore for all but the machines of the smaller capacities, which can be constructed more economically in the bipolar type.

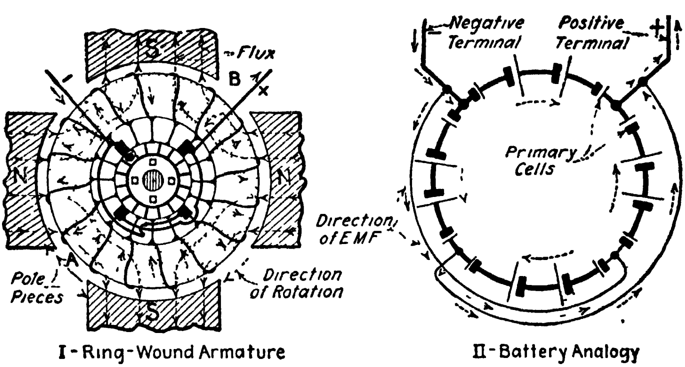


FIG 350 —Illustrating how the e m fs induced in the different coils of a multipolar generator armature combine to produce the brush voltage.

630. The magnetic circuit and generation of e.m.f. in multipolar machines are illustrated typically in Figs. 350 and 351, which indicate a four-pole generator. In Fig. 350,I is suggested how the e.m.fs. are induced in the different armature coils which cut the flux emanating from the different pole pieces. The e.m.f. directions shown by the dotted arrows may be verified by application on the hand rule of Arts. 462 and 550. The e.m.fs. combine to constitute the brush e.m.f. in much the same way as described in Art. 617 in connection with the bipolar machine armature. The battery analogy of II may serve better to illustrate how the e.m.fs. of the different coils combine. The e.m.fs. directed from the armature coils to the commutator combine at two locations and those directed from the commutator toward the coils diverge at two other locations. Hence—with a four-pole

machine—four brushes must be used—two of them positive (+) and two negative (−). Except with machines having certain seldom-used types of armature windings (Art. 638), generators always have the same number of sets of brushes as they have

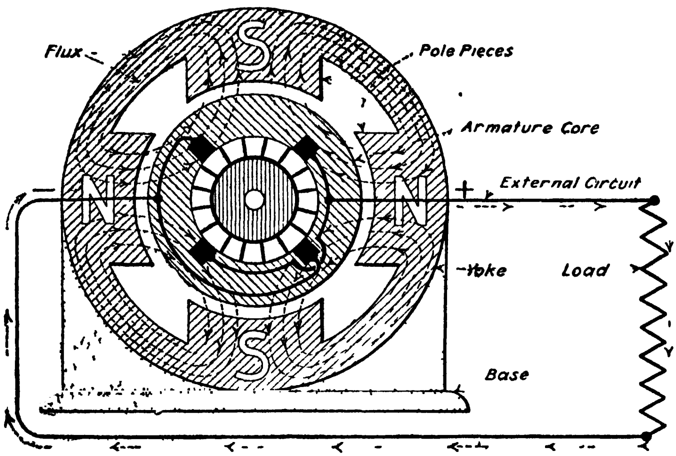


FIG 351 —Magnetic circuit of a four-pole generator

poles. The positive and the negative sets of brushes are suitably interconnected and connected to the external circuit as illustrated

631. The Economics of the Multipolar Design.—Multipolar machines are, in general, more economical in constructive material, largely because of the effective distribution of flux which they

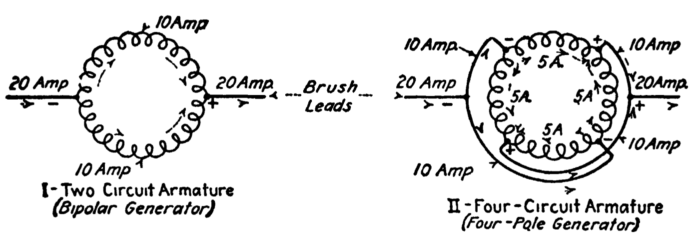


FIG 352 —Showing division of current in a two-circuit and a four-circuit armature.

provide, than are those of the bipolar type. Because of their greater areas of exposed surface per unit output, they radiate the heat due to losses more effectively. They weigh less per unit output. However, if an endeavor is made to construct a machine, of a given output, with too many poles, the increased cost of manufacture will obviously overbalance the saving in material.

It is for this reason that the smaller machines are usually made bipolar.

632. Smaller armature conductors usually feasible with generators of the multipolar type (see following note).—With a bipolar generator (Fig. 352,I), each armature conductor (closed-coil winding) must carry one-half the line current. Obviously, in machines of considerable power the line current will be very great, which would necessitate excessively large armature-conductor currents in large bipolar machines. In a four-pole machine (Fig. 352,II) the line current passes through the armature via four paths. Hence, in a four-pole machine, the armature-conductor current is but one-half that in a two-pole machine or one-quarter of the total current. There are always (except as detailed in the following note) as many armature paths as there are poles. It is evident that the greater the number of poles a machine has, the smaller, relatively, may its armature conductors be. The smaller the conductors—within reasonable limits—the more readily are they wound and handled. Also, the greater the number of poles, the smaller the current through each set of brushes, which facilitates sparkless commutation.

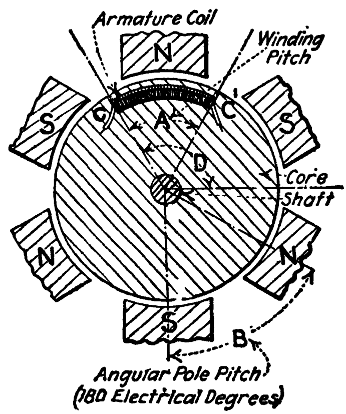


FIG. 353.—Showing spread or winding pitch of a drum-armature coil.

NOTE.—Machines with wave-wound armatures (Art. 638) may have only two paths through the armature, regardless of the number of poles. Hence the information of the above article may not apply to wave-wound armature machines.

633. Lower Peripheral Speeds Are Possible with Multipolar Machines.—When any coil of a bipolar generator has been rotated through 360 electrical deg., it has also passed through 360 actual degrees. But when any coil of a four-pole generator (Fig. 350,I) has been rotated through 360 electrical degrees, it has passed only from *A* to *B*, through 180 actual degrees. And when a coil of six-pole generator has been rotated through 360 electrical degrees (angle *D*, Fig. 353) it has passed through only 120 actual

degrees. It follows that, with the same flux per pole and a given angular velocity or rotational speed, a multipolar generator will induce a higher e.m.f. than will a bipolar machine. Furthermore, with the same flux per pole, the same e.m.f. may be induced with lower angular velocities in multipolar machines than in bipolar. It would not be feasible to build machines of ordinarily high voltage and of large capacity in the bipolar type. Then armatures would necessarily have to be large and could not be safely rotated at speeds great enough to induce, in a bipolar field, the e.m.fs. required. Instead of increasing the speeds, in the large machines the number of poles is increased, which is equivalent to increasing the flux.

SECTION 34

DIRECT-CURRENT ARMATURE WINDINGS

634. Armature windings of modern machines are in almost every case of the drum type (Art. 608), hence only these will be discussed. It is obvious (see Figs. 329 and 335) that the e.m.f. in one side of a drum-winding coil must be from front to rear at instants when the e.m.f. in the other side of the coil is from rear to front, so that the two e.m.fs. will act in unison and combine instead of opposing and tending to neutralize one another. It follows then that the coils on a drum-armature core must be so arranged that two inductors of any coil will never be passing under like poles at the same instant. The coil sides should therefore be separated by an angle (A, Fig. 353) about equal to the angle B between pole center lines. Angle A is called the angular pitch or spread of the coils or the winding pitch. Angle B is the angular pole pitch. The subject of armature windings is an extensive and rather involved one, hence only the essential fundamentals can be discussed herein.

635. Types of Windings: Lap and Wave Windings.—There are many possible types of windings, but it is seldom that any but the lap winding (parallel or multiple drum winding) or the wave winding (two-circuit series drum winding) are used.

The lap winding is ordinarily used for all direct-current and for some alternating-current armatures. It has the advantages of adaptability and simplicity but is not readily adapted for the inducing of high e.m.fs. With it there are always as many studs of brushes—hence current paths through the armature—as there are poles. A lap winding can be distinguished from the fact that the rear-end connections lap over one another, as shown by the black-line coil in the winding development of Fig. 354. Coils for lap windings are shown in Fig. 331. With a lap winding there are always as many conducting paths as there are poles.

The wave winding is used for direct-current drum armatures where a lap winding would not provide a sufficiently high e.m.f. or would require an excessive number of fine-wire turns per coil. With a wave winding (Fig. 355) only two brush studs are necessary, but more, up to a number equal to the number of poles, can be used. There are always only two circuits in

multiple, irrespective of the number of poles. This type of winding can be distinguished from the fact that the rear-end connections form geometric

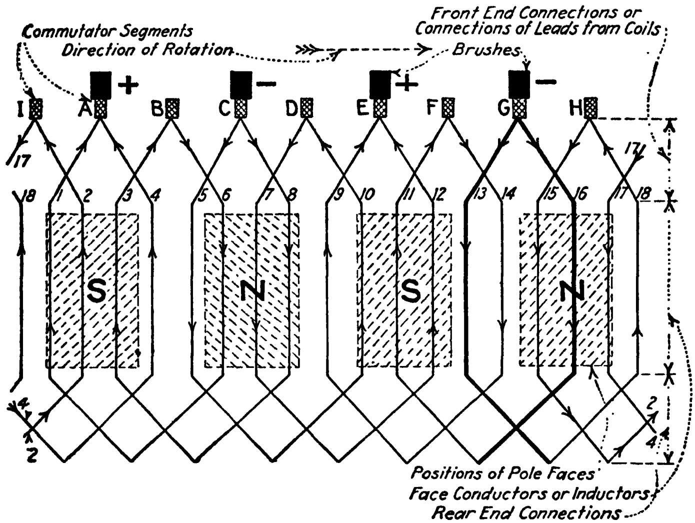


FIG. 354.—Lap winding. Development of a typical winding.

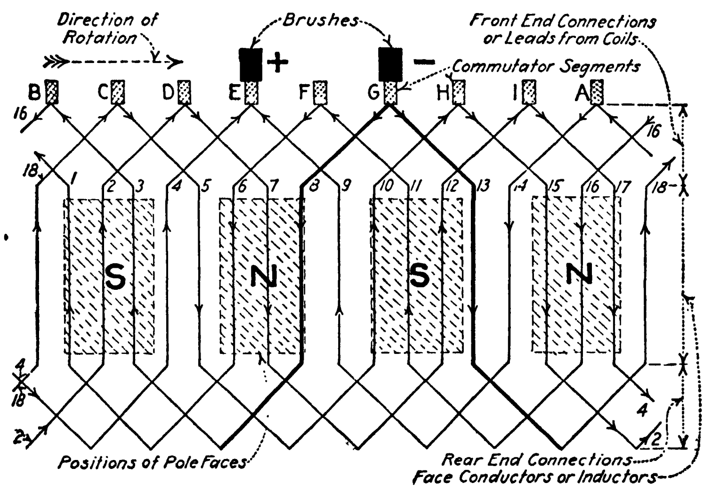


FIG. 355.—Wave winding. Development of a typical winding.

waves from coil to coil, as indicated by the black-line coil in Fig. 355. A coil for a wave winding is shown in Fig. 331.

636. Developments or diagrams of armature windings may be either plane developments, which represent the winding as it

would appear (Figs. 354 and 355) if it were unrolled from the core and laid out flat on a plane surface, or star developments (Figs. 356 and 357), which delineate the winding somewhat as it might appear if it were possible to strip it from the core and flatten it out by a pressure, applied along the axis (shaft) of the armature core. A plane development has the disadvantage that it does not show the armature-conductor circuits as entirely closed. In the star development this disadvantage is corrected.

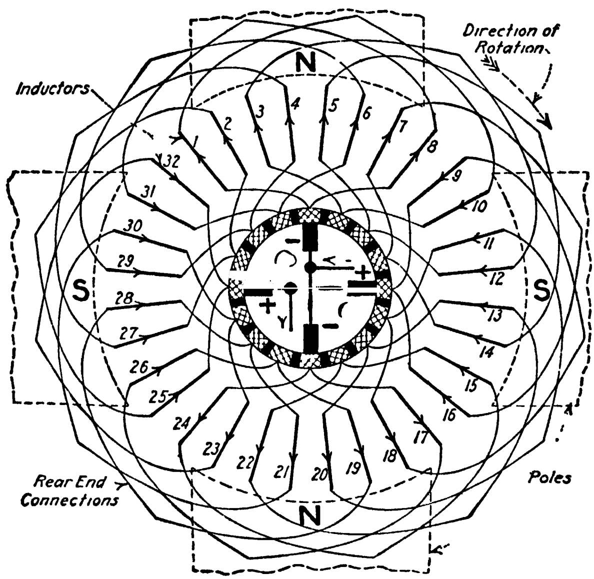


FIG 356 —Lap-winding for drum armature having 32 inductors (16 coils).

637. E.m.f. and Current in the Lap Winding (Art. 635).—The plane development of a lap winding for a four-pole generator is shown in Fig. 354. There are 18 inductors or 9 coils. With the direction of rotation as given, the e.m.f. directions will be as shown (hand rule, Art. 462) by the arrowheads. One side of each coil is under a *N* pole and the other side under a *S* pole. If the winding is traced out, starting at inductor 1, it will be found that all of the inductors are in series and, in effect, constitute a closed helix. The e.m.f. directions determine the brush locations. Wherever the e.m.fs. in the two conductors connecting to a segment are directed into or out of that segment, a brush should be

located at that segment. The star development of Fig. 356 shows a lap winding, having 32 inductors (16 coils), for a four-pole machine. The e.m.f. directions will be as shown determining the brush locations at the segments indicated. From this diagram it is readily apparent that there are as many current routes through the armature as there are poles (Art. 632).

638. E.m.f. and Current in a Wave Winding.—Consider the diagrams of Figs. 355 and 357. The directions of the e.m.fs. in

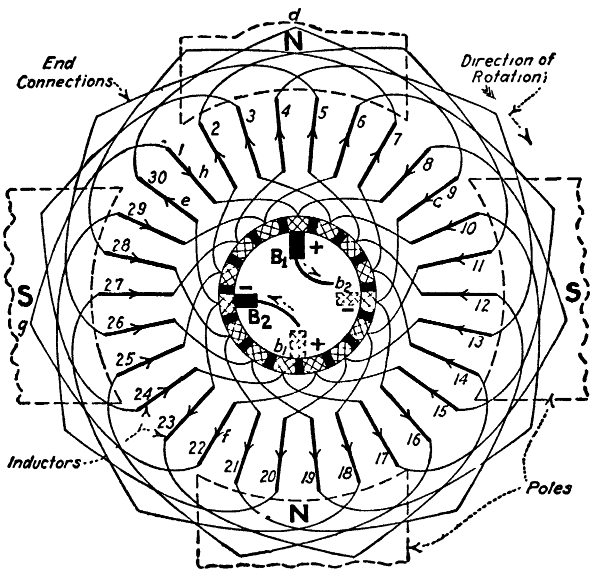


FIG. 357.—Wave winding for a drum armature having 30 inductors (15 coils).

the inductors (shown by the arrowheads) directly under the poles can be readily determined (hand rule, Art. 462), and those in the other conductors must correspond. Only two brushes (B_1 and B_2, Fig. 357) are necessary, but brushes may be, and are, where the total current is large, located at each neutral plane. All of the conductors are in series, and there are really, regardless of the number of brushes used, only two current paths through a wave winding.

NOTE.—Why the addition of two more brushes b_1 and b_2 at the neutral planes in Fig. 357 will not affect the situation may be explained thus: The added brush b_2 is, at the instant shown, in metallic connection with B_2 through the low-resistance path b_2cedB_2 provided by this armature coil.

The effect is the same as if B_2 and b_2 were the same brush. Furthermore, b_1 and B_1 are connected directly by the coil b_1fghB_1, producing a similar condition. The generator would also operate as well if b_1 and b_2 were provided and B_1 and B_2 omitted as when only B_1 and B_2 are used. In practice, where only two brushes are used, they are usually located above the horizontal diameter of the commutator, in which position they will be most available.

SECTION 35

DIRECT-CURRENT GENERATOR VOLTAGES, RATINGS, AND EFFICIENCIES

639. To compute the voltage induced in the armature of a bipolar, direct-current generator, the formula shown below can be used. The formula has already been given (Art. 579) for the computation of the e.m.f. induced in any rotating coil. That which follows is merely a modification of the rotating-coil one to adapt it specifically to the armature of a bipolar-frame machine. The formula (144) of Art. 579 for any rotating coil is

$$E_A = \frac{\phi_T \times C \times \text{r.p.m.}}{100{,}000{,}000 \times 60} \text{ (volts)} \qquad \text{(144 bis.)}$$

But in a new formula, let ϕ_P = flux per pole. Also let C_T = the *total* number of inductors or sides of coils on the armature. Obviously, in a two-pole machine $\phi_T = 2\phi_P$. And, in a two-pole machine, the inductors per pole, $C = C_T \div 2$. Then substituting these new symbols in the above equation

$$E = \frac{2\phi_P \times C_T \times \text{r.p.m.}}{100{,}000{,}000 \times 2 \times 60} = \frac{\phi_P \times C_T \times \text{r.p.m.}}{100{,}000{,}000 \times 60} \text{ (volts)} \qquad (145)$$

Wherein E = average e.m.f. induced in the armature of any direct-current bipolar generator.
ϕ_P = total useful flux or lines of force per pole.
C_T = total number of inductors or sides of coils on the armature; in drum armatures this is equal to twice the number of loops.
r.p.m. = number of revolutions per minute.

Example.—What average e.m.f. would be induced in the loop of Fig. 295, if it were rotated at the rate of 1,800 r.p.m. and the flux per pole was 400 kilolines, that is 400,000 lines? *Solution.*—Obviously the total number of inductors is 2. Now, substituting in formula (145),

$$E = \frac{\phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60} = \frac{400{,}000 \times 2 \times 1{,}800}{100{,}000{,}000 \times 60} = 0.24 \text{ volt}$$

Example.—The armature of a bipolar generator has 104 loops on its armature, and the flux pole is 2,270,000 lines. What e.m.f. will be induced in its armature if it is driven at the rate of 1,500 r.p.m.? *Solution.*—If the number of loops is 104 the number of inductors is 104 × 2 = 208. Now, substitute in formula (145)

$$E = \frac{\phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60} = \frac{2{,}270{,}000 \times 208 \times 1{,}500}{100{,}000{,}000 \times 60} = 118 \text{ volts}$$

640. To compute the e.m.f. induced in any direct-current generator armature the following formula, the derivation of which follows from that of Art. 639, can be used:

$$E = \frac{P \times \phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60 \times m} \text{ (volts)} \tag{146}$$

Wherein the symbols have the same meanings as in Art. 639, except P = the number of field poles. m = the number of parallel conducting paths between the positive and negative brush sets; that is, $C_T \div m$ is the number of inductors or armature conductors in series between positive and negative brush sets.

Example.—A four-pole, lap-wound armature has 200 inductors. The flux per pole is 3,000,000 lines. It is rotated at the rate of 1,200 r p.m. What e.m.f. is induced in it? *Solution.*—In a lap-wound armature there are as many conducting paths as poles (Art. 635). Substitute in formula (146)

$$E = \frac{P \times \phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60 \times m} = \frac{4 \times 3{,}000{,}000 \times 200 \times 1{,}200}{100{,}000{,}000 \times 60 \times 4} = 120 \text{ volts}$$

Example.—The armature of a four-pole generator has a wave winding and comprises 200 inductors. It is rotated at a speed of 1,200 r.p.m. in a field of 3,000,000 lines per pole. What e.m.f. is induced in this armature? *Solution.*—There are always 2 conducting paths through a wave winding (Art. 635). Substituting in formula (146),

$$E = \frac{P \times \phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60 \times m} = \frac{4 \times 3{,}000{,}000 \times 200 \times 1{,}200}{100{,}000{,}000 \times 60 \times 2} = 240 \text{ volts}$$

Note that the e.m.f. with this wave or two-circuit series winding is twice that of the similar generator of the preceding example with the lap winding.

641. Armature resistance is the resistance of an armature as measured between brushes of opposite polarity. The *brush contact resistance* (Art. 642) is not included in armature resistance. The conductors of an armature constitute a divided circuit (Art. 218) as shown in Fig. 358. Hence, the resistance of

an armature may be computed by treating it as a divided circuit In an armature for a two-pole generator there are two paths in parallel. In the armature of a four-pole generator (lap winding only) there are four paths in parallel, and so on.

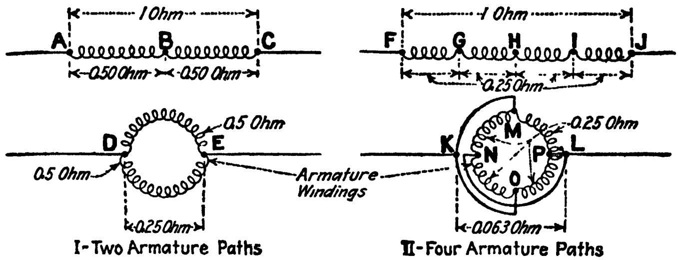


FIG. 358.—Illustrating armature resistance.

Example.—If a coil of wire AC (Fig. 358,I) having a resistance of 1 ohm is formed into a winding on a bipolar-machine armature, the resistance of each of the two paths will be $1 \div 2 = 0.5$ ohm. Then the resistance of these two paths in parallel—there are two paths in parallel through a bipolar-machine armature—would be $0.5 \div 2 = 0.25$ ohm. Hence the armature resistance of this armature is 0.25 ohm. Similarly, if this 1-ohm coil is formed into the winding for a four-pole armature, as at II, the armature resistance from the positive brush terminal K to the negative brush terminal L would be 0.063 ohm.

Example.—If the armature resistance of a four-pole generator is 0.20 ohm, what would be the resistance of the conductor comprising it if it were in one continuous length, like FJ (Fig. 358,II)? *Solution.*—Since there are four paths in parallel, each section of conductor constituting one of these paths must have a resistance of $0.20 \times 4 = 0.80$ ohm. Then these four sections, if arranged in series, would have a resistance of $0.80 \times 4 = 3.2$ ohms.

642. Brush resistance and brush-contact resistance are always present. The brush resistance is the resistance of the brush itself and obviously varies with the volume and shape of brush, just as does the resistance of any conductor. Brush-contact resistance is the resistance which occurs at the plane of contact between brush and segment. The drop in voltage due to the combined effects of brush and brush-contact resistance may be about 1 or 2 volts in average machines under normal load.

Example.— Experience indicates that the brush resistance and brush-contact resistance combined will vary from about 0.2 to 0.4 ohm per sq. in. of each brush contact. The resistance varies with the current density through the brush and contact and with the circumferential speed of the commutator.

643. Terminal E.m.f. The brush e.m.f. of a generator is the e.m.f. that the machine impresses on its brushes as distinguished from the e.m.f. induced in the armature, which is the armature e.m.f. The brush e.m.f. may differ from the armature e.m.f. because of $I \times R$ drop (Art. 151) in the armature winding. When the generator is not carrying load, the brush e.m.f. and the armature e.m.f. will be practically the same. The terminal e.m.f. is that which the machine impresses on the line. In a shunt-wound generator the brush e.m.f. is the same as the terminal e.m.f. In compound- and series-wound machines the terminal e.m.f. will be slightly lower, at any instant, than the brush e.m.f. because of IR drop in the series windings.

644. The e.m.f. impressed on an external circuit by a generator, that is, the terminal e.m.f., is the same as that induced only when there is no current in the armature, that is, only when all external paths from the brushes are open. When there is a current in the armature there will always be an internal IR drop (Art. 151) in voltage in it. The drop is due to the armature resistance (Art. 641) and the brush and brush-contact resistances (Art. 642). Numerically this internal drop, in volts will be equal to (current in amperes) $\times$ (ohms armature resistance + ohms brush and brush-contact resistance). The e.m.f. impressed on the line will be smaller than the induced e.m.f. by the amount of this $I \times R$ drop.

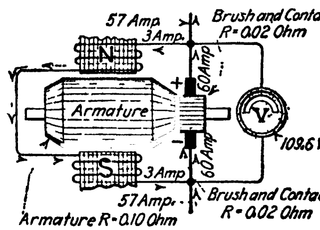


FIG. 359.—Effect of armature resistance on terminal e.m.f.

Example.—Assume the armature of Fig. 359 to be that of the 1,500 r.p.m. bipolar generator above (Art. 639). Then the e.m.f. induced by the armature is 118 volts If the armature resistance is 0.1 ohm and the brush and contact resistance for each brush is 0.02 ohm, what will be the terminal e.m.f. with a current of 60 amp. in the armature? *Solution.*—The drop in the armature will be 0.1 ohm $\times$ 60 amp. = 6.0 volts. The drop due to brush and contact resistance will be 2 $\times$ 0.02 ohm $\times$ 60 amp. = 2.4 volts. Then the total drop is 6.0 volts + 2.4 volts = 8.4 volts. Hence the terminal voltage at this load will be 118 volts − 8.4 volts = 109.6 volts.

645. The Resistance of the Entire Circuit through Which Its E.m.f. Impels Current Determines the Current in an Armature.—

This follows from Ohm's law (Arts. 151 and 565). The e.m.f. induced in the armature forces the current to circulate. The resistance determines the rate of flow or amperes. Consider an example:

Example.—The armature of Fig. 360 is inducing an e.m.f. of 125 volts. It is impelling a current through a circuit comprising the following components: (1) Armature winding, 0.04 ohm. (2) Two brushes and contact

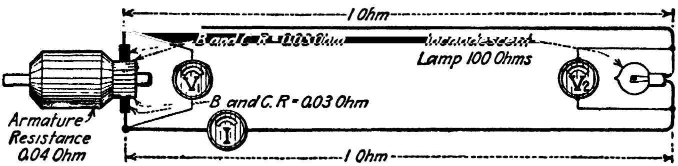


FIG. 360.—Illustrating how the resistance in the circuit determines the current in the armature of the generator.

surfaces, 0.03 ohm each. (3) Two line wires, 1 ohm each. (4) Incandescent lamp, 100 ohms. What is the current in the circuit? *Solution.*—The total resistance in circuit is:

Armature..................................	0.04 ohm
Two brushes @ 0.03 ohm each...............	0.06 ohm
Two line wires @ 1 ohm each................	2.00 ohms
One incandescent lamp......................	100.00 ohms
Total resistance in circuit..................	102 10 ohms

Then, applying Ohm's law: $I = E \div R = 125$ volts $\div$ 102.1 ohms $= 1.22$ amp., which is the current I in the armature, line wires, and lamp.

Example.—The voltage at V_1 or the terminal voltage in the above problem would be the induced e.m.f. less the armature and brush resistance. Then $V_1 = 125$ volts $-$ [1.22 amp. $\times$ (0.04 ohm $+$ 0.06 ohm)] $= 125$ volts $-$ 0.12 volt $=$ 124.88 volts. Hence voltmeter V_1 should read 124.88 volts.

Example.—The voltage V_2 at the lamp would be the voltage V_1 less the IR drop in the line wires. Or $V_2 =$ 124.88 volts $-$ (1.22 amp. $\times$ 2 ohms) $=$ 124.88 volts $-$ 2.44 volts $=$ 122.44 volts. Hence V_2 would read 122.44 volts.

646. Capacity of a Direct-current Generator.—If the prime mover driving a generator has ample power, the current which the machine will impel may be materially increased over that for which the machine is rated. However, such overloading is possible only to a certain extent, because the capacity of a machine is limited by one or all of three considerations:

1. Excessive sparking at the commutator. If the current through the brushes is much greater than that which they were designed to carry, they

will probably spark badly, and they may flash over, that is, an arc may form between brushes of opposite polarity.

2. Excessive internal drop or excessive $I \times R$ voltage drop in the armature. Since $I \times R$ drop in the armature decreases the terminal voltage of the generator, if the armature current is too great, the voltage which the machine impresses on its line terminals will become so low that it will not impel a sufficiently large current through the devices connected to the external circuit to operate them properly; the speeds of the motors will become too low, and the incandescent lamps will become too dim.

3. Excessive heating. The heating varies directly as the square of the current (Art. 298). If the current in a machine is too great, its conductors will become so hot that their insulations will be injured or burnt out.

NOTE.—Permissible temperature rises in electrical machinery are determined by the temperatures the insulating materials will safely withstand, since the other materials in a machine are metals which may be subjected to very high temperatures without damage. Ordinary combustible insulations such as silk, cotton, paper, and the like should never be subjected to temperatures greater than that of the boiling point of water—100°C. (212°F.). Mica, asbestos, and the noncombustible materials used for insulation may, without injury, be subjected to temperatures as great as 125°C. (257°F.).

Generator ratings are now universally based upon the standards established by the National Electrical Manufacturers Association.[1] Temperature ratings are based on a standard ambient (surrounding) temperature of 40°C. (104°F.) when the cooling medium is air. Direct-current generators (exclusive of engine type) will operate at full load continuously with a temperature rise not exceeding 40°C. in the windings and 55°C. on the commutator. Direct-current engine-driven generators are based on constant operation at full load with the above indicated temperature rises, and they will carry 1.25 per cent full load for 2 hr., with a rise not exceeding 40°C. in the windings and 55°C. on the commutator.

647. The rating of a direct-current generator is the kilowatts power (Art. 181) load that the machine will carry continuously without excessive (1) heating, (2) sparking, or (3) internal voltage drop, Art. 644. If a load greater than that for which it is rated is imposed on a machine for an extended period, it will probably give trouble, through one of the three above-mentioned causes. Nearly all generators are so designed and rated that they will carry some overload for an hour or so without injury. Every constant-potential (Art. 581) direct-current generator is designed to maintain practically constant some certain terminal voltage when operated at its rated speed. It follows that the current circulated by the machine will vary directly with the kilowatts load on the machine—and inversely as the resistance

[1] "NEMA Motor and Generator Standards."

of the external circuit. Because of the electrical losses in the machine—$I^2 \times R$ (Art. 189), eddy-current (Art. 541), and hysteresis losses (Art. 327)—a generator always produces more power than it delivers to the external circuit [see "Efficiency" (Art. 197)].

Example.—An 80-kw., 220-volt, 1,800-r.p.m. generator is one which will, when its armature is rotated at 1,800 r.p.m., maintain 220 volts at its line terminals if the connected load does not greatly exceed 80 kw. This means that this machine can continuously and satisfactorily circulate a current of 80,000 watts ÷ 220 volts = 364 amp.

648. The losses in a direct-current generator—or in any generator—may be divided into: (1) mechanical losses and (2) electrical losses. Furthermore, the electrical losses may be broadly subdivided into iron losses and copper losses.

1. The mechanical losses include journal or bearing friction, brush friction—the friction of the brushes on the rotating armature—and windage. Windage is the friction of the rotating armature on the air in which it is moving. These mechanical losses may vary with the speed of the machine, but the rotational speed of a constant-potential generator is practically constant. It follows that mechanical losses are practically constant at all loads. A portion of the power input to the machine is consumed in supplying these losses, and its net output is decreased accordingly.

2. The electrical losses.—Under iron losses may be grouped the hysteresis (Art. 325) and eddy-current (Art. 541) losses in the armature and field cores. Under copper losses may be grouped the $I^2 \times R$ losses (Art. 189) in the armature and field windings, the eddy-current losses in the copper conductors, and the losses in the controlling rheostats. Brush contact (Art. 642) $I^2 \times R$ loss is an electrical loss but is sometimes classed in with the copper losses. Most of the electrical losses increase with the load.

649. Determination of the Efficiency of a Direct-current Generator.—Efficiency (see Art. 197) is the ratio of the electrical power delivered by the generator to the mechanical power received by it. Hence, the efficiency of a generator may be expressed:

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} \text{ or } = \frac{\text{output}}{\text{input}} \qquad (147)$$

Probably the most convenient method of determining the efficiency of a generator consists in driving the generator (Fig. 361) with an electric motor of known efficiency. The electrical power output of the generator in watts can be readily ascertained

by multiplying the current and voltage readings obtained from an ammeter (I_G) and a voltmeter (E_G) connected in the generator circuit. The mechanical power input to the generator can be found by measuring the electrical input to the driving motor (by multiplying together the ammeter, I_M, and voltmeter, E_M, readings) and then multiplying this result by the motor efficiency at this load. Their product will be the electrical input to the

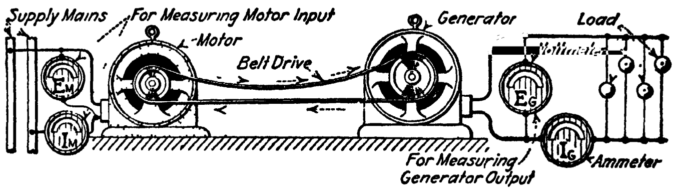


FIG. 361.—Illustrating method of determining the efficiency of a generator.

generator in watts, it being assumed that the belt losses are negligible, which they usually are in practical work. The efficiency will be the ratio of the watts output to the watts input.

Examples.—Large generators have higher efficiencies than small ones and the efficiency of any generator will vary with the load, because the losses (Art. 648) vary with the load. A good 5-kw. generator will have a full-load efficiency of about 82 per cent; a 100-kw. generator, 91 per cent; and a 1,000-kw. generator, 93.5 per cent. As an example of how efficiency varies with the load: For a 100-kw. machine the efficiency at full-load is 91 per cent; ¾ load, 90.5 per cent; ½ load, 89 per cent. See the author's "American Electricians' Handbook" for complete table showing full and partial load efficiencies for generators ranging from 5 kw. to 1,000 kw. in capacity.

650. The output of any direct-current generator is equal in watts (power output) to the product of the voltage and current delivered by the machine. This follows from the statements of Art. 186. That is,

$$P_W = E \times I \text{ (watts)} \tag{148}$$

hence

$$E = \frac{P_W}{I} \text{ (volts)} \tag{149}$$

and

$$I = \frac{P_W}{E} \text{ (amp.)} \tag{150}$$

Wherein P_W = power output of the generator, in watts.
E = e.m.f. impressed by the machine on its external circuit, in volts.
I = current, in amperes, flowing through the terminal lines of the machine.

It follows that

$$P_K = \frac{E \times I}{1,000} \text{ (kw.)} \tag{151}$$

hence

$$E = \frac{P_K \times 1,000}{I} \text{ (volts)} \tag{152}$$

and

$$I = \frac{P_K \times 1,000}{E} \text{ (amp.)} \tag{153}$$

Wherein the symbols have the same meanings as above except: P_K = power output of the generator in kilowatts.

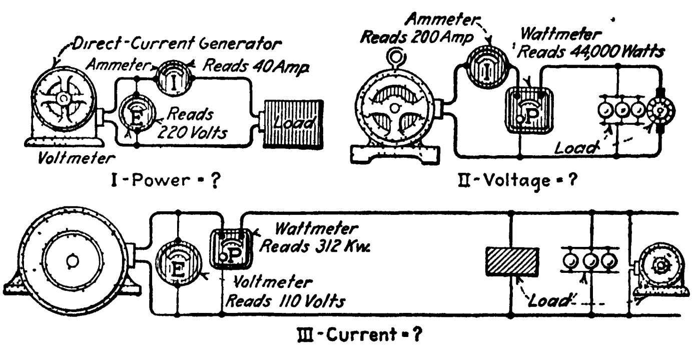


FIG. 362.—Examples in the computation of power output of direct-current generators.

Example.—If a voltmeter (Fig. 362,I) connected across the terminals of a direct-current generator reads 220 volts and an ammeter cut in the line reads 40 amp., the watts power output of the machine is $P_W = E \times I = 220 \times 40 = 8,800$ watts. In kilowatts the power output would be $P_K = (E \times I) \div 1,000 = (220 \times 40) \div 1,000 = 8,800 \div 1,000 = 8.8$ kw.

Example.—If a generator (Fig. 362,II) is developing 44,000 watts and the current through its terminals is 200 amp., the terminal voltage of the machine will be $E = P_W \div I = 44,000 \div 200 = 220$ volts. If the wattmeter reads 44 kw., then the voltage would be computed thus: $E = (P_K \times 1,000) \div I = (44 \times 1,000) \div 200 = 44,000 \div 200 = 220$ volts.

Example.—The current through the terminals of the direct-current generator of Fig. 362,III, which is developing 312 kw. at a pressure of 110 volts, would be $I = (P_K \times 1{,}000) \div E = (312 \times 1{,}000) \div 110 = 312{,}000 \div 110 = 2{,}840$ amp.

651. The output of a direct-current generator in horsepower equals its output in watts divided by 746 (Art. 188) or its output in kilowatts divided by 0.746. Thus

$$HP = \frac{P_W}{746} \text{ (hp.)} \tag{154}$$

and

$$P_W = HP \times 746 \text{ (watts)} \tag{155}$$

or

$$HP = \frac{P_K}{0.746} \text{ (hp.)} \tag{156}$$

and

$$P_K = HP \times 0.746 \text{ (kw.)} \tag{157}$$

Example.—If a generator is developing 2,000 watts, it is developing $HP = P_W \div 746 = 2{,}000 \div 746 = 2.68$ hp. Or if a generator is developing 85 kw., it is developing $HP = P_K \div 0.746 = 85 \div 0.746 = 114$ hp.

Example.—A generator developing 24 hp. is producing $P_W = HP \times 746 = 24 \times 746 = 17{,}900$ watts. Or this same generator is producing $P_K = HP \times 0.746 = 24 \times 0.746 = 17.9$ kw.

652. The power input of a generator, that is, the mechanical power required to drive the generator at any load is equal (Art. 649) to the power output of the generator at the given load divided by the efficiency of the generator at that load. The efficiencies of the different kinds of generators at different loads are tabulated in the author's "American Electricians' Handbook." Stated as a formula:

$$P_I = \frac{P_O}{\mathsf{E}} \text{ (input)} \tag{158}$$

or

$$\mathsf{E} = \frac{P_O}{P_I} \text{ (efficiency, decimal)} \tag{159}$$

and

$$P_O = P_I \times \mathsf{E} \text{ (output)} \tag{160}$$

Wherein P_I = power input to the generator.
P_O = power output of the generator.
E = efficiency of the generator, expressed as a percentage.

NOTE that P_I and P_O must always be expressed in the same units, which may be either watts, kilowatts, or horsepower

Example—If the power output of a generator is 90 kw and its efficiency at that output is 80 per cent, the corresponding input is $P_I = P_O \div E = 90 \div 0.80 = 112.6$ kw.

Where the electrical power output in kilowatts is known and it is desired to ascertain the mechanical power input in horse-

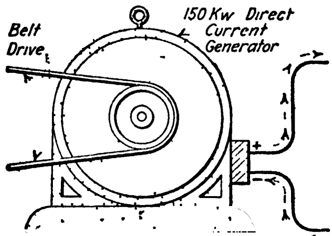


FIG 363—What power is necessary to drive generator?

power necessary to drive the machine at the given load, the following formula may be used:

$$HP_I = \frac{P_{OK} \times 100}{0.746 \times E} = \frac{P_{OK} \times 134}{E} \text{ (hp.)} \qquad (161)$$

or

$$P_{OK} = HP_I \times E \times 0.00746 \text{ (kw.)} \qquad (162)$$

and

$$E = \frac{P_{OK} \times 134}{HP_I} \text{ (efficiency, in per cent)} \qquad (163)$$

Wherein HP_I = power input, in horsepower, necessary to drive the generator at a given load.
P_{OK} = the given load output of the generator, in kilowatts.
E = efficiency of the generator at a given load output, in per cent.

Example.—What horsepower is required to drive the 150-kw. generator of Fig. 363, when it is developing its full rated load of 150 kw.? The

machine has a full-load efficiency of 91.5 per cent. *Solution.*—Substitute in the formula (161)

$$HP_I = \frac{P_{OK} \times 134}{\mathsf{E}} = \frac{150 \times 134}{91.5} = 219 \text{ hp.}$$

Example.—If the power input to a generator which has an efficiency of 80 per cent is 70 hp., the output of the machine will be (formula 162) $P_{OK} = HP_I \times \mathsf{E} \times 0.00746 = 70 \times 80 \times 0.00746 = 41.8$ kw.

Example.—If a generator is developing an output of 246 kw. and its power input is then 364 hp., what is its efficiency at this load? *Solution.*—Substitute in formula (163) $\mathsf{E} = (P_{OK} \times 134) \div HP_I = (246 \times 134) \div 364 = 32{,}964 \div 364 = 90.5$ per cent efficiency.

SECTION 36

DIRECT-CURRENT GENERATOR CHARACTERISTICS

653. Characteristic curves of generators are curves which show how the e.m.fs. developed by the machines vary with the load—that is with the current output. Figure 370, which follows, indicates typical examples for machines of the different classes. A characteristic curve of a machine offers a graphic statement of its peculiar qualities or traits. It is therefore obvious that a general familiarity with the typical characteristics of the generators of the different classes is valuable.

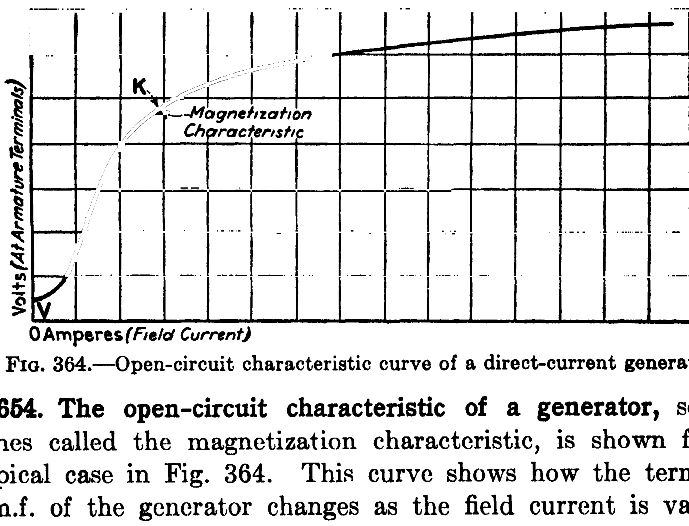


FIG. 364.—Open-circuit characteristic curve of a direct-current generator.

654. The open-circuit characteristic of a generator, sometimes called the magnetization characteristic, is shown for a typical case in Fig. 364. This curve shows how the terminal e.m.f. of the generator changes as the field current is varied. In taking data for plotting a curve like this, the field should be separately excited (Fig. 314). Current values, plotted horizontally, are the readings of an ammeter connected in the field circuit. Voltage values, plotted vertically, are the readings of a voltmeter connected across the armature terminals as the armature is rotated at the rated speed of the machine.

NOTE.—With zero current in the field, there is a small voltage OV developed. This is due to residual magnetism (Art. 593). As the field current

is increased, the voltage increases. For low field currents the voltage rises rapidly as the field current is increased. However, beyond the knee K of the curve a considerable increase in field current is required to effect a small increase in voltage. At about the point K the magnetic circuit becomes saturated (Art. 270). Obviously the open-circuit characteristic curve is merely a reproduction, in a rough way, of the magnetization curves of Fig. 162 which should be reviewed.

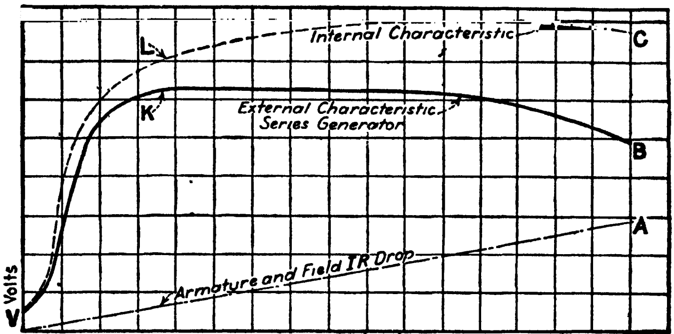


FIG. 365.—Characteristic curves of a series-wound generator.

655. The Characteristics of a Series Generator (review Art. 595 on the "Series Generator").—The curve is shown in Fig. 365. Figure 366 indicates how the instruments might be connected in the circuit to obtain data for plotting the curves; current and voltage values are plotted as in the preceding article. It is

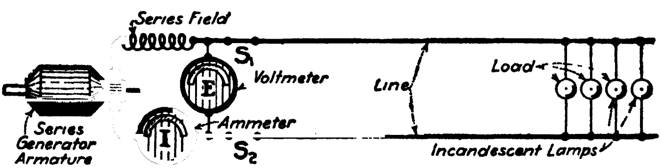


FIG. 366.—Series-wound generator with instruments for taking characteristic-curve data in circuit.

assumed that the armature is being rotated at constant speed. With the external circuit open at switches S_1 and S_2, there would be no field current, and the voltage impressed on the external circuit across S_1S_2 would be only (OV) due to residual magnetism. If now S_1 and S_2 be closed, a current will be circulated in the external circuit. As more load—lamps or other devices—is connected across the line, the external-circuit resistance is decreased. This permits a greater current to flow, which

strengthens the series field, effecting the induction of a greater e.m.f. The voltage impressed on the external circuit rises, as *VK*. However, with too great a current, the internal *IR* drop becomes excessive, and as the current is further increased through decreased external-circuit resistance, the terminal voltage will drop off as from *K* to *B*. The curve *VKB* is the external characteristic of the machine.

The curve *OA* shows the internal *IR* drop in the field coils and armature. The resistance of these members remains constant; hence the drop due to them increases directly with the load. This drop, in addition to that in the external circuit, must be overcome by the e.m.f. induced in the armature of the machine. Therefore, if the internal *IR* drop curve *OA* be added vertically to the external $I \times R$ drop curve (for that is what it is) *VKB*, their sum will be the curve *VLC*, which indicates the internal characteristic of the generator—that is, how the e.m.f. induced in the armature varies as the current in the circuit changes. The internal-characteristic curve of the machine would be identical with its magnetization curve were there no armature reaction (Art. 620).

If the resistance of an external circuit fed by a series generator is too low, an excessive current will circulate and the terminal e.m.f. of the machine will decrease excessively because of great internal *IR* drop and armature reaction. On the other hand, if the external-circuit resistance is too high, a current sufficient to magnetize the fields will not circulate, and hence the generator will not operate.

NOTE.—Series generators are sometimes called constant-current generators, not because they inherently produce constant currents but because they are inherently adapted for the development of the e.m f. for constant-current series circuits. See Art. 595 on the "Series Generator" and Art. 581 on "Constant-current Generators."

656. The Characteristics of the Shunt-wound Generator.—A typical characteristic curve is shown in Fig. 367. Figure 368 indicates how the measuring instruments should be connected for obtaining the data which is plotted into curves as described in preceding articles.

When a shunt-wound generator (Art. 596) is rotated at its rated constant speed, its field immediately builds up (Art. 593) even though switches S_2 and S_3 are open and there is no current

in the external circuit. This is shown in the curve. If the external circuit now be closed (by closing S_2 and S_3) the armature e.m.f. will circulate a current in it. As the external-circuit resistance is decreased the line current increases; hence the armature current increases. This causes a drop in the voltage

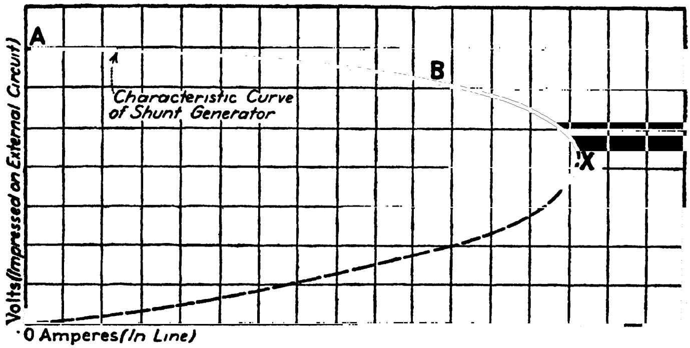


FIG. 367.—Characteristic curve of a shunt-wound generator.

impressed on the brushes because of three conditions: (1) The internal armature $I \times R$ drop (Art. 151) is increased. (2) The back ampere turns (Art. 623), causing armature demagnetization, are increased. (3) These two effects decrease the brush e.m.f., which decreases the field current, causing a still further lowering of the terminal or brush e.m.f.

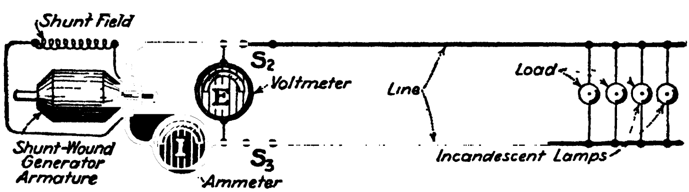


FIG. 368.—Shunt-wound generator with instruments for taking characteristic-curve data in circuit.

As the load on the generator is increased, the voltage impressed on the external circuit continues to drop, as shown in Fig. 367. If the external-circuit resistance is so greatly decreased that the line current exceeds the value X, the voltage decreases very rapidly, and the impressed e.m.f. falls to zero. This is because the field-core magnetization falls to a value below the knee of the curve (Art. 270) when the voltage falls to the value X so that, as the field-magnetization current decreases, the flux and e.m.f.

diminish very rapidly. That external-circuit resistance which is low enough to cause a shunt generator to demagnetize thus rapidly is the critical resistance for that machine.

NOTE.—The point x, Fig. 367, where the voltage of a generator drops off rapidly with increase in load, is beyond the operating range AB of the machine and hence is of theoretical interest only.

Examples.—A shunt-wound generator may be considered a constant e.m.f. generator and hence under certain conditions may be applied for the services outlined in the example under Art. 581. But although shunt-wound generators develop fairly constant terminal voltages even under varying loads, they do not operate so satisfactorily as compound-wound machines (Art. 597), nor do they produce as constant voltages. Shunt-wound generators have, in practice, been almost entirely superseded by the compound-wound machines.

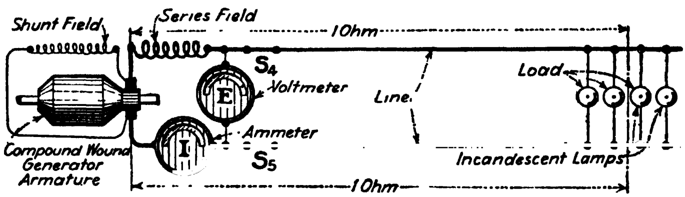


FIG. 369.—Compound-wound generator with instruments for taking characteristic-curve data in circuit.

657. Characteristics of the Compound-wound Generator.—As outlined in Art. 597, a compound-wound machine is a combination of a series and a shunt machine. Hence, its characteristic is the resultant of the characteristics of the shunt and series generator. Figure 369 shows the arrangement of instruments for taking the data for plotting the external characteristic curve and Fig. 370 delineates the curve. The dashed line shows how the terminal voltage impressed on the line would vary with the load if the machine had only its shunt-field winding.

If, however, a series-field winding of the proper number of turns be incorporated, then the terminal voltage will be as indicated by the full line. The dot-dashed line shows how that portion of the terminal voltage due to the series winding increases as the load increases. The height of the full-line curve above the horizontal reference line is at any point equal to the sum of the heights of the dashed and the dot-dashed curve. This series-winding curve rises very gradually (instead of abruptly, as does the series-generator curve of Fig. 365) because the series winding of a compound machine has relatively few—or the equivalent of

relatively few—turns. The magnetization due to the series winding can be so proportioned that it will compensate or more than compensate for the tendency toward decrease in voltage due to internal $I \times R$ drop (Art. 643) and armature reaction (Art. 620).

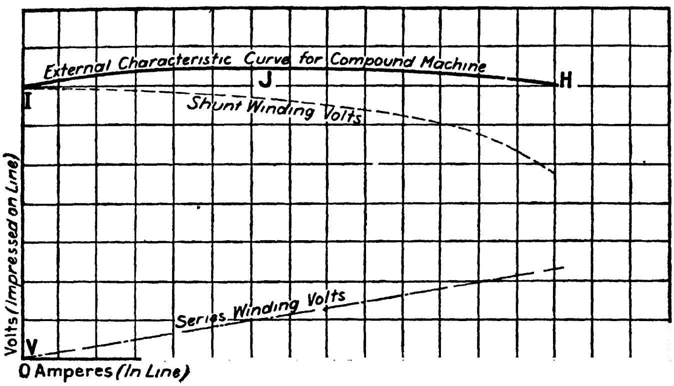


FIG. 370.—Characteristic curves of compound-wound generator.

Examples.—Compound-wound machines are now used almost invariably where a constant-e.m.f. direct-current generator is required. They are utilized in railway work and for electric lighting and power transmission where it is not necessary to distribute the energy over great distances. A compound-wound generator is a good example of a constant-e.m.f. (Art. 581) generator.

658. A flat-compounded generator is one having the number of its series turns so proportioned that the terminal voltage is maintained (without field rheostat regulation) practically constant at all loads within its range. Usually in these machines the full-load voltage H (Fig. 370) is the same as the no-load voltage I; and the voltage J at intermediate points is a trifle higher. It is difficult if not impossible so to proportion the field windings that the characteristic curve will be absolutely flat throughout the entire range of the machine.

659. An overcompounded generator is one having the number of its series turns so selected that its full-load voltage is greater than its no-load voltage. Overcompounding is necessary where it is desirable to maintain a practically constant voltage at some specified point out on the line distant from the generator. Overcompounding compensates for line drop. The

accepted standards[1] for overcompounding of generators provide for voltages as follows:

Generator	Terminal voltages	
	No load	Full load
General purpose generators...........	120	125
	240	250
	575	600
Generators for coal-mine service.......	250	275

Example.—Assume that each of the line wires of Fig. 369 has a resistance of 1 ohm, that each of the lamps constituting the load should have 100 volts impressed across its terminals, and that each lamp takes 1 amp. Then with one lamp burning the line drop would be 1 amp. × 2 ohms = 2 volts. Hence, with only one lamp burning, the generator terminal e.m.f. would have to be 100 + 2 = 102 volts. But with four lamps burning, the line drop would be 4 amp. × 2 ohms = 8 volts; with four lamps burning, the generator terminal e.m.f. would have to be 100 + 8 = 108 volts to ensure 100 volts across the lamps at the end of the line. Note that the generator e.m.f. must be increased as the load connected to it increases if the voltage at a given point out on the line is to be maintained constant. That is, the generator must be overcompounded. In the imaginary case just described, assuming that 4 amp. (or 4 amp. × 100 volts = 400 watts) is the rated capacity of the generator, it should be so overcompounded that its no-load terminal e.m.f. would be 100 volts and its full-load e.m.f. would be 108 volts.

660. Voltage regulation[2] of a direct-current generator is the final change in voltage with constant field rheostat setting when the specified load is reduced gradually to zero, expressed as a per cent of rated load voltage, the speed being kept constant. This regulation is usually stated by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads. Voltage regulation refers only to the changes in voltage that occur when the load on a generator changes gradually because of conditions within and brought about by the machine itself.

Example.—The regulation of a shunt-wound generator is poor (Fig. 367) because its terminal voltage drops off considerably at full load. The regu-

[1] "NEMA Motor and Generator Standards."

[2] The American Institute of Electrical Engineers, "American Standards for Rotating Electrical Machinery."

lation of a flat-compound generator (Fig. 370) is practically perfect because its terminal voltage is the same at full as at no load.

661. Voltage control of generators refers to changes in impressed voltage effected by attendants manipulating field rheostats or to changes effected by automatic control apparatus. Regulation (Art. 660) is due to the machine itself. Control is due to some external source.

Example.—The terminal voltage of a shunt or compound generator can be controlled by adjusting the shunt-field rheostat. If the resistance in the shunt-field circuit (Art. 596) is diminished, the shunt-field current increases and the terminal voltage is correspondingly increased; the reverse is also true. If the speed of the prime mover—engine, water wheel, or the like—changes, the terminal voltage of the generator it is driving will change. Where such speed changes are not too frequent the terminal voltage of the machine can frequently be maintained reasonably constant by suitable adjustments of the field rheostat. Automatic voltage regulators (the operation of which is explained in the author's "Central Stations") can be purchased which will maintain the voltage of a machine practically constant regardless of wide variations in speed and load.

SECTION 37

DIRECT-CURRENT MOTOR PRINCIPLES

662. An electric motor (Fig. 371) is a device for converting electrical energy into mechanical energy. Note that a motor is just the opposite of a generator as defined in Art. 546. A motor will supply mechanical power when a current due to some external source of e.m.f. is forced through it. The motion of a motor is due to the reaction between: (1) the current flowing in a set of conductors mounted on an armature; and (2) a magnetic field in which the conductors and their armature rotate. The motor exerts its mechanical effort or torque as a pull on a belt or thrust on a gear or a twisting force on a shaft.

663. How a direct-current motor converts electrical energy into a mechanical energy is explained in detail in articles which follow. Briefly the situation is this:

Explanation.—A direct-current motor is the same as a direct-current generator in construction. When a motor is connected to a source of e.m.f. —for example to a generator as in Fig. 372—the e.m.f. developed by the generator will impel a current through the motor armature and a current through the motor field windings. Certain electromagnetic reactions (Art. 469) then occur between the armature and the field which cause the motor armature to rotate and pull its load.

The motor will itself, in a way to be described (Art. 672), tend to govern the intensity of the current which flows through it. When pulling a small load, the current taken by a motor will be small. When pulling a large load, the current taken by a motor will be correspondingly large. That is, with the impressed e.m.f. remaining steady, the current taken at any instant will be almost exactly proportional to the load which the motor is pulling at that instant.

It has been shown that power in watts = volts × amperes (Art. 186). This holds true for a motor. The amperes input to the motor multiplied by the voltage at the motor terminals equals the electric power taken by the motor. The mechanical power delivered by the motor is somewhat smaller than the input power because there are certain losses in motors similar to those in generators (Art. 648). But if the motor-power input be multiplied by the efficiency (Arts. 197 and 716) of the motor, the product will be the power available at the motor pulley for doing mechanical work.

See following Arts. 715 to 717 for formulas and examples in further explanation of this principle.

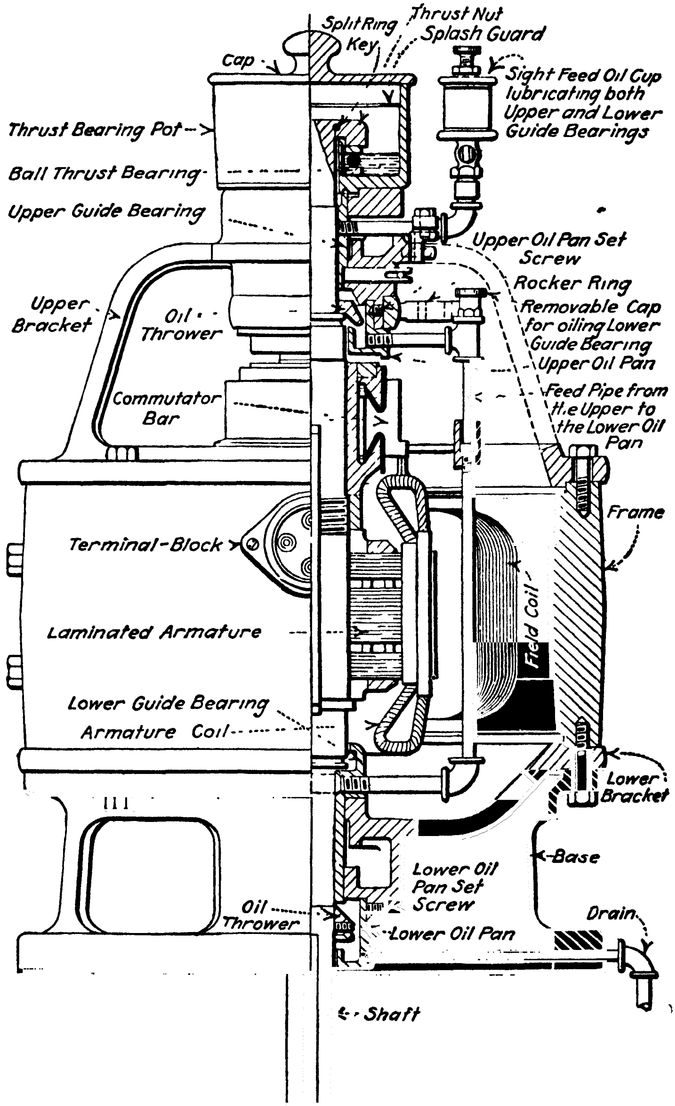


FIG. 371.—Typical sectional view of a vertical direct-current motor.

664. The principle of operation of an electric motor is based on the phenomena described in Art. 469, where it is shown that

there is always an electromagnetic force acting on any current-carrying conductor located in a magnetic field. This can be readily demonstrated with the simple apparatus shown in Fig. 374. Figures 233 and 373 picture this action on one conductor in a field. Now if a conducting loop like that of Fig. 375 be

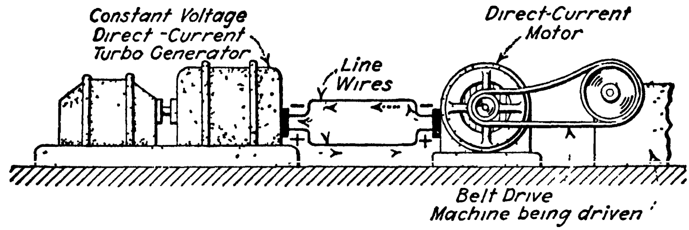


FIG. 372.—Illustrating the conversion of electrical energy into mechanical energy by the use of an electric motor.

mounted on an axis O (Fig. 376) in a magnetic field and current be forced through the loop, electromagnetic forces develop, tending to cause the loop to rotate.

Explanation.—As explained in Art. 469, when a conductor carrying current is placed in a magnetic field, the resultant field—that due to a combination of the original field and the field due to the current in the conductor—is stronger on one side of the conductor than on the other. This produces an excess of flux lines (field strength) on one side of the conductor and a deficit on the other side as shown in Fig. 376. The flux on one side of the conductor is distorted—bunched. The distorted flux lines, displaying their characteristic rubber-band-like tendency (Art. 65), endeavor to straighten out, thus forcing the current-carrying conductor to move in a direction at right angles to the field. Another way of stating this same fact is: The current-carrying conductor tends to move from the strong field to a weaker one.

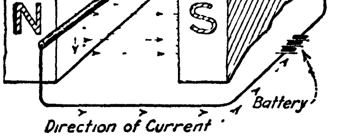


FIG. 373.—Illustrating action of a current-carrying conductor in a magnetic field. The conductor will be forced *up* out of the field by the magnetic reaction.

With a conducting loop like that of Figs. 375 and 376 in a field, a current in the loop will produce magnetic interactions, as indicated in the illustration, between each of the sides of the loop and the field. Both sides of the loop act in unison and tend to produce rotation of the loop around its axis O. The directions of the flux about the sides of the loop of Fig. 376 may be verified by applying the hand rule of Art. 462.

When the loop has rotated until it has assumed a position at right angles to the direction of the field, the tendency to cause its rotation becomes zero as shown in Fig. 377,I. However, in an actual motor, when a given loop rotates into the position of I, its commutator (Art. 566) reverses the current in it and the flux is then distorted somewhat as shown at III. With the

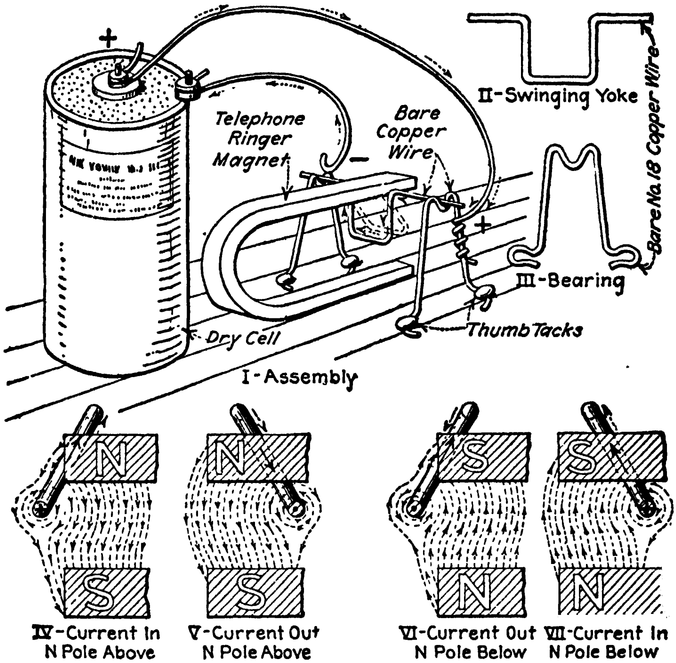


FIG. 374.—Showing how a current-carrying conductor is thrust from a magnetic field.

With the arrangement shown and one dry cell, the thrusting effect of a magnetic field on a current-carrying conductor can readily be demonstrated. Try it first as at I; the yoke will be forced from the field as at IV. Then reverse the direction of current through the swinging yoke and the yoke will be thrust in the opposite direction as at V. Now turn the magnet over and repeat the experiment as at VI and VII.

conditions as shown at III there would be no tendency for the loop to rotate in either direction. It would be on a dead center. But in actual motors there would be other loops on the armature in about the position of Fig. 376 which would force the loop of III to move in the direction of the arrow from the dead-center position; once displaced from this position, it would continue to rotate as hereinbefore described.

Another explanation (sometimes offered that does not, however, tell the entire story) which accounts for the rotation of a current-carrying loop in a magnetic field is this: Consider that each loop (Fig. 377,II) constitutes an electromagnet (Art. 236) having at one face a *N* pole and at its other

face a S pole. Then the N pole of the loop is always attracted by the S pole of the motor-field magnet and repelled by N pole of the field magnet. In Fig. 376,II, the S pole of the loop is attracted by the N pole of the field magnet and repelled by the S pole of the field magnet; thus the loop is forced into the position of Fig. 377,I.

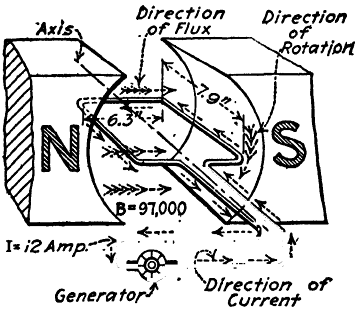


FIG. 375.—Illustrating action on a current-carrying loop in a magnetic field. Loop will rotate in clockwise direction. Right-hand side will go down and left-hand side will go up.

It would remain held in this position, but at this instant the commutator (Art. 566) reverses the current through the loop as shown in Fig. 377,III. Now the S pole of the loop is repelled by the S pole of the field magnet and attracted by the N pole of the field magnet, insuring a continuation of the rotation. This explanation follows from Maxwell's law (Art. 232) which states that: Any current-carrying coil will tend to assume such a position that the flux enclosed by it will be a maximum and all in the same direction.

A simple electric motor (Fig. 378) which will operate on a dry cell can be constructed in a few minutes. This little machine will afford a splendid demonstration of the principles hereinbefore outlined.

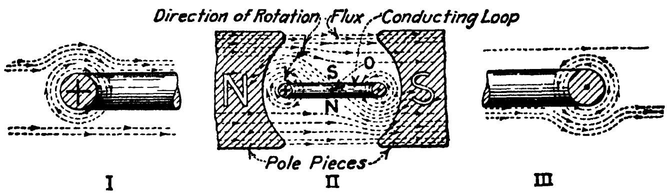


FIG. 376.—Showing how flux distortion tends to produce rotation of a loop, which carries current, located in a magnetic field.

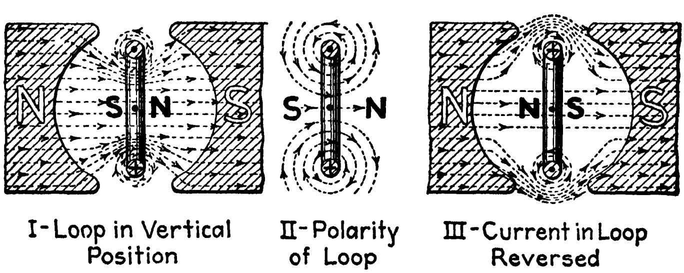


FIG. 377.—Flux about a loop in a magnetic field through which current is being forced.

665. Any Electric Generator Will Operate as a Motor.—It follows therefore that the constructional details of motors are in

general the same as those of generators as described in preceding sections. For certain special motor applications, such as those in railway, mining, and hoisting work, it is necessary that very compact and sturdy construction be used and that the motors be of the enclosed type to insure against the entrance of dirt Such motors are different in mechanical construction from ordi-

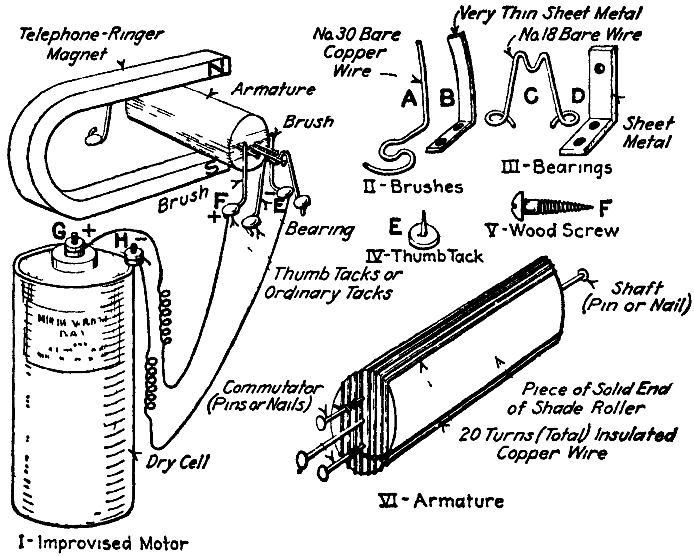


FIG 378—Showing how a demonstration electric motor can be made. This experimental motor can be easily made from materials usually available. It will operate on one dry cell It demonstrates the fundamental principles. Reverse the direction of current through armature, by transposing the connecting wires at E and F or at G and H and note that the direction of rotation of the armature is thereby changed Reverse the direction of the field, by turning the magnet upside-down and note that this will also reverse the direction of rotation.

nary generators, but electrically they are essentially the same as generators and could be used as such. Many motors have two-circuit or series-armature windings (Art. 635) which require only two sets of brushes regardless of the number of poles This type is used in railway and automobile service since it permits the two sets of brushes to be located in readily accessible positions.

666. The force exerted on a current-carrying conductor in a uniform magnetic field (at right angles to the direction of the field) which tends to move the conductor in the direction indicated by the left-hand rule of Art. 550, may be computed:

$$F_g = 0.000{,}62 \times I \times L \times B \text{ (grains)} \qquad (164)$$

or

$$F_{lb.} = 0.000{,}000{,}088 \times I \times L \times B = \frac{I \times L \times B}{11{,}300{,}000} \text{ (lb.)} \qquad (165)$$

$$F_{lb.} = \frac{8.85 \times I \times L \times B}{100{,}000{,}000} \text{ (pounds)} \qquad (166)$$

Wherein F_g = force, in grains, exerted on the conductor.
$F_{lb.}$ = force, in pounds, exerted on the conductor.
I = current flowing in the conductor, in amperes.
L = length of the conductor, in inches, in the uniform magnetic field, at right angles to the direction of the field.
B = flux density of the magnetic field, in lines per square inch.

(Note that there are 7,000 grains in 1 lb.).

Example.—What force, in grains, is exerted on the conductor shown in Fig. 373? The current in the conductor is 40 amp., the flux density is 129,000 lines per sq. in., and the length of the conductor in the field is $5\frac{3}{4}$ in. *Solution.*—The conductor in this case would be forced down (hand rule, Art. 462 and Fig. 275) and with a force of $F_g = 0.000{,}62 \times I \times L \times B = 0.00062 \times 40 \times 5.75 \times 129{,}000 = 18{,}400$ grains.

The force exerted, in pounds, on the conductor would be $F_{lb.} = 0.000{,}000{,}088 \times I \times L \times B = 0.000{,}000{,}088 \times 40 \times 5.75 \times 129{,}000 = 2.6$ lb.

Example.—What is the torque (Art. 194) or tendency to rotate developed by the loop in Fig. 375? Each side of the loop is 7.9 in. long. The diameter of the loop is 6.3 in. The flux density of the field is 97,000 lines per sq. in. and the current flowing in the loop is 12 amp. *Solution.*—Substitute in formula (165) to determine the force exerted on each side of the loop

$$F_{lb} = \frac{I \times L \times B}{11{,}300{,}000} = \frac{12 \times 7.9 \times 97}{11{,}300} = 0.82 \text{ lb.}$$

This force is exerted on each side of the loop; hence the force exerted on both sides of the loop is $2 \times 0.82 = 1.64$ lb. The force is exerted 3.2 in. $\div$ 12 = 0.26 ft. from the axis. Hence the torque is 0.26 ft. $\times$ 1.64 lb. = 0.43 lb.-ft. torque.

667. The total torque exerted by all of the conductors on a motor armature (Fig. 114) may readily be ascertained by using formula (167). The force acting on *one* current-carrying conductor tending to move it in a magnetic field may be computed as suggested in Art. 666. When a number of conductors are arranged into an armature winding, and current is forced

through the winding by an external e.m.f., a magnetic force is exerted by the field on each conductor. The forces on all of the conductors act in unison and thus produce a torque (Art. 194) on the armature shaft. This total torque is—with a given current flowing—independent of the speed of the armature.

$$T = \frac{P \times \phi_P \times C_T \times I_A}{85{,}200{,}000 \times m} \text{ (lb. at 1 ft. radius)} \qquad (167)$$

or

$$T = 0.000{,}000{,}01175 \times P \times \phi_P \times C_T \times I_A \div m \qquad (168)$$

The above equation may be often written conveniently as, $T =$ *constant* $\times \phi \times I_A$. The value of the constant will be determined by the construction of motor, but will always remain the same value for any given motor.

wherein T = total torque exerted by the armature, in pounds at 1 ft. radius.

P = number of poles of motor.

ϕ_P = total useful flux per pole, i.e., number of lines of force per pole that cut armature conductors.

C_T = number of active inductors on the armature.

I_A = total current, in amperes, forced through the armature from the line.

m = number of armature paths between the brushes.

Example —What would be the torque exerted by a four-pole motor armature having 80 armature conductors with a current of 20 amp in the armature circuit? The flux per pole is 400,000 lines. The armature has a multiple winding *Solution* —In a multiple-wound armature there are as many armature paths as poles; hence in this case there are four paths. Now, substituting in formula (167), $T = (P \times \phi_P \times C_T \times I) \div (85{,}200{,}000 \times m) = (4 \times 400{,}000 \times 80 \times 20) \div (85{,}200{,}000 \times 4) = 7.5$ lb.-ft. torque.

668. Counter e.m.f. is a phenomenon which occurs in electric motors and which should be thoroughly understood. When any conductor cuts flux there is an e.m.f. induced in the conductor, Art. 451. This is just as true of the conductors of a machine when it is operating as a motor as when it is running as a generator, because in both cases the armature conductors are rotating in a magnetic field. Furthermore, the e.m.f. induced in the armature conductors in a motor is in the same direction as that induced in the machine if operated as a generator. But this

e.m.f. induced in a motor armature is in a direction opposing (but is never so great as) the e.m.f. which is impressed on the armature and which causes it to rotate. The hand rules (Figs. 227 and 228) confirm this statement. It is also in conformity with Lenz's law (Art. 470). Since this induced e.m.f. is in a direction opposing the impressed e.m.f. and since it tends to impel a current in a direction opposite to that of the current which causes the motor armature to turn, it is termed the counter or back e.m.f. of a motor.

669. The effect of counter e.m.f. is to limit the current in a motor armature. The resistance of any motor armature is always very small. Frequently it is much less than an ohm. Hence, it is evident that if the normal line e.m.f. is impressed directly on an armature of a motor, an excessive current will flow unless there is something to limit the current to a reasonably safe value. As will be shown, resistance is inserted in series with an armature to limit the current while the armature is being started (Art. 670). The counter e.m.f. developed by the armature limits the current after the armature has commenced rotating. Obviously, when a motor armature is at rest the counter e.m.f. developed by it is zero. When the armature starts to turn it commences to induce a counter e.m.f. and as it speeds up the counter e.m.f. increases.

When an armature is at rest the intensity of the current that will flow in the armature is determined solely by Ohm's law ($I = E \div R$) by the impressed voltage and the resistance in ohms of the armature as shown in the following example. When the armature is rotating, the counter e.m.f. induced, since it is in a direction opposite to that of the impressed e.m.f., has the effect of decreasing the effectual impressed e.m.f. That is, the e.m.f. which is actually effectual in forcing current through the armature is the difference between the impressed e.m.f. and the counter e.m.f. As shown in the following example, this difference—that is, the effectual e.m.f.—may be but a few volts, even when the impressed e.m.f. is several hundred volts.

NOTE.—The action of motor counter e.m.f. may be better understood by a consideration of Fig. 379, where a 100-volt generator is shown, diagrammatically, forcing current through a rotating armature.. It is assumed that the generator and motor are very close together and that the connecting leads are large so there is practically no voltage drop in them. The voltage

impressed on the armature is, then, as represented by the length of the heavy dashed line, 100 volts. The counter e.m.f. induced by the rotating armature is (dot-and-dashed line) 90 volts and is in the opposite direction to the impressed e.m.f. Then obviously the effectual e.m.f., the pressure available to force current through the armature, is 100 volts − 90 volts = 10 volts as represented by the short black line.

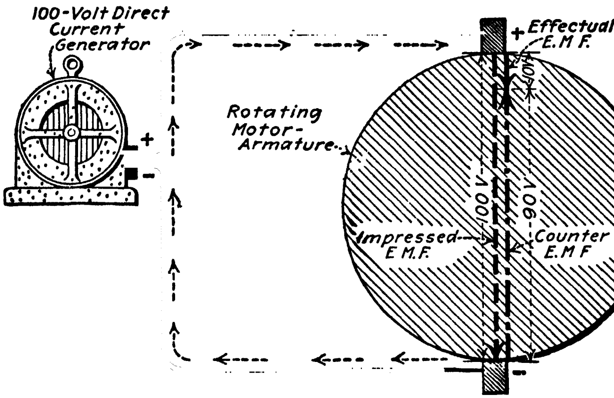


Fig. 379.—Illustrating impressed, counter, and effectual e.m.fs. of a motor.

Example.—A 500-volt motor armature (Fig. 380) has a resistance of 0.2 ohm. (1) What current will flow in the armature if it is restrained from rotating and the rated voltage of the machine, 500 volts, is impressed across it? (2) If when rotating at its full-load speed the armature induces a counter e.m.f. of 490 volts, what current will then be forced through the armature? *Solution.*—(1) With the armature at rest its resistance alone limits the current through the armature and in this case the current would

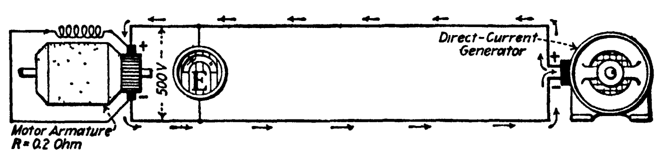


Fig. 380.—Direct-current generator forcing current through a direct-current motor armature.

then be $I = E \div R$ = 500 volts ÷ 0.2 ohm = 2,500 amp. (2) With the armature inducing a counter e.m.f. of 490 volts, the effectual e.m.f. would be 500 volts − 490 volts = 10 volts. Then the current through the armature would be 10 volts ÷ 0.2 ohm = 50 amp.

670. The function of a starting resistance (this matter is more fully treated in Art. 692) is to limit the armature current while the motor is being started and before it has attained its rated speed. Such resistances are usually arranged in series

with motor armatures about as shown in Fig. 381. When the motor is connected to the line, all of the resistance is in series with the armature. The resistance may be great enough to limit the starting current to any desirable value. However, as the armature attains speed, the resistance is cut out, step by step, until, when the armature is running at the speed at which it was designed to operate, all of the starting resistance will be cut out of the circuit and the armature will then be connected directly across the line.

671. The Counter E.m.f. Induced in Any Direct-current Motor Armature Is the E.m.f. Which the Same Armature Would Develop If Operated in a Generator at the Same Speed and in the Same Flux.—It follows from this that the formulas of Arts. 639 and 640 may be applied for computing the counter e.m.f. of a motor. Thus, the e.m.f. induced in the armature of any direct-current motor may be computed by using the following formula, which is identical with the generator formula of Art. 640

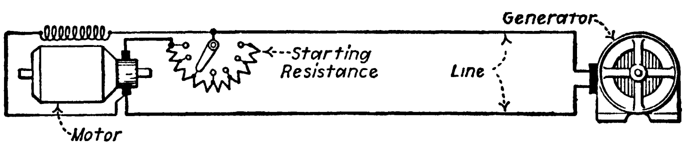


FIG. 381.—Showing a motor-starting resistance.

$$E_B = \frac{P \times \phi_P \times C_T \times \text{r.p.m.}}{10^8 \times 60 \times m} \text{ (volts)} \qquad (169)$$

Wherein E_B = the counter or back e.m.f. induced in the motor armature, in volts.

P = number of field poles of the motor.

ϕ_B = the flux or number of lines of force per pole entering or leaving the armature.

C_T = the total number of inductors (Art. 578) on the armature.

m = the number of parallel conducting paths between positive and negative brush sets (Art. 635).

r.p.m. = the number of revolutions per minute of the armature.

672. The speed of any direct-current motor is, at any instant, determined largely by the counter e.m.f. which it develops. The tendency of a direct-current motor is always to operate at

a speed such that the sum of the armature IR_A drop and the counter e.m.f. will just equal the impressed e.m.f. (Art. 669). At light loads the current, I, will be small. Armature resistances are small. Hence, at light loads, the armature drop, $I \times R_A$, will be small. Therefore, at light loads, E_B will be large and may almost (but can never quite) equal the impressed e.m.f., E. Therefore, when a motor is pulling a light load its armature will tend to rotate at a higher speed than when it is pulling a heavy load. How the speeds of series-, shunt-, and compound-wound motors vary, more or less, with their loads in accordance with this principle is discussed in Arts. 685, 701, and 708.

673. The essential formulas relating to counter e.m.f. are, it follows from the preceding discussion,

$$I_A = \frac{E - E_B}{R_A} \text{ (amp.)} \tag{170}$$

and, transposing,

$$E_B = E - (I_A \times R_A) \text{ (volts)} \tag{171}$$

and

$$E = E_B + (I_A \times R_A) \text{ (volts)} \tag{172}$$

Wherein I_A = current, in amperes, through motor armature.
E = e.m.f., in volts, impressed on the motor brushes.
E_B = counter or back e.m.f., in volts, induced in the motor armature.
R_A = armature resistance (Art. 641) of motor, in ohms.

Example.—What will be the current in a motor armature that has a resistance of 0.6 ohm, if it is inducing a back e.m.f. of 95 volts and the impressed e.m.f. is 100 volts? *Solution.*—Substitute in formula (170) $I_A = (E - E_B) \div R_A = (100 - 95) \div 0.6 = 5 \div 0.6 = 8.3$ amp.

Example.—What counter or back e.m.f. is being induced in a motor operating on an impressed e.m.f. of 220 volts and having an armature resistance of 0.3 ohm when the armature current is 20 amp.? *Solution.*—Substitute in formula (171) $E_B = E - (I_A \times R_A) = 220 - (20 \times 0.3) = 220 - 6 = 214$ volts.

674. The relations between the counter e.m.f. of a motor and the power developed by it are shown by the following formulas. As stated in Art. 673, equation (170), $I_A = (E - E_B) \div R_A$. Hence

$$I_A \times R_A = E - E_B \tag{173}$$

Now if every term of the equation be multiplied by I_A

$$I_A^2 \times R_A = (E \times I_A) - (E_B \times I_A) \qquad (174)$$

therefore

$$I_A \times E = (I_A^2 \times R_A) + (E_B \times I_A) \qquad (175)$$

Equation (175) shows that the total power input to a motor, $I_A \times E$, comprises two components: (*a*) The $I_A^2R_A$ power loss in the armature winding due to the current flowing through the winding. (*b*) The power, E_BI_A, due to the counter e.m.f. of the armature. (The power required to excite the field windings and the power required for other losses are here disregarded.) The power, $I_A^2R_A$, wasted in the armature winding is a total loss. Hence the power available for driving the motor shaft must be E_BI_A; hence it may be written that

$$P_M = E_B \times I_A \text{ (watts)} \qquad (176)$$

Wherein P_M = the mechanical power, in watts, developed by the motor, including that available at the motor shaft for driving the load and that expended in friction, windage, eddy-current, and hysteresis (Art. 648) losses.

It is obvious, then, from formula (176) that the power developed by a direct-current motor is determined by two factors: (1) the armature current; (2) the counter e.m.f. If, then, the counter e.m.f. of a motor is increased, the current remaining constant, the power of the motor is increased. The power increases directly as the counter e.m.f.

675. The relation between the direction of rotation of a motor, the direction of flux, and the direction of e.m.f. is the same as for a generator (Art. 558) except that in the case of a motor it is the impressed e.m.f. which is usually considered. The impressed e.m.f. is opposite in direction to the e.m.f. induced if the machine were operated as a generator. Hence, the left hand, as shown in Fig. 275,II, is used in determining the relations of the directions for a motor. The direction of armature current in a motor is always in the direction of the impressed e.m.f. It follows that the direction of current in a motor is opposite to that of the current in the same machine if operated as a generator.

676. The direction of rotation of a motor and the method of reversing the direction of rotation can be ascertained by apply-

ing the left-hand rule of Fig. 275,II. If a machine operating as a generator is being rotated in a certain direction, clockwise for example, precisely the same machine will, if a current is forced through it making it operate as a motor, rotate in the same clockwise direction (Art. 558). That is, if a machine is to be operated as a motor, the current must be forced through its armature in a direction opposite to the direction in which the current flows in the armature when the machine is operating as a generator.

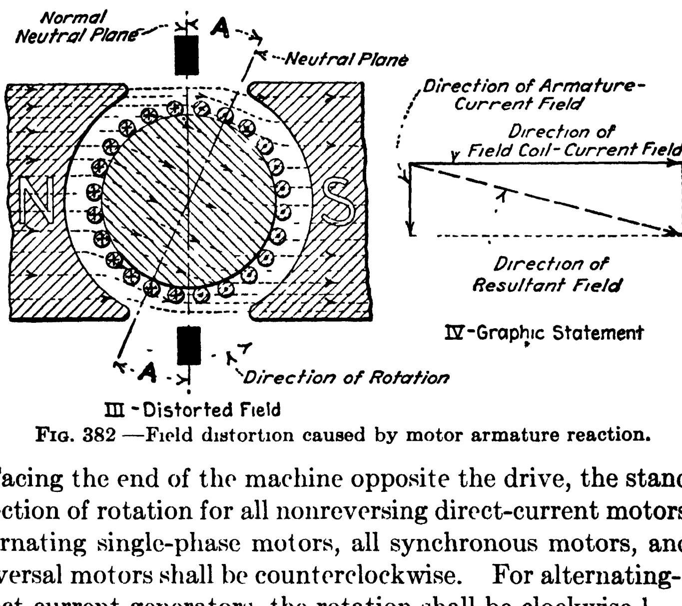


FIG. 382—Field distortion caused by motor armature reaction.

Facing the end of the machine opposite the drive, the standard direction of rotation for all nonreversing direct-current motors, all alternating single-phase motors, all synchronous motors, and all universal motors shall be counterclockwise. For alternating- and direct-current generators, the rotation shall be clockwise.[1]

NOTE.—A consideration of the hand rule of Art. 550 will indicate that the direction of rotation is determined by the relative directions of armature current and flux. If follows that to change the direction of rotation the direction of either the flux or of the armature current should be reversed. Hence, to reverse the rotational direction of a motor, reverse either the field-coil connections or the armature connections. Do not reverse both. Obviously, if both flux and armature current are reversed, their directions as related to one another remain the same and the change will have no effect on the rotational direction of the armature. Verify this statement by applying the left-hand rule (Fig. 275,II).

677. Commutation in motors involves the same principles as does commutation in generators (Art. 617). In this respect motor

[1] "NEMA Motor and Generator Standards."

commutation is similar to motor armature reaction. But with motor armature reaction the field distortion is back (Fig. 382), instead of ahead as in generators. Hence, in a motor the neutral plane is back of or behind the normal neutral plane (Art. 618). The brushes of a motor then are given a backward lead. The commutating plane of a motor is a trifle back of the neutral plane (Fig. 383), when the brushes are set in the plane of sparkless commutation (Art. 625). The motor brushes are thus located so that when any armature coil rotates into the location where commutation occurs there will then be no current flowing in the coil. That is, the brush short-circuits the coil (Fig. 383) before it rotates into the neutral location where it will cease to cut flux.

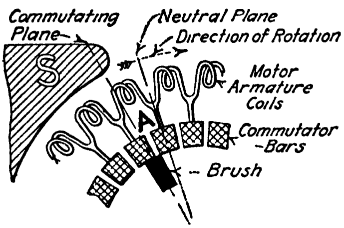


FIG. 383—Showing how the brushes of a motor should be shifted back of the neutral plane to promote sparkless commutation.

In a motor, the cutting of flux by an armature conductor induces a counter e.m.f. opposite in direction to the current which is flowing in the armature and which is causing the motor to rotate. Hence, if the brushes are shifted back of the neutral plane just the right distance, the counter e.m.f. induced by the armature conductors cutting flux will be just equal to the e.m.f. of self-induction developed when the short-circuited coil circuit (A, Fig. 383) is opened by the commutator bar B rotating from under the brush. As noted in Art. 501, the direction of an e.m.f. of self-induction is always such that it opposes any change. Therefore, in an armature coil, the e.m.f. of self-induction at the instant of commutation tends to maintain the current in the coil in the same direction as that in which it was flowing prior to the instant of commutation. For sparkless commutation, the motor counter e.m.f. should neutralize the e.m.f. of self-induction in the coil at the instant of commutation.

678. Commutating poles insure sparkless commutation in motors because their effect in a motor is essentially the same as in a generator and as described in Art. 628. Where motors are subject to extreme changes in load or to sudden reversal in rotational direction they are, if of ordinary construction, prone to spark. Furthermore, with motors of ordinary construction,

it is usually necessary to shift the brushes as the load changes, to insure minimum sparking. With well-designed commutating-pole motors sparking does not occur even when the motors are operating under heavy overloads or when they are suddenly reversed under load. It is not necessary or desirable to change the brush setting—the brush position—of a motor of this type after the best brush location has once been determined (see Art. 628). Figure 343 shows the appearance of a commutating-pole motor frame—the construction of the motors and generators of this type is practically identical.[1]

679. Armature reaction in motors is, as might be inferred from the fact that any generator will operate as a motor (Art. 665), similar to armature reaction in generators. However, the cur-

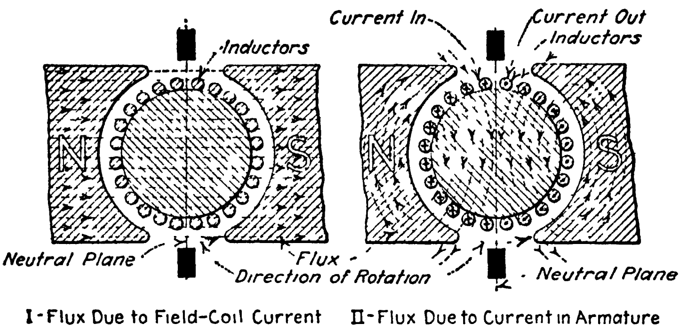


FIG. 384.—Showing the two fields which cause motor armature reaction and field distortion.

rent in a motor armature which is rotating, say clockwise, is in the opposite direction from the current in the armature of the same machine when it is operating as a generator and rotating clockwise. This renders motor armature reaction different in detail from generator armature reaction. However, the essential principles involved are those outlined under "Armature Reaction" in Art. 620.

Explanation.—Consider the generator armature which is shown diagrammatically in Fig. 340 and which is rotating counterclockwise: The current direction in the inductors on the right of the neutral plane is *in*, while that in the inductors on the left of the plane is *out*. Now if this same generator be operated as a motor by forcing a current through it, obviously the direction of current in the motor armature will be reversed. But the direc-

[1] For further information see Terrell Croft, "Electrical Machinery," McGraw-Hill Book Company, Inc.

tion of rotation of the armature will remain the same (hand rule, Fig. 275,II). Then, the current in those inductors on the motor armature on the right of the neutral plane will be *out* and that in the inductors on the left of the neutral plane will be *in*, as shown in Fig. 384,II, and in Fig. 382.

This reversal of direction of the armature current will magnetize the armature core in a direction opposite to the direction of magnetization of the generator armature core as shown in Fig. 384,II. Since the direction of the field-current field remains the same (Fig. 384,I) in a motor as in a generator the field of the motor will be distorted as shown in Fig. 382,III. Note that the field distortion in a motor is in the opposite direction to that in a generator. Then the neutral plane will be shifted *back* (against the direction of rotation), as shown in III.

The reader should compare carefully the illustrations showing generator armature reaction with those showing motor armature reaction. Note particularly that, in the diagrams illustrating these conditions which are given in this book, the direction of rotation of generator armature and of the motor armature is the same. With the generator the field is distorted *ahead;* in the motor the field is distorted *back*. Hence, the neutral plane is *ahead* of the normal neutral plane in a generator and *back* of the normal neutral plane in a motor.

680. A speed-torque classification of loads that may be driven by direct-current motors should be considered. Motors of the different types described herein (Art. 681) have different operating characteristics which determine their adaptabilities for different classes of work. The different kinds of loads which motors may be called upon to drive may be divided into three classifications thus:

1. Constant torque at variable speed.
2. Variable torque at constant speed.
3. Variable torque at variable speed.

1. Loads of constant torque at variable speed are those where the load-torque (Art. 194) is always practically constant, but where the speed at which the load is raised or driven must be varied. Cranes, hoists, and elevators offer loads of this character. Series motors (Art. 700) have characteristics which render them adaptable for driving such loads.

2. Loads of variable torque at constant speed are those imposed by machine tools such as lathes, planers, and saws, by line shafts, and by all other machinery the speeds of which must be maintained practically constant but the loads of which may be almost zero at one instant and considerable a few instants later or may vary more or less at different times. Shunt (Art. 684) and compound motors (Art. 708) are adapted for such services.

3. Loads of variable torque at variable speed are those such as imposed by railway traction loads. For example, in starting a car, the torque required is a maximum while the speed is a minimum. After the car has

been started the torque required decreases but the speed increases. Series motors (Art. 700) have characteristics which adapt them for such service.

681. Speed Characteristics. The three general types of motors, classified in accordance with the method whereby the field windings are connected, are the same as the three general types of generators, viz.: (1) shunt, (2) series, and (3) compound. Each of the three types is discussed in following articles. Just as the generators of the different types have different characteristics, the motors of the different types also have different characteristics. When one speaks of the characteristic of a generator it is ordinarily its voltage characteristic which is referred to, the speed of the machine remaining constant and the load varying. In speaking of the characteristic of a motor, it is the speed characteristic which is referred to, the impressed voltage remaining constant and the load varying. This follows because motors practically always operate from constant-voltage circuits while generators are practically always driven by constant-speed prime movers.

NOTE.—As will be shown, the speed characteristics of motors of the different types (shunt, series, or compound) are similar to the voltage characteristics of generators of the corresponding types. For example: A generator of a type which will maintain a practically constant voltage while the load which it is serving changes, its speed remaining constant, will, when operated as a motor, maintain a practically constant speed when its load varies, the impressed voltage remaining constant.

682. "Speed regulation" and "speed control" are two terms that have certain specific meanings just as do the terms "voltage regulation" and "voltage control" defined in Arts. 660 and 661 in connection with generators. Speed regulation refers to changes in speed caused by the interactions inherent to and within the motor itself as the load driven by it decreases or increases. Speed control refers to changes in speed effected by the hand or by the automatic manipulation of some device, usually external to the motor itself, whereby the speed of the motor is changed.

683. The speed regulation of direct-current constant-speed motors[1] is the change in speed when the load is reduced gradually from the rated value to zero with constant applied voltage and

[1] The American Institute of Electrical Engineers, "American Standards for Rotating Electrical Machinery."

field rheostat setting, expressed as a per cent of speed at rated load.

The regulation of a direct-current motor refers to the change between no-load speed and the speed obtained with unvarying load on the motor and does not relate to transient periods in which comparatively large fluctuations in speed accompany rapid changes in load.

The speed regulation of shunt-wound constant-speed continuous duty motors[1] from full load to no-load hot shall not exceed 12 per cent on motors $\frac{3}{4}$ to 5 hp., inclusive, and 10 per cent on larger motors, based on full-load speeds.

[1] "NEMA Motor and Generator Standards."

SECTION 38

THE SHUNT MOTOR

684. The shunt motor (Fig. 385) will be considered first because its characteristics adapt it to so many applications that it is used more frequently than are direct-current motors of any of the other types. Its most important characteristic is that with a constant (unvarying) impressed voltage it will maintain practically constant speed under wide variations in load. Why this is true will be explained. Commercial shunt motors are practically identical in construction with shunt generators, as described in Art. 596. Any shunt motor may be operated as a generator and vice versa. The characteristic curves of a shunt motor and of a series motor are shown side by side in Fig. 386 for comparison. In Fig. 387 the characteristic curves of a direct-current compound-wound motor are given.

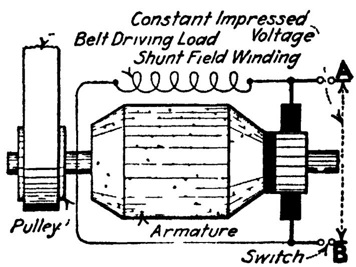


FIG. 385.—The elements of a shunt-wound motor

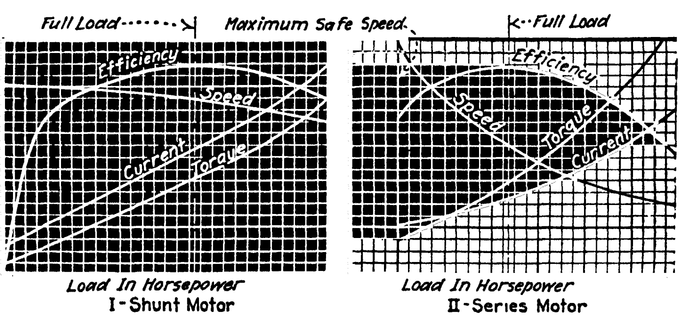


FIG. 386.—Characteristic curves of shunt-wound and series-wound motors.

685. The explanation of the speed regulation of the shunt motor, that is, why a shunt motor always maintains a practically constant speed even when its load varies widely, is this:

Explanation.—Consider the diagrammatic illustration of a shunt motor shown in Fig. 385, which indicates only the essentials. Assume that the switch is closed, impressing a constant voltage across AB. This will excite the field winding and its excitation current and field strength will remain constant as long as the impressed e.m.f. remains constant. Furthermore, current will be forced through the armature. (In practice, resistance is inserted in the armature circuit when a motor is started, as described in Art. 670, but for the present this feature will be disregarded.) Due to the interaction of the shunt-winding field and the field developed by the current in the armature conductors (Art. 664), the armature will start to rotate and it will speed up until it attains some certain speed such that the counter e.m.f. induced in it (Art. 669) plus the $I \times R_A$ drop in it just equals the impressed voltage. Obviously the speed thus attained will be determined by the design—field strength, number of armature conductors, etc.—of the motor in question.

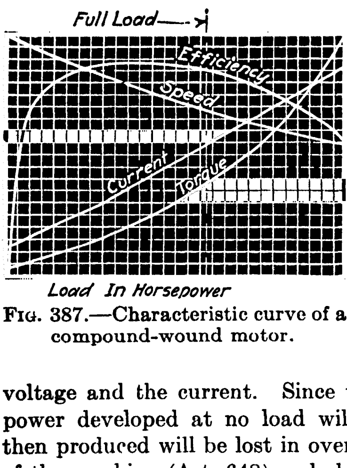


FIG. 387.—Characteristic curve of a compound-wound motor.

Practically, it may be assumed that the counter e.m.f. when the motor is operating at no load is equal to the impressed e.m.f. because, at no load, the $I \times R_A$ drop is very small. Hence, at no load, practically no current is forced through the motor by the voltage of the line. The power then developed by the motor, assuming for simplicity that it is 100 per cent efficient, will be, in watts (Art. 186), equal to the product of the impressed voltage and the current. Since the current at no load is very small, the power developed at no load will be correspondingly small. All that is then produced will be lost in overcoming the internal friction and windage of the machine (Art. 648) and electrical losses. None will be available at the motor shaft for pulling the load.

Now assume that the load which the motor is turning is increased considerably. The motor will slow down—not much, only a very little. The armature conductors will cut a smaller number of lines per second and hence will induce a smaller counter e.m.f. This will permit the forcing of the greater current, necessary to pull the load, from the line through the armature. However, as will be shown (Art. 686), a very slight decrease in armature speed (counter e.m.f.) allows a considerably greater current to flow through the armature, so that the motor will maintain practically the same speed when pulling this considerable load as it did when it was pulling no load at all. Again, the power developed by the motor will be equal to volts $\times$ amperes; but now the current is considerable, and the power developed will be correspondingly greater.

If the load on a shunt motor decreases, the armature will speed up a trifle, thus inducing a greater counter e.m.f. and effecting a material decrease in the current being forced through the motor. Furthermore, the power in

watts developed by the motor will also equal volts × amperes, but since the current is now smaller than before the power developed will be proportionately less.

Thus, it is apparent that a shunt motor will maintain practically constant speed as the load it is pulling varies and that it automatically regulates the intensity of the current forced through it at any instant in proportion to the load it is pulling at that instant. Numerical examples further illustrating this condition are given under Art. 686.

686. The automatic speed regulation of the shunt motor may be explained numerically thus: From Art. 666, formula (165), the force tending to thrust a current-carrying conductor from a magnetic field is

$$F_{\text{lb}} = 0.000{,}000{,}088 \times I_A \times L \times B \text{ (lb.)} \qquad (177)$$

In a shunt motor operating on a constant-voltage circuit the flux density, B, remains practically constant because the current through the shunt-field winding is always about the same. Furthermore, the length, L, of the armature conductors in the field always remains constant in a given armature. Obviously, if the torque of the motor is to vary at different loads, the armature current, I_A, must vary.

It has been shown in Art. 674 that the current through an armature is $I_A = (E - E_B) \div R_A$. Hence, the armature current I_A can be increased only by increasing E, decreasing E_B, or decreasing R_A. But on a constant-voltage circuit E is constant and in any given motor R_A is constant. It follows that if the armature current, hence torque, of any given operating motor is to be increased to drive a greater load, its counter e.m.f. E_B must decrease. The counter e.m.f. of a motor will be decreased when its armature rotates at a lower speed—this counter e.m.f. varies directly as the speed of rotation (Art. 671).

When, then, the load on a motor is increased, its speed and hence counter e.m.f. decrease somewhat and a greater current is forced through the armature from the line. Thereby the torque of the motor is materially increased and it pulls its load at a very slightly decreased speed. Consider the following example:

Example.—A shunt motor is connected across a constant-voltage 220-volt main. Its armature resistance is 0.5 ohm. It is operating at such a speed that its counter e.m.f. is 215 volts. Then the armature current will

$I = (E - E_B) \div R_A = (220 - 215) \div 0.5 = 5 \div 0.5 = 10$ amp. Now assume that an additional load is thrown on the motor which decreases its speed 1 per cent; then the counter e.m.f. will be decreased 1 per cent and will be $0.99 \times 215 = 213$ volts. Under these conditions the armature current will be $(220 - 213) \div 0.5 = 7 \div 0.5 = 14$ amp. Thus, with a decrease of only 1 per cent in speed, the armature current has been increased by $(14 - 10) \div 10 = 4 \div 10 = 0.4$ or 40 per cent.

The power being developed by the armature when the armature current was 10 amp. was 10 amp. $\times$ 215 volts = 2,150 watts or 2.88 hp. (Art. 186). But with the 14-amp. armature current the power is 14 amp. $\times$ 213 volts = 2,982 watts or 3.99 hp. Obviously in this case the power has been increased by $(3.99 - 2.88) \div 2.88 = 1.11 \div 2.88 = 0.38$ or 38 per cent. (The increase in power is not quite proportional to the increase in current shown in the above article because the I^2R_A watts power loss in the armature increases as the square of the armature current.) Thus, with a decrease in speed of only 1 per cent the power output of the armature has been increased approximately 38 per cent.

687. Speed control of shunt motors may be treated under two headings (1) field control, and (2) armature control. Field control is the most efficient and effective for reasons which will be given, hence will be considered first.

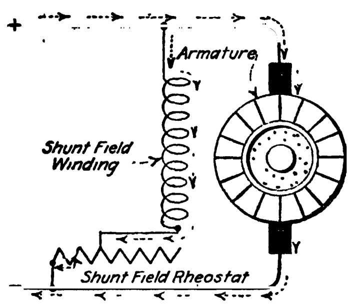


FIG. 388.—Showing a shunt-field rheostat for speed control of a shunt-wound motor.

688. Field control of shunt-motor speed is usually effected with a rheostat as shown diagrammatically in Fig. 388. By varying the resistance of this shunt-field rheostat, which is connected in series with the shunt-field winding across the constant-voltage main, the current through the shunt-field winding may be varied accordingly. The field of the motor may thereby be strengthened or weakened. Weakening the field of a shunt motor or of any other kind of motor causes its speed to increase; strengthening the field of a motor causes the speed to decrease. Note the following explanations:

Explanation.—The simplest explanation is: Consider the case of a motor armature, which is developing a constant power watts output at different speeds and which is fed from a constant-voltage source. The current input to the motor armature must then be the same at all speeds.

Likewise the counter e.m.f. must remain constant (Art. 671) if the current remains constant. Now if the field strength or flux is weakened, the armature must rotate more rapidly—must speed up—in order to cut the same

number of lines per second, that is, to induce the same counter e.m.f., as before.

NOTE.—With conditions as just described, that is, with constant power output and varying speed, the torque of the armature must vary inversely as the speed. Such a condition is seldom encountered in practice. The explanation of the condition which is usually encountered, that is, one where the driven torque remains practically constant and the horsepower output of the motor therefore varies with the speed, follows.

Explanation.—Where a shunt motor is driving a load of practically constant torque, a decrease in field current, and hence flux, will cause the counter e.m.f. to decrease as above described. Thereby, a greater current will be forced from the line through the armature. Now the instantaneous increase in armature current is much greater proportionately than the decrease in flux. Hence, the motor's torque (Art. 194) will for an instant before the armature has time to speed up be greatly increased and the armature speed will tend to increase proportionately. The following specific example illustrates numerically how the thing works out.

Example.—1.—Consider the case of a 15-hp., 220-volt shunt-wound motor, having an armature resistance of 0.2 ohm, which is pulling its full rated load at a speed of 1,000 r.p.m. and taking at this load and speed an armature current of 55 amp.

2.—Then note what happens when the field flux is decreased by 10 per cent.

With the motor operating at rated load and speed as above: The torque developed by the motor (Art. 195) will be

$$(a)\quad T = \frac{\text{hp.} \times 33{,}000}{2 \times \pi \times \text{r.p.m.}} = \frac{15 \times 33{,}000}{2 \times 3.14 \times 1000} = 78.6 \text{ lb. at 1 ft. radius}$$

The counter e.m.f. (Art. 673) is

$$E_B = E - (I_A \times R_A) = 220 - (55 \times 0.2) = 220 - 11.0 = 209 \text{ volts}$$

With the flux decreased by 10 per cent: At this instant the counter e.m.f. will then (Art. 673) obviously be 0.9 of what it was formerly, or it will be: $0.9 \times 209 = 188$ volts. Then the armature current at this instant will be (Art. 673)

$$I_A = \frac{E - E_B}{R_A} = \frac{220 - 188}{0.2} = \frac{32}{0.2} = 160 \text{ amp.}$$

That is, the instantaneous armature current is: $160 \div 55 = 2.9$ times or 290 per cent as great as was the former current. Now since the torque is proportional to the product of the flux and the armature current (Art. 667) the instantaneous torque has now been increased to

$$(b)\quad 78.6 \times (0.9 \times 2.9) = 205 \text{ lb. at 1 ft. radius}$$

Thus, the torque developed by the motor at this instant has been increased $205 \div 78.6 = 2.6$ times. It follows that the motor will speed up accordingly. But as it speeds up its counter e.m.f. will increase, so that the ultimate current is limited in value. Finally, its speed (with the constant

torque of 78.6 lb. ft. and a 10 per cent decrease in flux) will be increased 10½ per cent or will be 1,105 r.p.m. This can be verified by using formulas (146), (167), and (172). Most of the quantities in these formulas are constants for this example and hence cancel out.

689. Speed adjustment by field control is, in practice, very largely employed for a wide range of motor applications. Since shunt-wound motors are usually designed to operate at normal ratings with full field strength, this form of control is, in practice, used only as a method of speed increase. With noncommutating motors, sparking is likely to occur when the field is weakened. Their use as adjustable speed motors is therefore limited to a speed increase of but 50 or 75 per cent. For greater speed increases the motors should be of the commutating-pole type, since the use of commutating poles provides for the elimination of armature reaction (Art. 621). Standard motors of this type may be employed for speed increases by field weakening through ranges of 2 or 3 to 1; when provided with special fields, ranges up to 6 to 1 may be obtained.

It should be borne in mind that as the speed of a shunt-wound motor is increased by field weakening, the torque is proportionally decreased so that the horsepower output remains practically constant throughout the range. When ranges greater than standard are employed, the special construction involved may provide for horsepower ratings that are constant for the entire range, or the motors may be designed for reduced horsepowers at the higher speeds, depending upon the requirements of the application for which they are used.

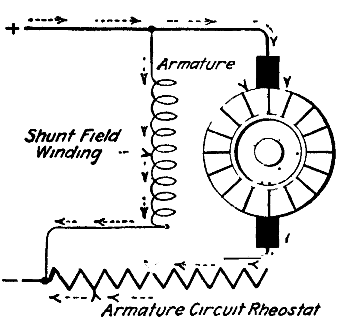


Fig. 389.—Showing armature-circuit rheostat for speed control of a shunt-wound motor.

Practically all commercial direct-current shunt-wound motors are now built with commutating poles.

690. Armature control of shunt-motor speed is effected by inserting a rheostat in series between the source of e.m.f. and the armature, as shown in Fig. 389, and then by varying the effective resistance of this rheostat. Thus the voltage impressed on the armature may be varied (decreased) but the flux due to the shunt-

field winding is not affected. Obviously, a decrease in the pressure across the armature decreases the armature current. Now (Art. 667), the torque of a motor varies as the product of armature current I_A and the flux ϕ_P. Hence, since ϕ_P remains practically constant, a variation in the armature current will produce a corresponding variation in torque developed and speed. Therefore, the speed of a shunt motor can be decreased by inserting resistance in its armature circuit.

691. There are two objections to armature control, and because of these it is seldom used except for the starting of shunt motors (Art. 693) and for certain special applications.

1. Armature control is wasteful, as with it there is a considerable $I^2 \times R$ power loss (Art. 189) in the control rheostat. This loss varies as the square of the armature current.

2. The speed regulation of an armature-controlled shunt motor operating at any speed less than maximum is poor. The speed regulation of a field-controlled shunt motor is good because with such a motor the armature is connected directly across a source of a constant e.m.f. The e.m.f. across an armature-controlled armature varies as the current through the armature and the voltage drop in the control resistance varies.

692. Resistance is required in series with the armature in starting a shunt motor, to prevent an excessive current from flowing through the armature before the armature has had time to commence rotation and thereby induce a counter e.m.f. This device and its function were described briefly in Art. 670. Resistors of practically the same construction, and connected in practically the same manner, are used with series and compound as with shunt motors. With a resistance in circuit the following formula will be true:

$$I_A = \frac{E - E_B}{R_A + R_R} \text{ (amp.)} \tag{178}$$

Wherein I_A = current, in amperes, through the armature.
E = e.m.f. impressed on the motor and rheostats, in volts (Fig. 390).
E_B = counter e.m.f., in volts, induced in the motor armature.
R_A = resistance of armature circuit, in ohms.
R_R = resistance of starting rheostat, in ohms.

NOTE.—At the instant when the motor is started, there is no counter e.m.f., that is, E_B is then zero. Therefore, the current through the armature at this instant is $I_A = E \div (R_A + R_R)$. Hence, at the instant of starting, enough starting resistance, R_R, must be cut in series with the armature to limit the starting current to a reasonable value, one which will not dangerously heat the machine and cause excessive sparking at the commutator. The maximum (Fig. 391) starting current permissible is usually assumed as that intensity which if exceeded would injure the motor windings, cause excessive sparking and pronounced line voltage drop. In practice the maximum starting current is limited to about 1.5 times normal full-load current. The minimum starting current permissible is that necessary to develop the required starting torque.

Frequently the total resistance of the rheostat is made such that with all this resistance (R_5, Figs. 390 and 391) in circuit with the armature, 50 per cent of the normal starting current will flow through the armature. That is, when the contact arm of Fig. 390 is on contact button 1, one-half the

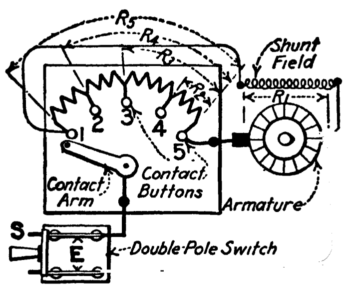


FIG. 390.—Simplified connections of a shunt-wound motor and starter.

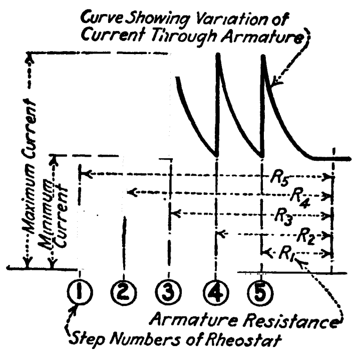


FIG. 391.—Indicating how the current through an armature may be limited with a starting rheostat.

normal starting current will flow as shown in Fig. 391. This current is, since it is only one-half normal, insufficient to start the motor, assuming that it must develop full-load torque to start.

When the arm is moved to button 2, the rheostat resistance then in series is such that the normal starting current flows through the armature as shown in Fig. 391. This may not be a great enough current to start the load. But when the arm is moved to button 3 the maximum permissible current will be forced through the armature from the line and the load should start. As the motor increases its speed its counter e.m.f. increases and the current decreases, as shown in the diagram of Fig. 391.

The contact arm is then shifted successively over the other contact buttons until it rests on button 5. Now all of the rheostat resistance is cut out, the only resistance in the armature circuit being that (R_1) of the armature itself, because the motor armature is now inducing a sufficiently high counter e.m.f. to itself limit the current through the armature.

693. A starter for a shunt motor or compound motor is a device wherein are assembled the devices essential for the starting of the motor and frequently those for the protection of the motor after it is in operation. The term "starter" as standardized by National Electrical Manufacturers Association denotes a controller designed for accelerating a motor to normal speed in one direction of rotation. A device designed for starting a motor in either direction of rotation includes the additional function of reversing and is designated as a "controller." Figure 392 shows diagrammatically a starter of the simplest type. The variations in practice to satisfy the many possible conditions are almost endless. See the author's "American Electrician's Handbook"

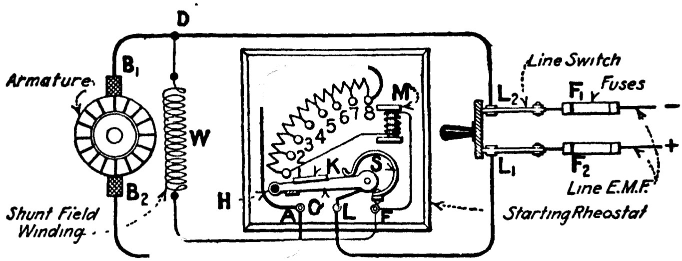


FIG. 392.—Showing connections between a shunt-wound motor and starter.

for detailed descriptions. A commercial shunt or compound motor starter usually comprises:

1. An adjustable resistor or starting rheostat pure and simple, the function of which is to limit the armature current at starting as described in Arts. 690 and 692.
2. A low-voltage protection device, described below.
3. An overload protection, described below; this device is frequently omitted.

694. The operation in starting a shunt motor is as follows: The line switch is closed, impressing the line voltage across brush B_1 and metallic swinging arm C (Fig. 392). Then the arm is moved up to contact with button 1. This connects both the armature and the field circuits to the line, and current is forced therefrom through each. The current through the armature, which is small, being limited by the starting resistance as described in preceding Art. 693, flows via the route: L_1, L, C, 1, 8, A, B_2, B_1, D, L_2. The path of the field current is: L_1, L, C, 1, M, F, W, D, L_2.

NOTE, then, that when the arm is on the first button, a small current flows through the armature and that the field winding is connected directly across the line. (Magnet winding *M* is in practice of very low resistance as compared with the resistance of the shunt-field winding.) So the field is strong—a maximum—and the armature current weak. Now the arm is moved successively over buttons 2, 3, 4, etc., with a pause at each button. The current through, and the torque of, the armature will be thereby gradually increased, as described in preceding Art. 670. The field is weakened slightly as the arm advances because of the insertion thereby of the armature resistance in the field circuit. This weakening of the field tends to assist in speeding up the armature (Art. 688). From 15 to 30 sec. should ordinarily be consumed in starting a motor.

Finally, the arm is moved to contact with button 8. It is held there by the attraction of magnet *M* on iron keeper *K*, which is attached to the arm. The field current actuates *M*, the functions of which are treated in detail below. With the arm on 8, the armature is connected directly across the line and the field winding is across the line in series with *M* and the starting resistance. The combined resistance of the starting resistance and *M* in series is small as compared with the resistance of the motor-field winding. Hence the shunt-field strength is little affected thereby.

695. To stop the motor, pull the line switch. Magnet *M* will then be demagnetized and spring *S* will force the arm back to its original position shown in the illustration.

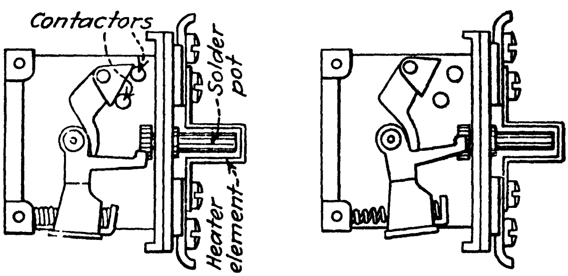
696. A low (or under) voltage protection is provided on motor starters to protect the motor and the circuits feeding it against an excessive rush of current in case the line e.m.f. is discontinued and is then again impressed on the motor without the starter handle having first been returned to the starting position.

Explanation.—Assume that the starter of Fig. 392 is not equipped with the low voltage protection magnet *M* and the spring *S*. Then, if lever arm *C* were moved into contact with button 8, the arm would remain in that position, even if the line e.m.f. were discontinued because of an accident or otherwise. Assume that the arm is in contact with 8, that the line e.m.f. is discontinued, and the motor armature slows down or ceases to rotate in consequence. Now if the e.m.f. is again impressed on the line, *C* still on 8, there would be a very large current forced through the armature because it would be turning slowly or be at rest and therefore inducing a small or no counter e.m.f. This excessive current would cause sparking at the commutator and might damage it or burn out the motor windings or the wiring.

But with the low voltage protection magnet *M*, arranged as in Fig. 392, when the line e.m.f. is discontinued and the armature ceases rotating, *M* is demagnetized and *C* is forced back into the off position by spring *S*. Then the circuit through the motor is open and the resumption of line e.m.f. can do no harm.

697. An overload protection on a starter is provided to prevent excessive current flowing through the motor or through the motor-circuit wiring, if the motor is overloaded or if a short circuit or similar trouble occurs on the motor circuit. Circuit breakers and fuses perform functions similar to that of the overload device and are frequently substituted for it, because they may protect both sides of the circuit (F_1 and F_2, Fig. 392).

698. There are two classes of overload protection devices which are designed for incorporation as integral parts of the starters with which they are employed. These are: (1) the thermal type, and (2) the solenoid and dashpot type. Of the thermal type, which is the type most generally used in connection with the starters for motors up to the higher ratings, there are two



I Closed II Open

FIG. 393.—Solder pot type of overload protection device.

forms known as: (*a*) the bimetallic strip, and (*b*) the solder pot (Fig. 393). In both forms, the circuit leading to the holding magnet *M* (Fig. 392) passes through a pair of contactors which are normally closed. The heating elements of these devices are in series with the armature circuit. When an excess of current passes through the heating elements, heat is produced in proportion to the load which is placed upon the motor. This heat actuates the device by causing the bimetallic strip to deflect or the solder to melt and thus release the contactors so that the circuit to the holding magnet is broken. The magnet being thus demagnetized, releases its hold on the starting arm *C* (Fig. 392) which flies back to open position and thus causes stopping of the motor.

In the solenoid and dashpot type the contactors are mounted above the plunger of the solenoid, and the armature circuit passes through the solenoid coil. Movement of the plunger is restrained

by a piston in a dashpot, but when the current becomes sufficiently strong to overcome the resistance of the dashpot plunger, the core rises sharply, striking a pin which in turn raises one spring of the contactor arm and thus breaks the holding circuit. The construction of this type of protection is shown in detail in Fig. 394.

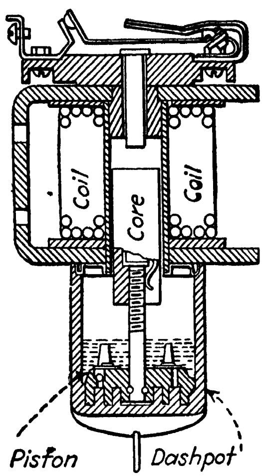


FIG. 394.—Section through solenoid and dashpot overload protective device.

699. The inverse time principle, which means that the quickness of operation is a factor of the volume of the overload current and the time during which it exists, is employed in both types of devices. Their action therefore corresponds to the conditions which govern the heating of the motor coils, that is, a short overload of considerable volume will not damage the motor and, similarly, a short overload will not cause the immediate operation of the protective device. On the other hand, if even a moderate overload is sufficiently prolonged it will cause damage to the windings and such an overload will also cause the protective device to function. On heavy overloads which would at once cause damage to the motor, these devices operate almost immediately.

SECTION 39

SERIES AND COMPOUND-WOUND MOTORS

700. The series motor is shown diagrammatically in Fig. 395. It consists merely of an armature and series-field winding, connected in series, and almost invariably operated from a constant e.m.f. source. A constant impressed e.m.f. is assumed in the following discussion. Obviously the current in the armature and in the field winding must, at any given instant, be the same.

701. The characteristics of the series motor are shown graphically in Fig. 386,II, where they may be conveniently compared with those of the shunt motor of I.

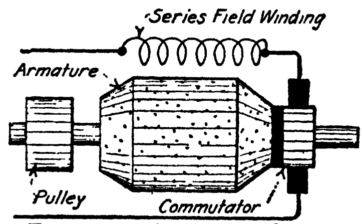


FIG. 395.—Series-wound motor diagram. A series-wound motor should never be belted to its load. See note of Art. 703.

The torque of a series motor, when the magnetic-circuit iron is being worked well below the saturation point (Art. 270), varies nearly as the square of the current. Thus, motors of this type have splendid starting torque. Torque (Art. 194) varies as the product of flux, ϕ_P, and armature current I_A. Now in a series motor, if the current is doubled, the flux is also doubled (approximately) since the armature current traverses the field coils. Hence the torque (proportional to: current $\times$ flux) is increased four times. When the load being driven by a series motor increases, the motor slows down, hence its counter e.m.f. decreases. Then its current increases and its torque increases as the square of the current.

The speed of a series motor varies (Fig. 386,II) with the load. For any given horsepower load which a certain series motor is driving, there will be a certain definite speed. With a heavy load, a series motor will operate at relatively slow speed. With a light load, it will operate at a high speed With no load, it will, theoretically, operate at an infinite speed, that is, it will run away. The explanation is this:

Explanation.—Any armature always tends to operate at such a speed that the sum of the volts lost in its armature, in IR_A drop, and the volts counter e.m.f. induced will equal the impressed voltage. At heavy loads the current, and consequently the IR_A drop, in a series motor armature will be large, and it is not necessary for the armature to rotate very rapidly to induce the required counter e.m.f. But suppose the load is thrown off

the motor. There now being no resisting torque, the motor speeds up, inducing a higher counter e.m.f. and decreasing the current through the field coil and armature. This weakens the field, causing a further increase in armature speed (Art. 688) which again decreases the counter e.m.f. With a series motor operating without load, this process continues until the armature bursts due to great centrifugal stresses.

702. A comparison of the characteristics of series and shunt motors is given graphically in Fig. 386. Since in the shunt motor the field is practically constant, its torque varies directly with the current (approximately). As shown in Art. 701 the torque of a series motor varies almost as the square of the current. The series motor has, then, a much greater starting torque (ability to rotate something) than has the shunt motor. Hence a series motor of a given size (that is, rating) will start a much greater load than will a shunt motor of the same rating. The shunt motor has a practically constant speed for all loads within its capacity (Art. 684), while the series motor has a certain speed for each load—its speed varies with the load.

703. The applications for which the series motor is adapted are limited because of its speed and torque characteristics which are described above. The series motor is not adapted to drive loads for which the power requirement varies, and which should be driven at a constant speed. The reason is that the speed of a series motor varies with the load.

NOTE.—Hence, a series motor is not, in general, adapted for machine-tool or line-shaft drives. If it is used for such, the speed will be low when the load driven is great and high when the driven load is light. Furthermore, since a series motor will run away at no load, it should be applied only where it can be geared or otherwise permanently connected to its load. Crane, hoist, and railway traction applications are good examples of services for which the series motor is well adapted, and it is also used in connection with machine tools for the performance of auxiliary functions such as the movement of heavy head and tail stocks, raising and lowering cross heads, and similar services where constant speed is not necessary but where great torque at low speeds is desirable.

704. In starting series motors, an adjustable resistance is connected, in series with the motor across the source of impressed e.m.f. (Fig. 396), to prevent an excessive rush of current at the instant of starting, while the armature is stationary and inducing no counter e.m.f. As the armature attains speed, the effective

resistance of the rheostat may be decreased by turning the handle. The function of this starting resistance is the same as that of the armature resistance used in starting a shunt motor, which is described in Art. 692.

705. The speed control of series motors is effected by changing the resistance in circuit between the motor and the source of constant e.m.f. The same rheostat (Fig. 396) may be used for this as is used for starting the motor, providing its resistors have sufficient current-carrying capacity to prevent their overheating in speed-control service. The method is somewhat similar to armature-speed control (Art. 690) for shunt motors. It is subject to the same disadvantages. When the effective resistance of the rheostat is varied, the voltage across the armature, E_A (Fig. 396), is varied accordingly and the armature then speeds up or slows down correspondingly until its counter e.m.f. equals (approximately) the e.m.f., E_A, impressed across the armature. Thus, although it is true that a series motor has a definite speed when driving any given load with a given e.m.f. impressed across its armature (Art. 701), its speed when it is pulling a given load may be altered by changing the e.m.f. across the armature.

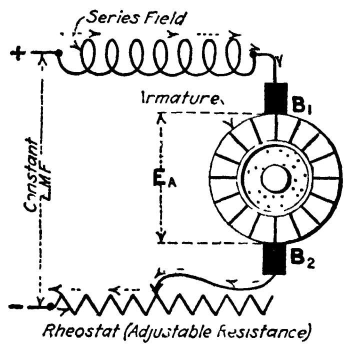


FIG. 396.—Resistance in series with a series-wound motor. See Fig. 401

706. Series motors are extensively used for driving electric cars because their torque and speed characteristics (Art. 701) render them especially fitted for this service. Shunt motors would not be well adapted for propelling cars as they would tend always to drive the car at about the same speed and on grades and in starting the motors would be greatly overloaded. Practically all of the cars in city service are driven by series direct-current motors. Many of the cars and locomotives in suburban and trunk line service are also driven by direct-current series motors, although alternating-current motors are being applied to an increasing extent for this work. Cars are equipped with two or four motors so that the torque may be applied at both trucks and so that series-parallel control (Fig. 397) may be used.

707. Series-parallel car control is used principally because it is economical. The controller located on the car platform and operated by the motorman is merely a rotary switch whereby the different series and parallel connections of the motors may be effected. Briefly, the operation is this:

Explanation.—Refer to Fig. 397. Advancing the controller handle to the first notch connects both motors, 1 and 2, in series, with all the starting resistance AC in series also. As the controller handle is swung around the starting resistance is gradually cut out, the car accelerating meanwhile, until both of the motors are connected directly across the line without any intervening starting resistance. Throwing the controller around another notch connects the motors in parallel between trolley and rail but again in series with all of the starting resistance $A'C'$. As the car accelerates the

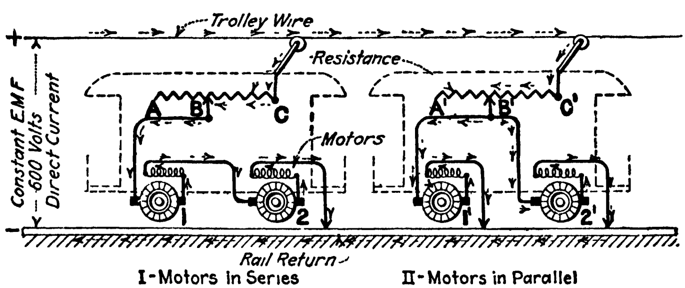


FIG. 397.—Showing how the two series motors driving a street car are first connected in series and then in parallel.

controller handle is swung on around until the motors are connected directly in parallel between trolley and rail.

The notches whereat the motors are connected without resistance: (1) in series between trolley and rail, and (2) in parallel between trolley and rail are termed running notches and are shown by longer raised marks on the controller top casting than are the others. When the controller handle pointer is over either of these marks no power is being wasted in starting resistance; all power taken from the line (except motor losses) is being expended by the motors in propelling the car.

Where the two motors are connected in series, only half the line current is required to start the car that would be necessary for starting with the motors in parallel. Assume that it takes 50 amp. through each motor to start the car. With the motors in parallel, 100 amp. would flow from the line, but with the motors in series only 50 amp. flows from the line. With the motors in parallel, resistance in series with them would be necessary to limit the current through each to 50 amp. But with the motors in series each motor takes the place of a starting resistance for the other. Thus the economy of series-parallel control is obvious.

708. Compound-wound direct-current motors partake of the characteristics of both shunt and series motors just as compound-wound generators (Art 597) partake of the characteristics of both series and shunt generators Figure 387 shows typical characteristic curves for a compound-wound motor A compound-wound motor may be made to approximate more or less the characteristics of a shunt or a series motor respectively by making the ampere turns of its shunt or series windings more or less powerful proportionately The elementary connections of a compound-wound motor are the same as those of Fig 318 for a compound-wound generator

709. A differential-compound motor has its windings connected as shown in Fig 398,*I* Its series-field winding "bucks"

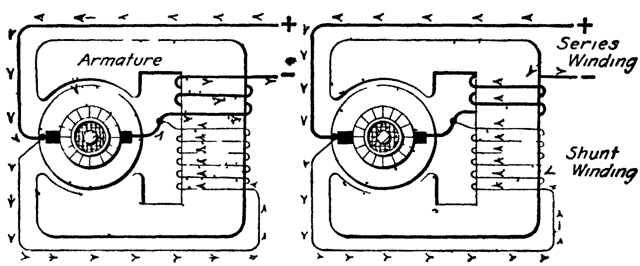


FIG 398 —Diagrams of differential and cumulative compound wound motors

or opposes its shunt-field winding A motor of this type can be arranged to operate at an almost exactly constant speed under varying load As the load increases and the motor tends to slow down the series-field magnetization increases and, since it opposes the shunt-field magnetization (which is normally greater than the series magnetization), the resultant field is weakened This would normally tend to cause the motor to speed up If the number of turns on the series field be properly chosen, the combined effects of increased load and weakened field will neutralize each other and the speed will remain unchanged By using more series-field turns it is possible to cause the motor to speed up with increased load The speed regulation of a differential-compound motor may be closer than that of a shunt motor but its starting torque is not so great

710. One of the chief disadvantages of the differential-compound motors is that they tend to "race" under very heavy

overload. They are also liable to start up in the wrong direction when the handle of the starting box is thrown to the first contact. The high inductance of the shunt field winding, because of its large number of turns, may so impede the rise of the shunt-field current that the current in the series field, which builds up much faster because of its smaller inductance, may overpower the magnetizing effect of the shunt field. Thereby the flux and the direction of rotation may be reversed. The series winding of a differential-compound motor should, preferably, be short-circuited when starting the machine.

711. A cumulative compound motor is so connected that its series turns augment the shunt-turns magnetization, as shown in Fig. 398,*II*. A motor thus connected will have strong starting torque—stronger than that of a shunt motor, but its speed regulation will not be so good. Motors of this type are used for shears, rolls, and similar applications where close speed regulation is not necessary but where great torque may be desirable at starting and while running. Frequently, motors of this type are merely series machines provided with sufficient shunt turns to prevent the motor from running away when operating at no load. They are then termed series-shunt-wound motors. The term compound-wound motor as usually used, applies to a cumulative compound machine.

712. The starting and control of compound motors are, in general, similar to those for shunt motors, as hereinbefore described. The same starting rheostats can be used for ordinary compound as for shunt motors. Ordinarily, the series winding has no special connection to the starting rheostat. Sometimes provision is made whereby the series winding may be shunted out of circuit after the motor has been started. The possible special control arrangements are too numerous to describe. Speed adjustment by field control is employed with compound-wound motors only to a limited extent. Any material weakening of the shunt field changes its relation to the series winding so that, as the weakening is increased, the motor partakes more and more of the series characteristics of poor speed regulation.

SECTION 40

DIRECT-CURRENT MOTOR POWER, CURRENT AND VOLTAGE RELATIONS

713. The commercial voltages used for direct-current motors as standardized by the National Electrical Manufacturers Association are 115 and 230. The lower voltage is ordinarily used for motors operating on lighting circuits; 230 volts is common for motors employed in industrial installations. Urban electric railways operate on a voltage of 600 volts, whereas voltages of 1,200 to 1,500 are often employed for interurban electric railways.

714. The rating of a motor is determined by the same factors which determine the rating of a generator as outlined in Art. 647. For example, a 10-hp. motor is one which will, when the voltage for which it was designed is impressed across its terminals, pull a 10-hp. load continuously without injury to itself. That is, the motor will carry its full rated load continuously with a temperature rise not exceeding 40°C. if it is of the open type, or 50°C. if it is semienclosed. It will also be capable of delivering 50 per cent overload torque momentarily but without temperature guarantees. General-purpose motors, if built in accordance with A. I. E. E. standards, will carry continuously a load of 1.15 times the rated load when operated at rated voltage and under usual service conditions. This factor of 1.15 is known as the *service factor.*

715. To find the power input, current, or impressed voltage of a motor, any two of these factors being known, the following formulas may be used. The power input to a motor, in watts, is equal to the current, in amperes taken by the motor, multiplied by the e.m.f., in volts, impressed across the motor terminals. This follows from the statements of Art. 186. Expressed as a formula,

$$P_{IW} = E \times I, \text{ or watts input} = \text{impressed volts} \times \text{amp.} \qquad (179)$$

hence

$$E = \frac{P_{IW}}{I} \text{ or, impressed volts} = \frac{\text{watts input}}{\text{amp.}} \qquad (180)$$

and

$$I = \frac{P_{IW}}{E} \text{ or, amp.} = \frac{\text{watts input}}{\text{impressed volts}} \qquad (181)$$

Wherein P_{IW} = the power input to the motor, in watts.
E = e.m.f. or voltage impressed across the motor terminals, in volts.
I = current flowing in the motor terminals, in amperes.

The power in watts thus obtained may be reduced to kilowatts or horsepower as directed in Art. 188.

Example.—If an e.m.f. of 110 volts (Fig. 399) is impressed across the terminals of a direct-current motor and the current through the motor is 10

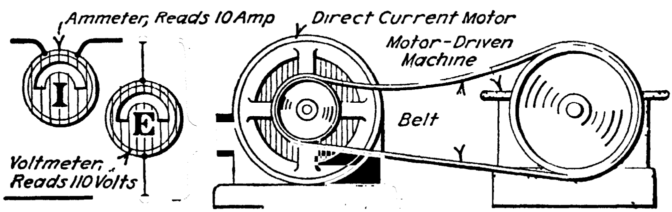


FIG. 399.—Illustrating power input to a motor.

amp., how many watts is the motor consuming? How many kilowatts? How many horsepower? *Solution.* Substitute in formula (179) $P_{IW} = E \times I = 110 \times 10 = 1{,}100$ watts. Then 1,100 watts is $1{,}100 \div 1{,}000 = 1.1$ kw. Also, 1,100 watts is $1{,}100 \div 746 = 1.48$ hp.

716. The relations between the kilowatt input, horsepower output, efficiency, impressed voltage, and current of a motor are indicated in the following formulas, all of which follow from Arts. 715, 188, and 197:

$$HP_O = \frac{P_{IK} \times \mathsf{E}}{0.746} = \frac{E \times I \times \mathsf{E}}{746} \text{ (hp.)} \qquad (182)$$

$$\mathsf{E} = \frac{HP_O \times 0.746}{P_{IK}} = \frac{HP_O \times 746}{E \times I} \text{ (efficiency)} \qquad (183)$$

$$P_{IK} = \frac{HP_O \times 0.746}{\mathsf{E}} \text{ (kw.)} \qquad (184)$$

$$E = \frac{HP_O \times 746}{\mathsf{E} \times I} \text{ (volts)} \qquad (185)$$

$$I = \frac{HP_O \times 746}{E \times \mathsf{E}} \text{ (amp.)} \qquad (186)$$

Wherein HP_O = horsepower output of motor.
P_{IK} = kilowatt input to the motor.
E = efficiency of the motor, expressed decimally, at the output HP_O.
E = the e.m.f. impressed on the motor, in volts.
I = current, in amperes, taken by the motor for the output HP_O.

Example.—Thus the output of a motor having an input of 2 kw. and an efficiency of 90 per cent would be $(P_{IK} \times \mathsf{E}) \div 0.746 = (2 \times 0.90) \div 0.746 = 1.80 \div 0.746 = 2.4$ hp.

Example.—If a motor at full load requires 12.2 kw. input and its efficiency at this load is 80 per cent, what is its output in horsepower at this load? *Solution.*—Substitute in formula (182) $HP_O = (P_{IK} \times \mathsf{E}) \div 0.746 = (12.2 \times 0.80) \div 0.746 = 976 \div 74.6 = 13.1$ hp.

Example.—If the e.m.f. impressed on a motor is 230 volts and the current taken by it is 55.5 amp. what horsepower is it delivering assuming that its efficiency at this load is 80 per cent? *Solution.*—Substitute in formula (182) $HP_O = (E \times I \times \mathsf{E}) \div 746 = (230 \times 55.5 \times 0.80) \div 746 = 13.7$ hp.

Example.—A 13-hp. motor has an efficiency of 80 per cent and is to operate on an impressed voltage of 230; what current will it take? *Solution.*—Substitute in formula (186) $I = (HP_O \times 746) \div (E \times \mathsf{E}) = (13 \times 746) \div (230 \times 0.80) = 9{,}698 \div 184 = 52.7$ amp.

717. The horsepower developed by any motor on the basis of its torque and speed can be readily computed. In Art. 716 the method of determining the power output of a motor, where its electrical input and efficiency are known, is outlined. In this article the method of directly determining mechanical power output is described. If the torque (Art. 194) being developed by the motor is known, this formula may be used:

$$\text{Hp.} = \frac{2 \times \pi \times \text{r.p.m.}}{33{,}000} \times T \text{ (hp.)} \tag{187}$$

Wherein hp. = horsepower being developed by the motor.
π = 3.1416
r.p.m. = revolutions per minute of motor.
T = torque being developed by the motor, in pound-feet.

Now T or torque equals (Art. 194) the product of force times lever arm. With a prony brake (Fig. 400) the torque being developed by a motor can be determined by measuring (1) the force exerted and (2) the lever arm length. Then, these factors being known, the horsepower developed may be computed with this formula:

$$\text{Hp.} = \frac{2 \times \pi \times \text{r.p.m.}}{33{,}000} \times r \times F \text{ (hp.)} \tag{188}$$

Wherein all of the symbols have the same meaning as above except, r = distance, in feet, from center of motor shaft to point of application of the resisting force on a prony brake lever arm. F = force or pull, in pounds, exerted on the prony brake lever arm at the distance r from the shaft center. A study of the following example will assist the reader in obtaining an understanding of this situation.

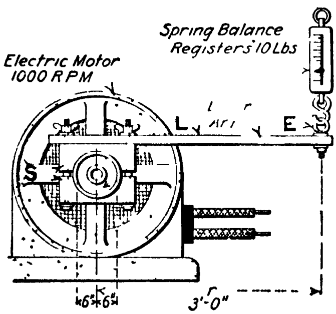


FIG. 400.—Prony brake arranged on an electric motor for a horsepower test.

Example.– The prony brake arrangement of Fig. 400 is a common form. The brake shown comprises a lever arm L at one end of which are arranged two wooden shoes, S, to clamp the motor pulley. At the other end is provided an iron eye E whereby a spring balance or scale may be attached to measure the force with which the motor pulley tends to turn the lever arm. Bolts through the shoes provide means for tightening them on the pulley, thus making the motor do more work and increasing the force, measured by the spring balance, tending to rotate the lever arm.

Assume that the motor is started and that the brake shoes are tightened sufficiently so that when the motor is running at 1,000 r.p m. the lever arm pulls down on the scale with a force of 10 lb. as shown. The length of lever arm, r, is 3 ft. Then the torque (Art. 194) is 10 lb. at 3 ft. radius or is 3 ft. $\times$ 10 lb. = 30 lb.-ft. torque. Or the torque is 30 lb. at 1 ft. radius or 60 lb. at 0.5 ft. radius. Since the motor is rotating at a speed of 1,000 r.p.m., a point on the circumference of the pulley travels, in feet per min. $2 \times \pi \times r \times$ r.p.m. $= 2 \times 3.14 \times 0.5 \times 1{,}000 = 3{,}140$ ft. per min. At its circumference the pulley is overcoming a resistance 60 lb. Hence, the motor is doing work at the rate of 60 lb. $\times$ 3,140 ft. per min. = 188,400 ft.-lb. per min. Since 1 hp. is developed when work is done at the rate of 33,000 ft.-lb. per min., the horsepower that this motor is developing is 188,400 ft.-lb. per min. $\div$ 33,000 = 5.7 hp.

Note that although the torque at the circumference of the pulley was taken in the example just recited, it is not at all necessary to take the circumference at this location. The torque exerted by the motor at any location may be taken provided the radius to that point is used instead of the radius of the pulley. From explanation just preceding it follows that the formulas given in the article above can be applied for computing horsepower output. Thus, for the example of Fig. 400, equation (188),

$$\text{Hp.} = \frac{2 \times \pi \times \text{r.p.m.} \times r \times F}{33{,}000} = \frac{2 \times 3.14 \times 1{,}000 \times 3 \times 10}{33{,}000} = 5.7 \text{ hp.}$$

SECTION 41

CHARACTERISTICS OF ALTERNATING CURRENTS

718. Alternating currents have been referred to briefly in preceding articles. These articles, an enumeration of which follows, should be reviewed before the reader proceeds. In Arts. 127 to 130 the difference between an alternating and a direct current was explained. It was shown, using Fig. 80 for an illustration, that a direct current always flows in the same direction (Art. 126), whereas an alternating current alternates in direction. For example, a 60-cycle current (Art. 127) alternates (Fig. 79) 120 times per sec. It flows in one direction for $\frac{1}{120}$ sec.; the next $\frac{1}{120}$ sec. it flows in the opposite direction, and it continues thus to alternate as long as it flows. A 25-cycle current (Fig. 77) alternates 50 times per sec.

719. The difference between alternating-current and direct-current phenomena is very pronounced. The fact that a direct current always flows in the same direction while an alternating current constantly alternates in direction, as hereinbefore described, is in itself a significant thing. But, as will be shown, because an alternating current constantly changes in direction and in intensity, certain effects, which do not exist with steady currents, with alternating currents become of great importance. These are the effects of inductance (Art. 508) and of permittance or, as it is frequently called, electrostatic capacity (Art. 793). Just how inductance and permittance in alternating-current circuits produce these important conditions will be shown in following articles. For the present, note the following:

1. The current in an alternating-current circuit may not be equal to the e.m.f. impressed on the circuit divided by the resistance of the circuit. That is, Ohm's law (Art. 151) does not apply to alternating-current circuits in the way in which it applies to direct-current circuits. However, it will be shown (Art. 782) that Ohm's law is also true for alternating-current circuits or any circuit but that it can not always be applied directly.

2. The sum of the voltage drops across the components of a circuit carrying alternating current may not equal the e.m.f. impressed on the

circuit. In a direct-current circuit the sum of the voltage drops does equal the impressed e.m.f. (Art. 209).

3. The sum of the currents in the branches of a divided, alternating-current circuit (Art. 218) may not be the same as the total current flowing into and out of the divided circuit. With direct currents, the sum of the branch currents equals the main current, as stated in Kirchhoff's law (Art. 223).

4. Alternating current may flow in an open circuit. Direct current can not flow in an open circuit—except possibly for an instant after e.m.f. is applied to or removed from the circuit. The alternating currents that flow in so-called open circuits are displacement or charging currents due to permittance.

720. The method of generating alternating currents was indicated in an elementary way in preceding Arts. 464 and 556. Attention was directed to the fact that an alternating current is precisely like any other kind of an electric current in one respect, namely: a voltage (e.m.f. or difference of potential) must be developed before an alternating current—or any sort of a current—can be forced to circulate in a circuit.

NOTE.—To impel an alternating current in a circuit an alternating e.m.f. must be provided. Then, when this alternating e.m.f. is impressed on a circuit, an alternating current, inversely proportional to the opposition ("Impedance," Art. 788) offered by the circuit, will be impelled in the circuit. Alternating e.m.fs. are induced in most cases, by moving conductors through flux. A machine whereby conductors are made to cut flux, in such a way that an alternating e.m.f. is impressed on the external circuit served by the machine, is an alternating-current generator or alternator.

It was explained in Art. 557 that the e.m.fs. induced in the armatures of all commercial direct-current generators are alternating e.m.fs. and that a commutator is required to commutate these alternating e.m.fs. into direct e.m.fs. Alternating e.m.fs. are induced in the secondaries of induction coils (Art. 500). Hence induction coils impel alternating currents in their secondary circuits.

721. A cycle is a complete set of values through which an alternating current or e.m.f. repeatedly passes, as shown in Figs. 77 and 79. A cycle comprises: (*a*) an increase (*OA*, Fig. 77) in current or e.m.f. from zero to a maximum, and a decrease (*AB*) from maximum to zero, with the current or e.m.f. in one (for example, the positive) direction; and (*b*) in the opposite (for example, the negative) direction, an increase (*BC*) in current or e.m.f. from zero to a maximum and a final decrease (*CD*) to zero. In the illustration the portion *OABCD* represents 1 cycle.

Examples.—The expression 60 cycles per sec. means that the current or e.m.f. referred to completes 60 cycles in 1 sec.; therefore $\frac{1}{60}$ sec. is required to complete 1 cycle (see Fig. 79; see also "Frequency," Art. 722). With a 25-cycle current, $\frac{1}{25}$-sec. is required to complete 1 cycle, as shown in Fig. 77.

722. The frequency of an alternating current or e.m.f. is the number of cycles that it completes in 1 sec. Figure 79 shows a curve of a 60-cycle alternating current and Fig. 77 shows a curve of a 25-cycle alternating current, both being plotted to the same time scale. Thus it may be stated that the frequency of a certain current or e.m.f. is 60 cycles per sec. or merely 60 cycles, or the frequency in another circuit may be 25 cycles.

723. The frequencies used in practice in the United States are 60 cycles (Fig. 79) and 25 cycles (Fig. 77). Higher frequencies such as 125 and 133 cycles were formerly employed. In European countries the use of 50 cycles predominates.

NOTE.—A frequency of 60 cycles is common—almost standard—in the United States for general lighting and power installations. A frequency of 60 cycles is usually considered preferable for electric lighting because, with lower frequencies, there may be a visible flickering of incandescent lamps. However, 25 cycles can be and is used for incandescent lighting. For long-distance power transmission a low frequency—for example, 25 cycles—is sometimes considered preferable to a higher one because the undesirable effects of inductance (Art. 508) and permittance (electrostatic capacity, Art. 793), increase as the frequency increases. However, it is now usually conceded that, all things considered, 60 cycles is ordinarily preferable to 25, even for power transmission, where the energy at the receiving end of the circuit is to be used for general power and lighting service. A few of the older central stations still generate at either 125 or 133 cycles, which were formerly in vogue. For single-phase railway work, 15 cycles has been used to a limited extent. See the author's "American Electricians' Handbook" and his "Electrical Machinery" for further information regarding the adaptability of different frequencies for different services.

724. An alternation is half a cycle, as shown in Figs. 77 and 79. There are two alternations in a cycle, one in the positive (Art. 559) and the other in the negative direction.

725. Phase.—Any point on an alternating-current wave is called a phase. To designate a certain phase of a wave, it must be specifically indicated in some way. Usually this is done by specifying the number of electrical degrees between the phase to be indicated and some reference point or phase on the wave. The reference point is, unless otherwise specified, **taken as the 0-degree point or phase—the starting point of the wave.**

Example.—In Fig. 401 (taking 0 deg. as the reference point or phase), A_1 and A_2 are 30-deg. phases, B_1 and B_2 are 90-deg. phases, C_1 is a 180-deg. phase, D_1 and D_2 are 240-deg. phases, E is a 360-deg. phase, etc.

It was explained in the example under Art. 565 that electrical degrees really refer to time. Hence, it follows that the term phase may also refer to time. Therefore, a phase may also be defined as the time instant when some maximum, zero or any other value is attained by the wave.

Example.—When two alternating currents attain their corresponding zero, maximum, and intermediate (not necessarily the same) values at exactly the same instants, they are said to be in phase. If the currents are not in phase they attain corresponding values at different instants.

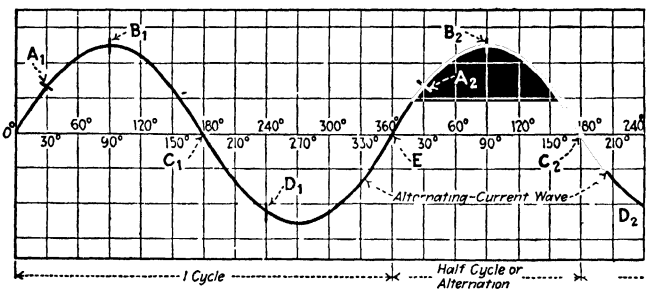


FIG. 401.—Illustrating the term phase.

NOTE.—Difference in phase is obviously the difference, usually stated in degrees, between two specified phases.

Example.—The difference in phase between points A_1 and B_1 in Fig 401 is: $90 - 30 = 60$ deg. The difference in phase between points B_1 and E is: $360 - 90 = 270$ deg.

NOTE.—The above-given definitions for phase are the technically correct ones. However, the word "phase," as loosely used, has other meanings Sometimes, each of the three wires of a three-phase circuit is called a phase wire—or for short, a phase. Also, any pair of wires of a polyphase (Art. 831) circuit, across which the normal voltage of the circuit should exist, is sometimes referred to as a phase of the circuit. Two-phase and three-phase currents and circuits are treated in following Arts. 835 and 844, respectively.

726. Alternating e.m.fs. and currents may be either single-phase or polyphase, depending on the construction and arrangement of the generator used for their production. Single-phase e.m.fs., currents, and principles will first be discussed; polyphase circuits and devices will be treated later (Art. 834).

727. The term single-phase implies a single, or only one, alternating e.m.f.—or current—in a circuit. That is, with a

single-phase e.m.f. or current there is at any given instant only one phase, as defined in Art. 725. With polyphase e.m.fs. or currents there is more than one phase at any given instant (Art. 834).

Example.—In the curve of a single-phase alternating current, shown in Fig. 401, there is at the 90-deg. instant but one phase, B_1. In the two-phase curve of Fig. 517 there are two phases at the 90-deg. instant. In the three-phase curve of Fig. 529 there are three phases at the 90-deg. instant. There are two phases at every instant, in a two-phase e.m.f. or current curve and three phases at every instant in a three-phase curve.

NOTE.—As will be shown later, under polyphase e.m.fs. and currents (Art. 834), there may be two or more alternating e.m.fs. impressed on, or two or more alternating currents in, the same polyphase circuit. A single-phase circuit ordinarily requires but two wires and is, in this respect, similar to a two-wire direct-current circuit. A polyphase circuit always has more than two wires.

Examples.—The alternating e.m.f. induced in the loop of Fig. 293 when it is rotated in the magnetic field is a single-phase e.m.f. and the current this e.m.f. impels in the external circuit is a single-phase current. Hence, curves like those of Figs. 291 and 402 represent graphically single-phase e.m.fs. and currents. A single-phase generator is one which impresses a single alternating e.m.f. on its external circuit. A single-phase circuit is one which has impressed on it, by its source of e.m.f., a single alternating e.m.f. Any alternating e.m.f. impressed on a two-wire circuit must be a single-phase e.m.f. and any alternating current in a two-wire circuit must be a single-phase alternating current.

728. An instantaneous value of an alternating current or its voltage is its value at some designated instant (phase) or, in other words, at some designated point in its cycle.

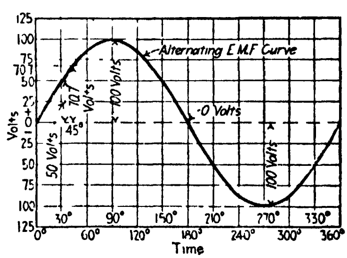


FIG. 402.—Illustrating instantaneous values of e.m.f.

Example.—In Fig. 402 the instantaneous e.m.f. at the 30-deg. phase is +50 volts (see Figs. 77 and 79 which indicate the actual time elapsing between different phases for 25- and 60-cycle e.m.fs.). At the 45-deg. phase the instantaneous e.m.f. is +70.7 volts. At the ¼-cycle or 90-deg. phase (Fig. 402) the instantaneous e.m.f. is +100.0 volts, which is also the maximum or crest e.m.f. in this case. At the 180-deg. phase the instantaneous e.m.f. is 0 volt. At the 270-deg. phase the instantaneous e.m.f. is −100.0 volts, etc. The symbol + means that the e.m.f. value following it is in the positive (Art. 559) direction. The symbol − means that the e.m.f. is in the negative direction.

729. The maximum value of an alternating e.m.f. or current (sometimes called the crest value) is the greatest value that it attains (Fig. 403). It is an instantaneous value. For example, the maximum value of the alternating current of the curve of Fig. 79 is 100 amp.; that of the curve of Fig. 82,I is 1 amp. With a sine-wave form, a maximum, positive-direction current flows for only an instant during one alternation of a cycle and a maximum, negative-direction current flows for an instant during the other alternation of the cycle.

NOTE.—The insulation of alternating-current apparatus must be designed to withstand successfully the maximum e.m.f. of the circuit. In spite of the fact that this maximum is (where the e.m.f. has a sine-wave form) impressed for only an instant during each alternation, the insulation may be broken down during these maximum instants if it is not of sufficient strength.

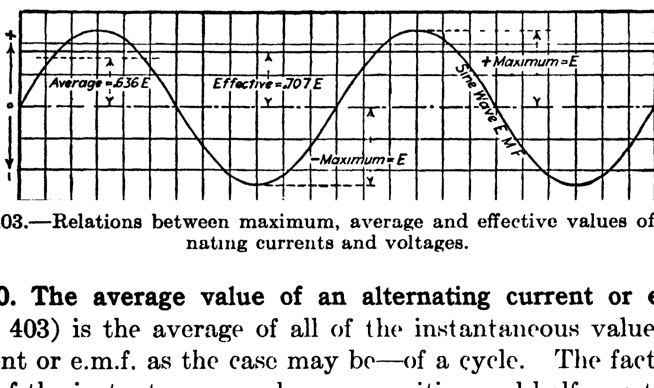


FIG. 403.—Relations between maximum, average and effective values of alternating currents and voltages.

730. The average value of an alternating current or e.m.f. (Fig. 403) is the average of all of the instantaneous values—of current or e.m.f. as the case may be—of a cycle. The fact that half of the instantaneous values are positive and half negative is disregarded in computing the average value. Through the application of the higher mathematics it can be shown that the average value of an alternating e.m.f. or current of sine-wave form = 0.637 × its maximum value. It follows that maximum value = average value ÷ 0.637 = 1.57 × average value. These relations are given in the formulas in Art. 737.

NOTE.—Average values are seldom used or referred to except in theoretical demonstrations, as for illustration in the proof under Art. 737. A practical man may never hear of average values in his everyday work.

731. The effective or virtual value of an alternating current is defined as that value (Fig. 403) which will produce the same heating effect as will the same intensity of direct current. An

alternating current is continually changing in intensity, within a certain range, from instant to instant. And it changes in direction at each alteration. These changes occur even if the load is constant. The table under Art. 130 shows how an alternating current thus varies.

Now these changes occur many times a second with the alternating currents which are used in practice. For example, with a 60-cycle current (Fig. 79) there are 120 alterations per sec. Commercial ammeters could not be made which would indicate at every instant these constantly changing values—even if it were desirable to have them do so. The mechanisms of the instruments could not respond to these ever-changing variations.

When an alternating current flows in an instrument designed for measuring alternating-current intensities, the pointer of the instrument will be deflected along its scale a certain distance and will remain steady at that location so long as the current remains constant, that is, so long as the maximum value of the current does not change during successive alternations. But if the alternating-current intensity decreases—if its maximum value decreases through successive alternations—the pointer will swing back showing a smaller deflection. If the alternating current increases, the deflection of the pointer will become greater than before.

It is evident, therefore, that by properly marking its scale, the ammeter could be calibrated to indicate either the average or the maximum value of the current passing through it. However, it is more convenient to have the instruments indicate effective values, as they are above defined.

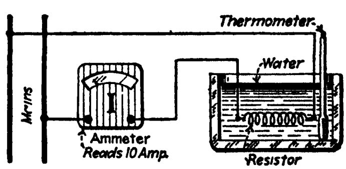


FIG. 404.—Illustrating the heating effects of direct and alternating currents.

Example.—If a direct current of 10-amp. intensity (as indicated by the ammeter, Fig. 404) flowed, say, for 1 hr., through the insulated resistor submerged in the water in the vessel, the heat, due to the $I^2 \times R$ loss (Art. 189) in the resistor, would raise the temperature of the water in the vessel. Assume that this direct-current heating effect raised the temperature 10°—from 70° to 80°F. Now assume that instead of direct current, an alternating current is forced through this same resistor submerged in this same vessel. Further, assume that the intensity of this alternating current is regulated by varying the applied e.m.f., until a constant alternating current flows through the resistor which will, in exactly

1 hr., raise the temperature of the water from 70 to 80°F.—the same temperature rise as before. Then this alternating current would have an effective value of 10 amp., because it has the same heating effect as a 10-amp. direct current. In other words, it would be an alternating current of 10 amp.

NOTE.—The ampere was used as the unit of direct-current intensity before alternating currents were employed. When then it became necessary to measure alternating-current intensities, instead of defining some new unit for measuring these intensities, it was decided to call the unit of alternating current also an ampere and to have it, insofar as possible, equivalent to the direct-current ampere.

Now alternating-current intensities can not be compared with direct-current intensities on the basis of their electrolytic effects. Why? Because the metal deposited electrolytically by an alternating current during any one alternation would be redeposited in the other direction, during the next alternation. Obviously, the net electrolytic effect of an alternating current is nil. Hence an alternating-current intensity or rate of flow, cannot be defined by its electrolytic effect as is the direct-current intensity. But the heating effects of alternating and direct currents can be compared readily—as shown in the above example. Hence it was decided to define the alternating-current ampere on a basis of its heating effect as hereinbefore specified.

732. Alternating-current Measuring Instruments Indicate Effective Values.—This is true of both ammeters and voltmeters. In speaking of alternating currents or e.m.fs., the values referred to are ordinarily, unless it is otherwise specified, effective values. The practical man deals almost exclusively with effective values.

733. The relation of the effective to the maximum value of an alternating current or voltage is shown in Fig. 403. It may be expressed thus: effective value = 0.707 × maximum value. It also follows that maximum value = effective value ÷ 0.707. Equations expressing these relations are given in a following paragraph (Art. 737).

Example.—What is the effective value of the alternating current of Fig. 79? *Solution.*—Its maximum value, reading from the curve, is 100 amp. Hence, to find its effective value,

$$\text{Effective value} = 0.707 \times \text{max. value} = 0.707 \times 100 \text{ amp.} = 70.7 \text{ amp.}$$

Example.—What is the effective voltage of a circuit that has a maximum voltage of 156 volts? *Solution.*—Substitute in the formula

$$\text{Effective value} = 0.707 \times \text{max. value} = 0.707 \times 156 = 110 \text{ volts}$$

Example.—If a voltmeter on an alternating-current circuit reads 2,200, what is the maximum instantaneous voltage? *Solution.*—Voltmeters

and ammeters always indicate effective values (Art. 732). Substitute in the formula

$$\text{Maximum value} = \frac{\text{effective value}}{0.707} = \frac{2{,}200}{0.707} = 3{,}110 \text{ volts}$$

734. Why the effective value equals 0.707 $\times$ the maximum value may be explained thus: The effective value of an alternating current is, as stated in Art. 731, determined by its heating effect. Now the heating effect of a current of electricity is proportional to the square of the current, in amperes (Art. 189). It follows that the heating effect of an alternating current at any instant is proportional to the square of the current at that instant. Now if the squares of the current intensities of an alternating current for a great number of equidistant instants over an alternation be computed and their average taken, this resulting value will be the average or mean square of all of these instantaneous currents. Then to ascertain the equivalent steady current that would produce the same heating effect as all of the instantaneous currents of different values, the square root of the mean square is taken. It will always be found, if the problem is solved by the approximate method just described, that this square root of the mean square for currents of sine-wave form is equal to 0.707 of the maximum value. It can be readily shown, by the application of the exact methods of the higher mathematics, that the value 0.707 is absolutely correct for currents of sine-wave form.

Example.—Consider the alternating-current sine curve of Fig. 405. The maximum current is 100 amp. The mean square of the current values at 9 different equidistant instants is 5,001.40. The square root of 5,001.40 is 70.72 amp. which is the effective current in accordance with this *approximate* solution. Actually the effective current, with a maximum of 100 amp., would be 70.71 amp. Hence, the above approximate solution gives a value just a trifle high. If current values are taken 2 deg. apart over an alternation, instead of 20 deg. apart, as in the example just solved, the effective value will come out almost exactly 70.71 amp. Obviously it is not necessary to consider values over an entire cycle, since the values for one alternation are the same as those for another, assuming that the maximum current remains the same. That is, the values for the first half cycle will be the same as those for the second half cycle.

735. The effective value of an alternating e.m.f. is that value which will propagate an alternating current of corresponding

intensity. The meaning of this statement can be understood from a consideration of the following numerical example.

Example.—If, in a circuit (Fig. 406,I) comprising resistance only and having a resistance of 10 ohms, an effective alternating current of 6 amp. flows, the effective e.m.f. impelling the current is (Ohm's law) $E = I \times R =$

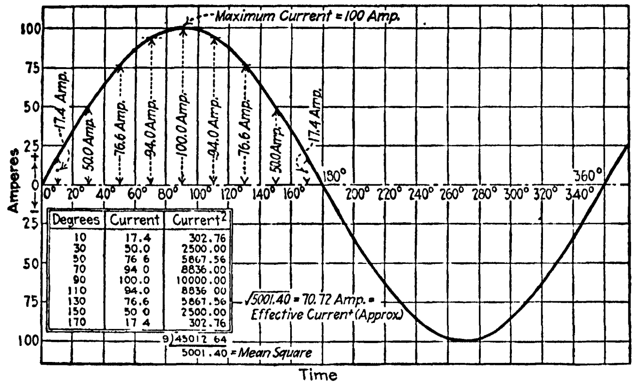


FIG. 405.—Illustrating approximate method of determining the effective value of an alternating current when its maximum value is known.

$6 \times 10 = 60$ volts. If an effective current of 15 amp. flows in this same circuit (Fig. 406,II) then the effective e.m.f. impelling it must be $15 \times 10 = 150$ volts.

736. The relation of the effective value of an alternating e.m.f. to its maximum e.m.f. is numerically the same as the relation of effective and maximum current values. It is always the e.m.f. which originates or impels a current, and a current is

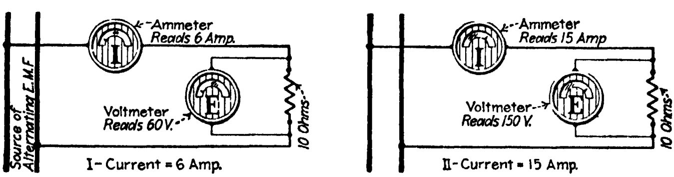


FIG. 406.—Effective e.m.fs. of 60 and 150 volts respectively impelling effective currents in alternating-current circuits.

always proportional to its e.m.f. If an alternating current has a sine-wave form, the e.m.f. which impels it must, obviously, have a sine-wave form. Hence, the same numerical relation must hold between maximum and effective values for both voltages and currents.

Hence, formula (191): effective e.m.f. = 0.707 × maximum e.m.f. It follows that maximum e.m.f. = effective e.m.f. ÷ 0.707 = 1.41 × effective e.m.f. (formula (195)).

Equations indicating these relations are given in the following Art. 737.

737. The numerical relations of maximum, effective, and average values to one another are shown in the following equations. They have been thus collected for ready reference:

Maximum value = 1.41[1] × effective value (189)
Maximum value = 1.57 × average value (190)
Effective value = 0.707 × maximum value (191)
Effective value = 1.11 × average value (192)
Average value = 0.637 × maximum value (193)
Average value = 0.901 × effective value (194)

Therefore:

$$E_M = 1.41 \times E_E, \text{ or, } I_M = 1.41 \times I_E \tag{195}$$
$$E_M = 1.57 \times E_A, \text{ or, } I_M = 1.57 \times I_A \tag{196}$$
$$E_E = 0.707 \times E_M, \text{ or, } I_E = 0.707 \times I_M \tag{197}$$
$$E_E = 1.11 \times E_A, \text{ or, } I_E = 1.11 \times I_A \tag{198}$$
$$E_A = 0.637 \times E_M, \text{ or, } I_A = 0.637 \times I_M \tag{199}$$
$$E_A = 0.901 \times E_E, \text{ or, } I_A = 0.901 \times I_E \tag{200}$$

Wherein E_M = maximum e.m.f., in volts.
E_E = effective e.m.f., in volts.
E_A = average e.m.f. in volts.
I_M = maximum current, in amperes.
I_E = effective current, in amperes.
I_A = average current, in amperes.

[1] The value 1.41 represents approximately the square root of two ($\sqrt{2}$).

SECTION 42

ALTERNATING-CURRENT GENERATOR PRINCIPLES AND CONSTRUCTION

738. Practical Alternating-current Generators May Be Divided into Two General Classes Thus.—(1) revolving-armature machines; (2) revolving-field machines. As stated in Art. 571, the armature is the portion of a generator in which the e.m.f. is induced. The field is the structure which provides the magnetic field, the flux of which cuts or is cut by the armature inductors, whereby the e.m.f. is induced. Each of the two classes is briefly treated below.

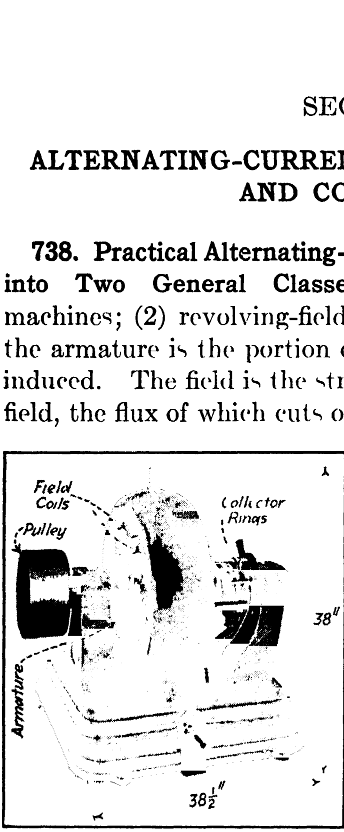


FIG. 407.—Revolving-armature belted alternator 25-kva., three-phase, 17½-kva., single-phase. (*General Electric Company*.)

739. The construction of revolving-armature alternators is shown in Figs. 407 and 408. In general they are quite similar to direct-current generators except that collector rings (Fig. 409) are substituted for the commutator. The fields must be excited with direct current, for which an independent, small direct-current generator—an exciter—is ordinarily required. Frequently the exciter-armature winding is wound on the same core with the alternator winding and a separate commutator provided for it. With such an arrangement, or when an independent exciter is directly connected to the alternator, the revolving-armature alternator is then self-contained. Revolving-armature alternators are frequently built on the same frames as are used for direct-current generators.

740. A Telephone Generator or Magneto Is a Revolving-armature Alternating-current Generator.—This machine is described and illustrated in Art. 580. The magnetic field in this

machine is produced by permanent magnets which are described in the data on "Magnetism" in Art. 89.

741. The application of revolving-armature alternators is, in practice, limited, because revolving-field machines (Art. 744) are in the larger capacities, less expensive, and in any case

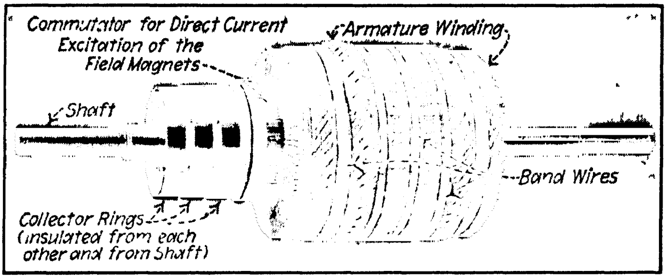


FIG. 408.—Complete armature of a revolving-armature alternator. The three collector rings indicate that this is for a three-phase machine.

have better inherent characteristics. It is seldom that revolving-armature alternating-current generators are made for capacities exceeding 25 kva. (three-phase; 17½ kva., single-phase) or for voltages exceeding 600. Their inherent regulation is poor, and the alternating voltage is impressed on the armature and

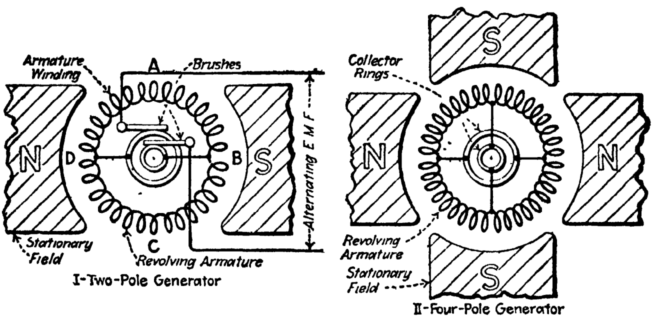


FIG. 409.—Showing how revolving-armature windings are connected to collector rings in revolving-armature, alternating-current generators (single-phase).

collector rings, which are difficult to insulate economically for high voltages. In revolving-field machines the revolving parts are not subject to high or line voltages.

742. The explanation of the operation of a revolving-armature alternator is as follows:

Explanation.—When the armature is in the position indicated in Fig. 409,I, the e.m.f. impressed on the rings is zero. The e.m.f. induced at this instant in the inductors between B and C is equal and opposite to that induced in those between C and D. Also, the e.m.f. induced in winding BA neutralizes that in DA, so there is no e.m.f. across the rings. But when the armature is rotated 1/4 revolution, then, at that instant, the e.m.fs. in windings DAB and DCB are a maximum and maximum voltage is impressed on the rings. When the armature has been rotated through another 1/4 revolution the e.m.f. across the rings is again zero. When it has been rotated through still another quarter the e.m.f. is again a maximum, but it is now in the opposite direction. These statements may be verified by applying the hand rule for determining the direction of an induced e.m.f. Thus as the armature is rotated it impresses an alternating e.m.f. on the external circuit connected to its collector rings.

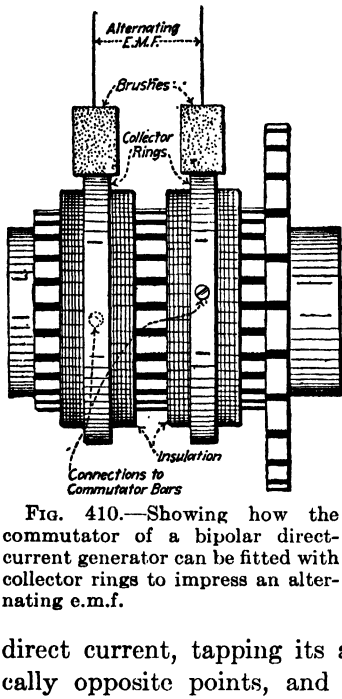


FIG. 410.—Showing how the commutator of a bipolar direct-current generator can be fitted with collector rings to impress an alternating e.m.f.

743. A Revolving-armature Alternating-current Generator Can Be Arranged from any Direct-current Generator (Fig. 410). It was shown in Art. 557 that the e.m.f. induced in a loop or coil rotated in a magnetic field is an alternating e.m.f. Hence, a (bipolar) direct-current generator may be modified into an alternating-current machine by separately exciting (Art. 591) its field with direct current, tapping its armature winding at two diametrically opposite points, and connecting each of the taps to a collector ring, as shown in Fig. 409,I. When an armature thus connected is rotated in the field, an alternating e.m.f. will be impressed across the collector rings. The collector rings may be attached to (but insulated from) the commutator of the armature as shown in Fig. 410. When a machine is built as an alternator, obviously only the collector rings are necessary; a commutator is not then required.

With a multipolar-field machine, there should be as many equidistantly spaced taps from the armature winding to the collector rings (Fig. 409) as there are poles. Alternate taps connect to

the same ring The information given in this article applies to single-phase generators. For polyphase machines it must be modified as elsewhere suggested.

Example —Figure 409,II shows the revolving-armature winding of a four-pole generator tapped at four equidistant points and connected to two collector rings so as to impress an alternating e m f across the collector rings The brushes are, for simplicity, omitted

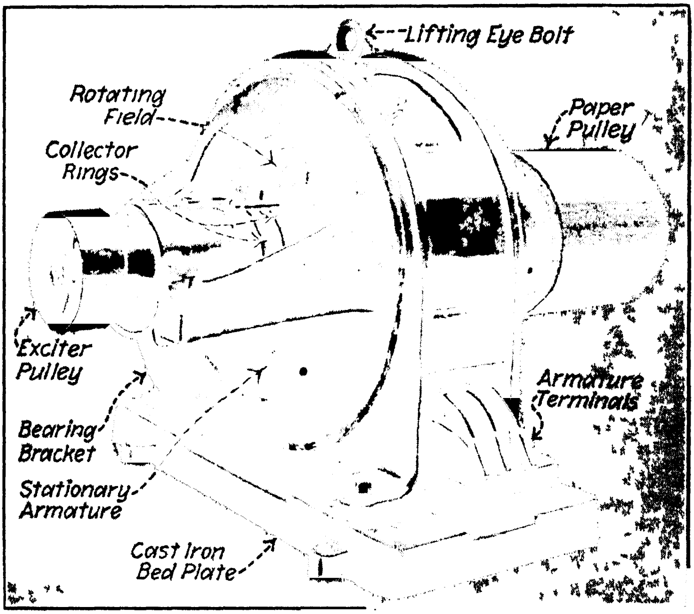


FIG 411 —A revolving-field belt-driven alternator 150-kva, 2300-volt, three-phase 900-r p m (*General Electric Company*)

744. Revolving-field alternators are the most important commercially, for reasons suggested in Art. 741. All modern machines of large capacity are of this type. Figures 411 and 412 show typical horizontal, revolving-field machines. Figure 413 details the general construction of all revolving-field alternators; however, some of the mechanical features there shown are incorporated only in vertical machines

745. The elementary revolving-field alternating-current generator is diagramed in Fig 414. The revolving field is produced by the rotating electromagnet *M*, which is energized with direct current circulated by a couple of dry cells. The dry cells are

equivalent to an exciter. The stationary conducting loop L is equivalent to a stationary armature winding. Note that this arrangement is similar to the elementary generator of Fig. 274 except that, in that illustration, the armature is rotated and the field is stationary.

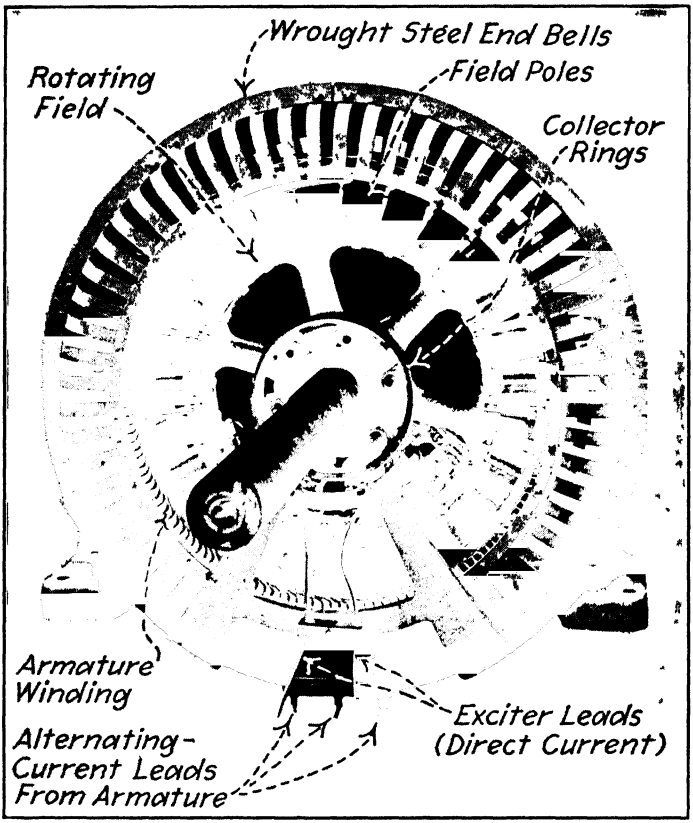


FIG. 412.—A 26-pole revolving-field engine-type alternator. (*Westinghouse Electric & Mfg. Company.*)

Explanation.—As the field M is rotated (clockwise) its flux cuts the stationary conductor L. At the instant pictured, the flux from the S pole is cutting the top side of the loop and that from the N pole the bottom side of the loop. An e.m.f. is being induced, and a current will flow (from front to rear in the top side of the loop) in the direction shown by the full-line arrows (hand rule, Art. 462). When M has been rotated $\frac{1}{4}$ revolution to a horizontal position, then at that instant, no flux will be cutting L and the e.m.f. induced in it will be zero. At the instant after M has been rotated

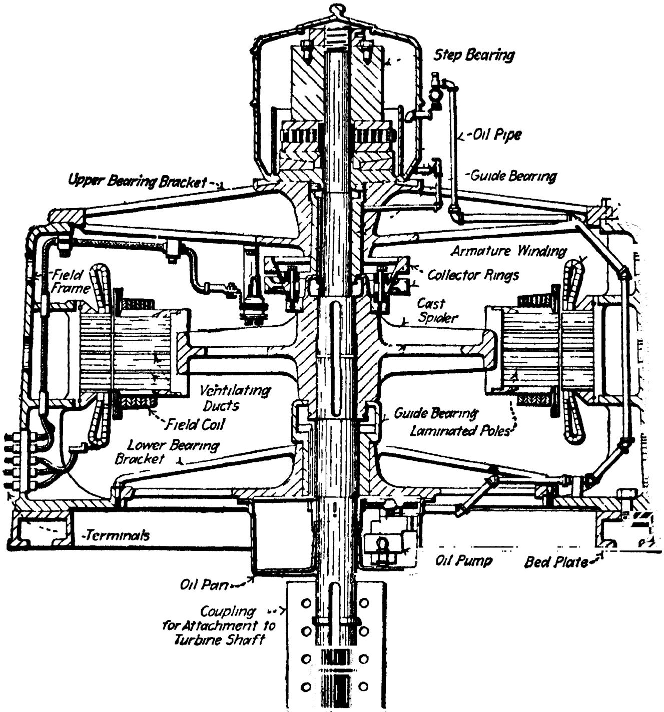


FIG. 413.—Showing construction of a vertical revolving-field, alternating-current generator for water-wheel drive

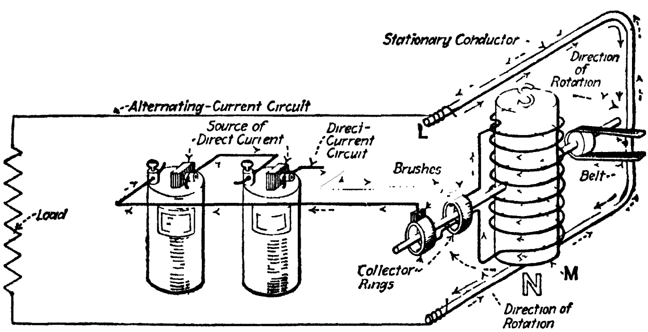


FIG. 414.—Demonstrating how alternating e.m f. is induced in a stationary conductor by a rotating field.

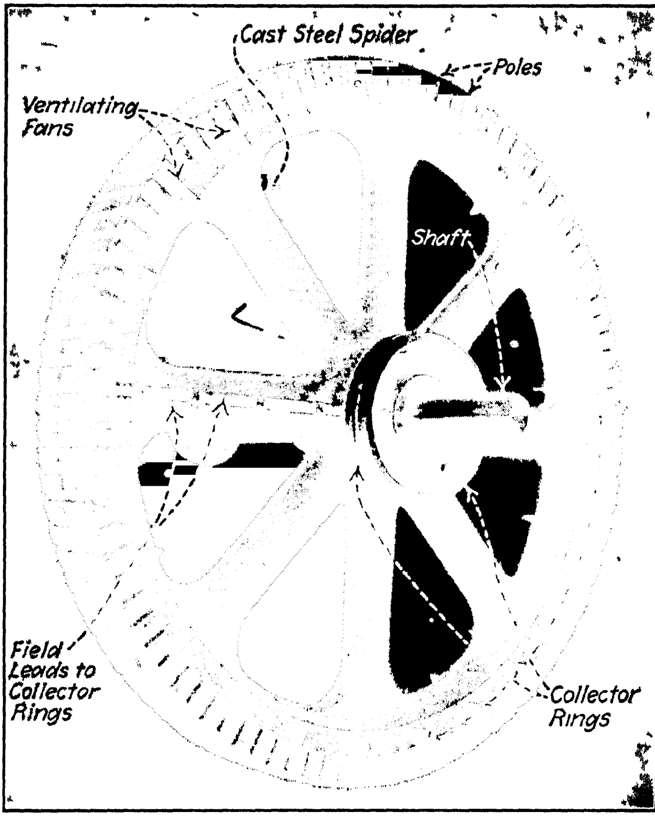


FIG. 415.—A 100-pole revolving field for a slow-speed alternator. (*Westinghouse Electric & Mfg. Company*)

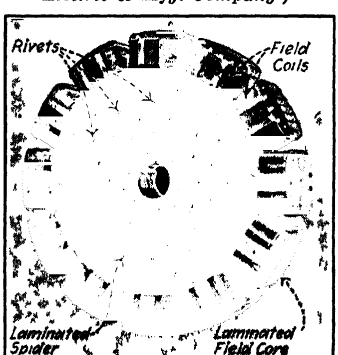


FIG. 416.—A 12-pole revolving field for a high-speed alternator (*Westinghouse Electric & Mfg. Company.*)

another ¼ revolution, its S-pole flux will be cutting the bottom side of the loop and the N flux the top side. Now the e m f and the current it impels in the loop will be (from rear to front in the top side) as shown by the dotted arrows Thus an alternating e m f has been induced in the loop as the field has been rotated This e m f has impelled an alternating current So long as the rotation of the magnet M is continued, the alternating current will flow in the external circuit

746. The construction of revolving fields for alternators is indicated in Figs. 415 and 416. Where the machine is to rotate

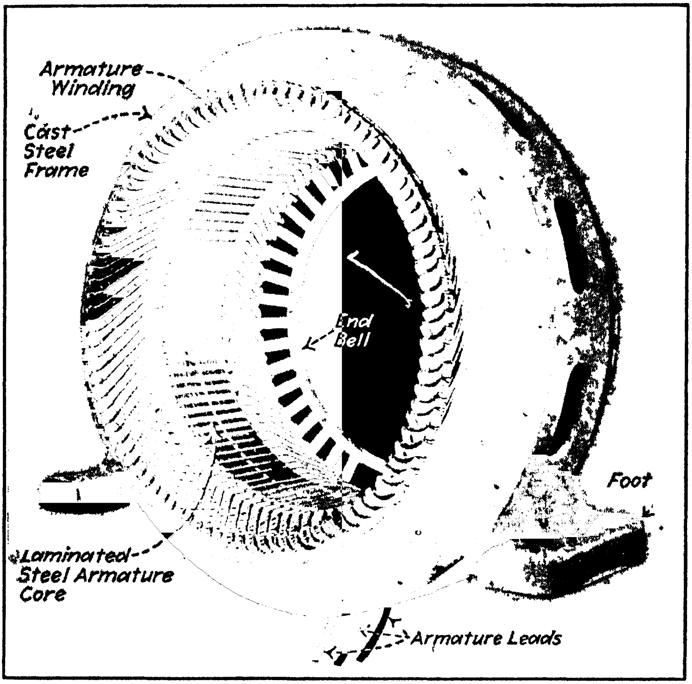


FIG. 417.—Stationary armature structure for an alternator.

at high speed the spider is usually built up of steel-plate laminations. Cast-steel or cast-iron spiders are used for machines for the lower speeds. The field coils are wound on laminated steel cores and are frequently insulated with noncombustible materials so that they can be overloaded without damage.

747. Armature construction and windings of revolving-field alternators are shown in Figs. 417 and 418. The armature coils are, instead of being wound on projecting poles as shown in

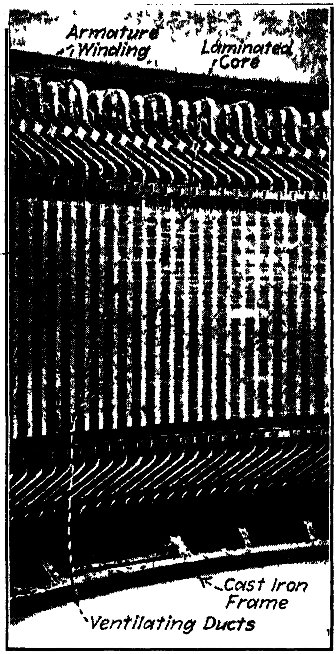


FIG. 418.—Portion of the armature structure of a revolving-field alternator. (*Westinghouse Electric & Mfg. Company.*)

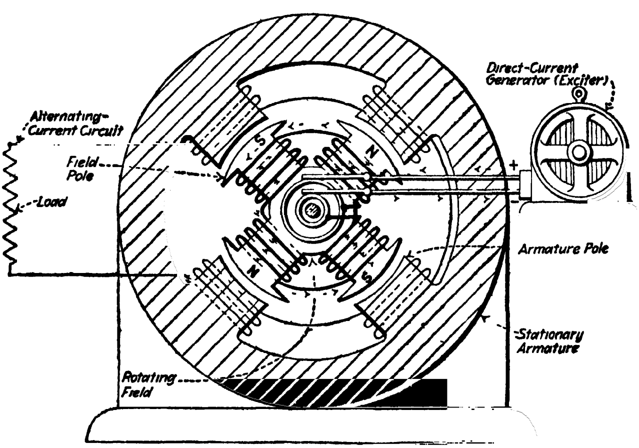


FIG. 419.—Diagram of a four-pole revolving-field generator, single phase (Strictly, the pole faces should be no wider than the armature coils to insure correct generation of alternating e.m.f.)

Fig. 419, arranged in slots along in the armature core as illustrated (Figs. 417 and 418). This arrangement is the most economical and effective. The armature cores are built up from laminations or punchings in somewhat the same way as are the armature cores (Art. 606) of direct-current generators. How-

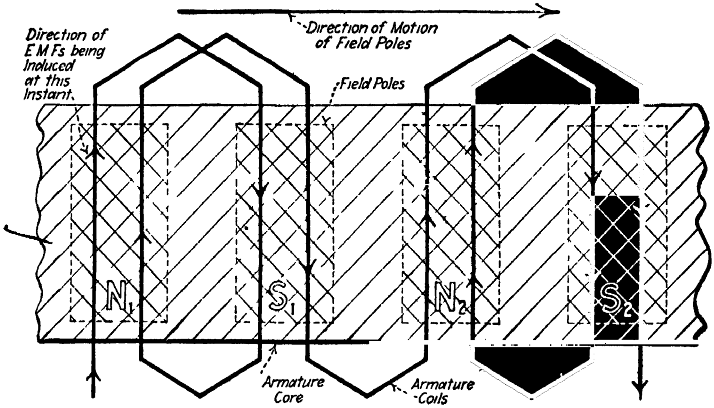


FIG. 420.—Development of an armature winding for a four-pole alternator. (This may be considered as showing either a rotating-field winding or a rotating-armature winding. For a rotating-armature winding the direction of motion of the armature would then be the opposite of the direction shown to produce the e.m.f. directions indicated.)

ever, the punchings for alternating-current stationary armatures form rings similar to that of Fig. 327, except that the winding slots are on the inside of the ring for the alternating-current machines. Form-wound coils (Art. 612) are used in modern machines. Figure 420 gives an idea as to how the armature coils are arranged.

SECTION 43

HOW ALTERNATORS DEVELOP E.M.FS.

748. The explanation of the induction of e.m.f. on the basis of cutting flux, by an alternating-current generator in which the inductor coils are in slots distributed along in the surface of the armature, will now be considered in connection with Figs. 421 to 434. The essential principles are the same as those already cited in Arts. 551 and 742. However, just what happens is not always readily apparent to the student; hence this

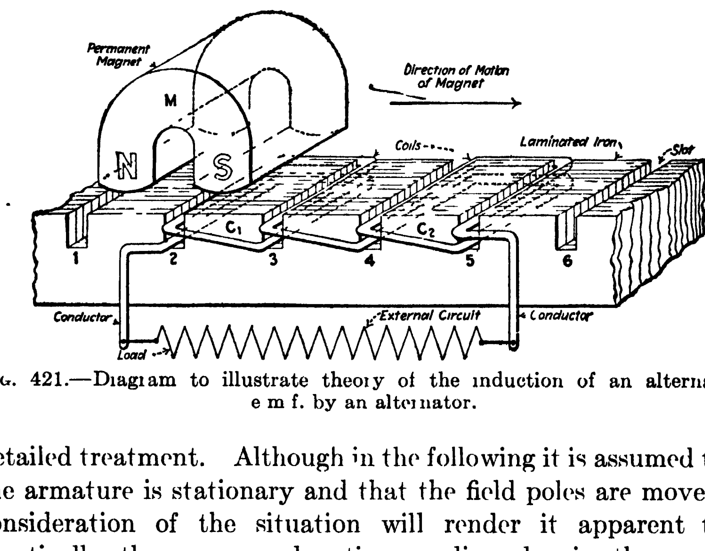


FIG. 421.—Diagram to illustrate theory of the induction of an alternating e m f. by an alternator.

detailed treatment. Although in the following it is assumed that the armature is stationary and that the field poles are moved, a consideration of the situation will render it apparent that practically the same explanation applies also in the case of moving-armature (revolving-armature) machines.

Explanation.—Consider the apparatus of Fig. 421. A permanent horseshoe magnet (Art. 89), M, provides a magnetic field as shown in Fig. 422. The magnet may be moved longitudinally, just above but not touching the stationary laminated iron slab, which has conducting coils mounted in, but insulated from, slots in its surface. The iron slab may be considered as the equivalent of the developed (Art. 636) armature core of an alternator. The arrangement of the armature coils represents a typical one for alternators but is not the only way in which the coils may be arranged. However,

the general principle to be described holds for all practicable arrangements of coils.

Now if the permanent magnet, M,—the equivalent of field magnets of a revolving-field alternator—is pushed in either direction along over the coils, as shown in Figs. 421 to 434, a sine-wave alternating e.m.f. will be induced in the coil inductors, as will be shown. Obviously, an alternating current will then be impelled in the external circuit connected to the inductors.

An alternating e.m.f. will be induced in the inductors if the permanent magnet is started from the right end and swept toward the left, or if it is started at the left end and swept toward the right. In the following explanation it will be assumed that the magnet is moved from left to right.

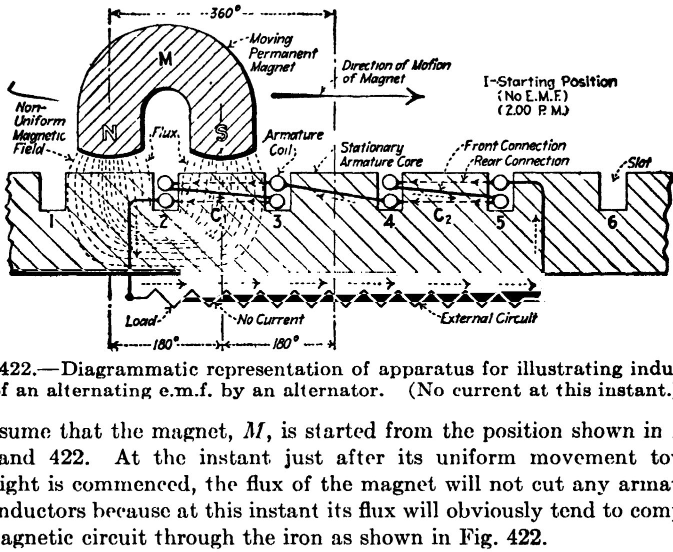


FIG. 422.—Diagrammatic representation of apparatus for illustrating induction of an alternating e.m.f. by an alternator. (No current at this instant.)

Assume that the magnet, M, is started from the position shown in Figs. 421 and 422. At the instant just after its uniform movement toward the right is commenced, the flux of the magnet will not cut any armature-coil inductors because at this instant its flux will obviously tend to complete its magnetic circuit through the iron as shown in Fig. 422.

Hence, at this instant, no e.m.f. will be induced in the armature inductors. However, as the movement of the magnet is continued, its flux starts to cut the inductors. The flux from the magnet poles, as they are moved toward the right, starts cutting the inductors gradually because the poles should be so proportioned, as described hereinafter (Art. 749), that the flux is weak toward the sides of the poles and strongest toward their centers. The instant that the flux commences cutting the inductors then an e.m.f. will be induced in the inductors. The e.m.f. induced at any instant will, as has been shown, be proportional to the rate of cutting at that instant (Art. 474). The directions of the e.m.fs. will be as shown in Fig. 423 for the conditions of that instant. Verify the e.m.f. directions, using the right-hand rule for determining the direction of an induced e.m.f. of Art. 462.

At the instant just after the magnet has been moved through 30 electrical degrees (Fig. 405) a flux of considerable strength is cutting the inductors in slots 2 and 3. Therefore, an e.m.f. proportional to this rate of cutting is induced at this instant in these inductors. The e.m.f. induced at this

instant should be proportional to the vertical distance *AB* (Fig. 423) if the device (shown in Fig. 421) has been correctly designed to induce a sine-wave e.m.f.

Note that the e.m.fs. induced in the inductors in slot 2 are in series with and in the same electrical direction as those induced in the inductors in

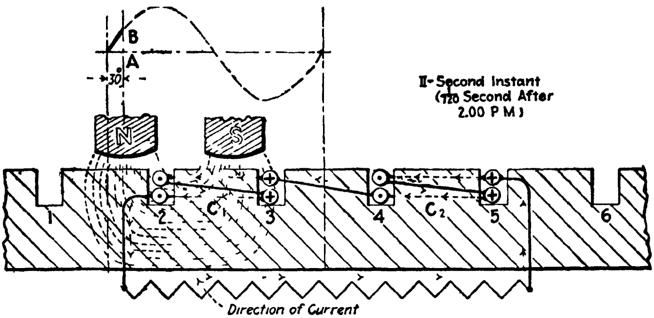


FIG. 423.—Field magnet has been shifted through 30 electrical degrees. (Current has started to flow in the positive direction in the external circuit.)

slot 3. The total e.m.f.—the sum of that induced in the inductors in slot 2 and that induced in the inductors of slot 3—impels a current through the external circuit This current is also forced through the inductors in slots 4 and 5, since they are in series with the other inductors, although there is no e.m.f being induced in these inductors (in 4 and 5) at this (Fig. 423) instant. Note that the current in the external circuit, at this instant, is

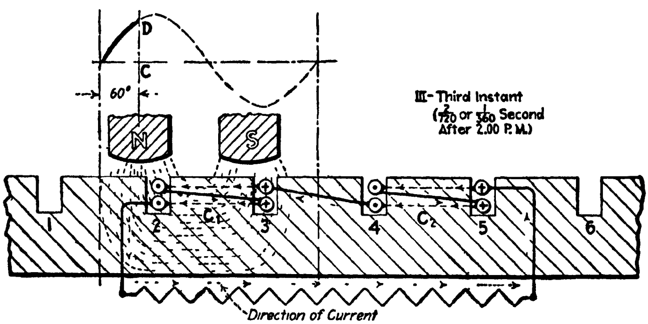


FIG. 424.—Field magnet has been shifted through 60 electrical degrees. (A greater current is at this instant flowing in the external circuit in the positive direction.)

from left to right—which we shall designate as the positive direction (Art. 558). The length of the dotted arrows, which indicate current direction, is proportional to the current intensity at this instant.

As the movement of the permanent magnet, *M*, toward the right is continued, the flux will continue to cut the inductors in slots 2 and 3. The

e.m.f. will become greater from instant to instant because, while the magnet is being shifted toward the right, the flux cutting the inductors becomes stronger as the centers of the magnet poles approach positions directly over the inductors in the slots. At the instant of Fig. 424 the magnet has been shifted through 60 electrical degrees. The e.m.f. being induced at this

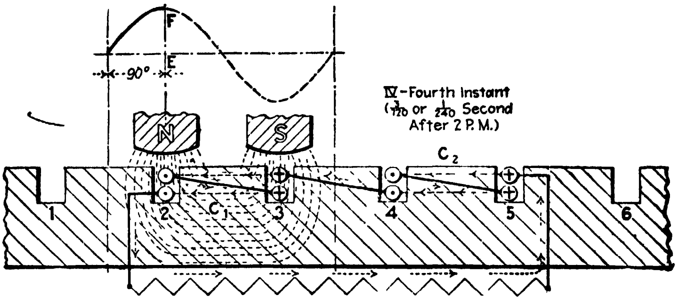


FIG. 425.—Field magnet has been shifted through 90 electrical degrees. (Current has, at this instant, attained its maximum value in the external circuit in the positive direction.)

instant is proportional to the vertical distance *CD*. The e.m.f. is still in the positive direction (hand rule, Art. 462) and the current forced by it through the circuit is in the positive direction, as represented in direction and in intensity by the dotted arrows.

As the uniform movement of the magnet, *M*, toward the right is continued, the position of Fig. 425 is soon attained. At this instant the inductors

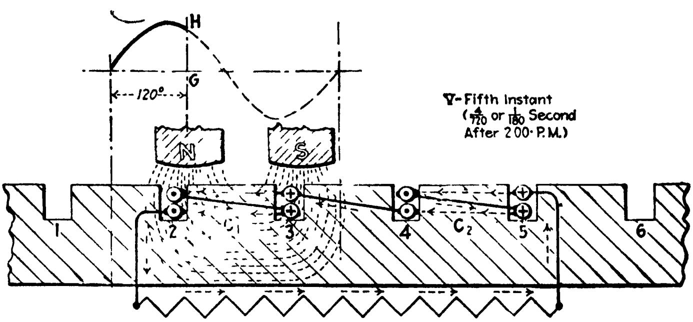


FIG. 426. —Field magnet has been shifted through 120 electrical degrees. (Current is now decreasing but is still flowing in the positive direction in the external circuit.)

are being cut by the field that is directly under the centers of the magnets where the field is strongest. Hence, at this instant, a maximum e.m.f. proportional to *EF* is being induced. The current at this instant is therefore a maximum—and it is in the positive direction.

Now as the transition of the magnet, *M*, is continued, the inductors are being cut by a weaker flux, hence the e.m.f. now decreases from instant to

instant. At the instant of Fig. 426, the e.m.f. is proportional to only *GH*. At the instant of Fig. 427 the e.m.f. is proportional to *IJ*. The current is obviously now also decreasing from instant to instant—but is still in the positive direction.

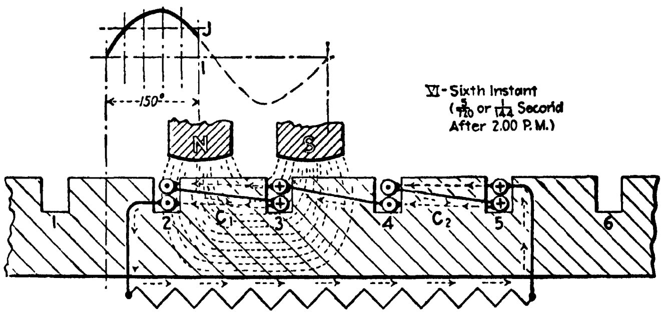


FIG. 427.—Field magnet has been shifted through 150 electrical degrees. (Current at this instant is still decreasing but is still flowing in the positive direction.)

When the position of the instant of Fig. 428 is reached, no flux is cutting the inductors. Hence no e.m.f., as indicated at *K*, is being induced and there is no current in the circuit. Note that the e.m.f. and the current have gradually increased to a maximum and decreased gradually to zero, in the positive direction *JK*, as shown by the sine curve *XBDFHJK*, while the

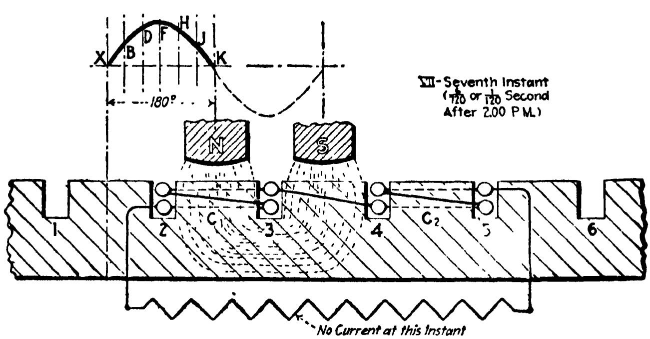


FIG. 428.—Field magnet has been shifted through 180 electrical degrees. (No e.m.f. is being induced at this instant, hence there is no current in the external circuit.)

magnet, *M*, has been shifted through 180 electrical degrees. The e.m.f. has passed through one alternation.

As the movement is continued, the flux will now cut the inductors of slots 3 and 4. But note (Fig. 429) that the flux is now cutting the inductors in a direction opposite to that obtaining previously. It follows that the e.m.f. and current will be in a direction opposite to that obtaining during

the first 180 deg. At the instant of Fig. 429, after the magnet has been shifted through 225 deg., the e.m.f. has now increased in the negative

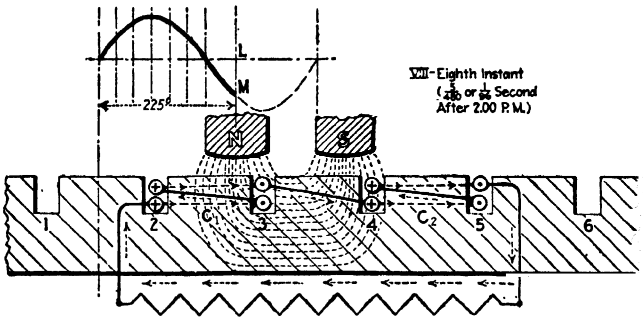


FIG. 429.—Field magnet has been shifted through 225 electrical degrees. (E.m.f. and current are now in the negative direction in the external circuit and the current is, at this instant, increasing in this direction.)

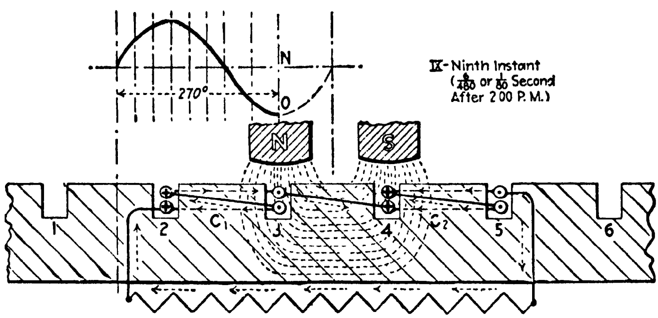


FIG. 430.—Field magnet has been shifted through 270 electrical degrees.

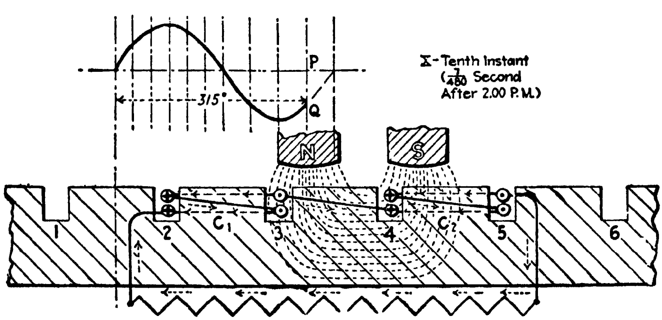


FIG. 431.—Field magnet has been shifted through 315 electrical degrees.

direction so as to be proportional to *LM*. The dotted arrows represent the direction and intensity of the current at this instant. (The pictures are

now showing conditions at 45-deg intervals instead of at 30-deg intervals as during the first 180 deg

Figure 430 pictures conditions at the 270-deg instant The e m f and current have increased to a maximum in the negative direction The e m f

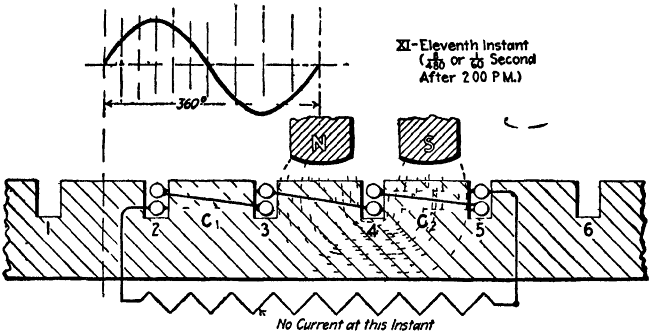


FIG 432 —Field magnet has been shifted through 360 electrical degrees

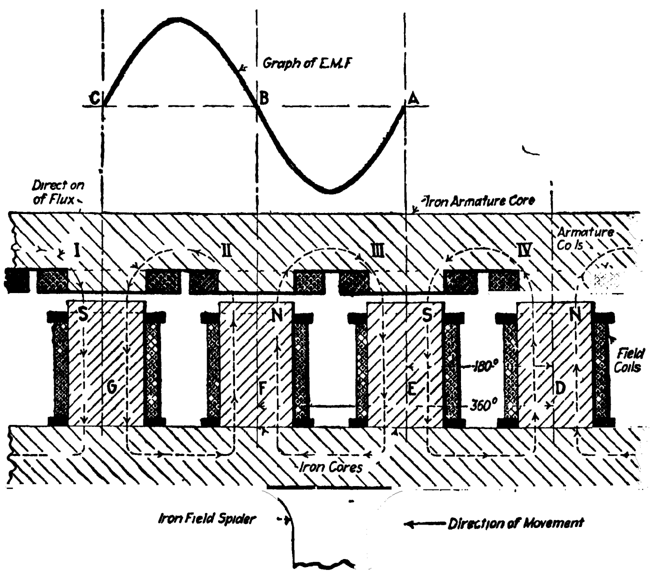


FIG 433 —Development of armature and field indicating method whereby an alternating e m f is induced

at this instant is proportional to NO Now as movement is continued, the e m f induced will decrease gradually, as shown by the sine curve until, at the instant shown in Fig 431, it will have decreased to a value proportional to PQ E m f and current are in the negative direction

Figure 432 diagrams conditions after the magnet has been shifted through 360 degrees. The e.m.f. and current have again decreased to zero at this instant. A cycle has been completed.

If it is assumed that the permanent magnet, M, was moved at such a uniform rate as to induce a 60-cycle e.m.f., it would require just $\frac{1}{60}$ sec. to move it through the 360 electrical degrees constituting a cycle. If the magnet were started (Fig. 422) at just 2 o'clock, Fig. 432 would (for a frequency of 60 cycles) represent conditions $\frac{1}{60}$ sec. after 2 o'clock. The intervening illustrations illustrate correspondingly the conditions at the different time instants as noted.

Consideration of the facts will then render it obvious that if a number of armature coils are arranged in an armature core and a suitable number of electromagnetic poles are arranged (as shown in Fig. 433) so that the poles may be swept past the coils, alternating e.m.fs. will be induced in these coils when the poles are shifted. By forming the developed armature into a ring and by mounting the poles on a wheel (Fig. 415) to rotate within the armature ring, an arrangement whereby the poles may be continuously swept past the coils is provided. Such an arrangement would constitute a revolving-field alternator (Fig. 412).

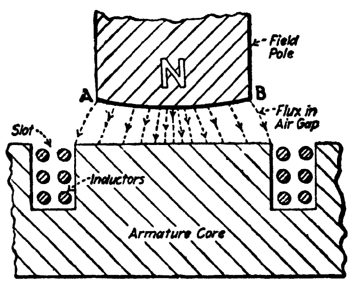


FIG. 434.—Showing how the density of the flux under a field pole may vary.

749. The distribution of flux under a field-pole face is shown in Fig. 434. Note that in this illustration the flux is much denser at the middle of the pole than at the sides. Rounding off the pole face, AB, as illustrated obviously tends to produce this condition. As is noted in the explanation under Art. 748, such a nonuniformity of field, if consistently planned, assists toward the production of a sine-wave form e.m.f. If the flux in the air gap were uniform all along under the pole face, the e.m.f. induced in an inductor, when the pole was swept past the inductor, would tend to have a flat-topped form somewhat like that of Fig. 82,I rather than a sine-wave form. In fact the wave form of the e.m.f. induced by an alternator can be controlled to a considerable extent by varying the contour of the pole face. The wave form is also dependent on the arrangement of the inductors in the armature slots. Distributing the inductors along in the armature

surface (rather than concentrating them at a few locations) tends toward the production of a sine-wave e.m.f.

750. The simplest explanation of the induction of an alternating e.m.f. in a coil, past which a magnet pole is moved, follows from the rule of Art. 463. While a moving field pole is approaching a position just opposite a stationary coil, it is obvious that then the flux enclosed by the coil is increasing. Hence, an e.m.f. is then induced in one (clockwise or counterclockwise) direction in the coil (Fig. 435). When the pole is moving away from the coil, the flux is decreasing, which effects the induction of an e.m.f. in the opposite direction. Thus, as the pole approaches, moves

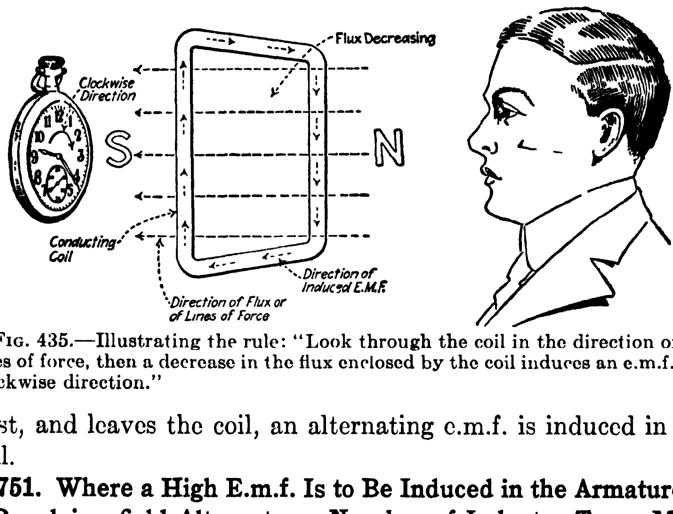


FIG. 435.—Illustrating the rule: "Look through the coil in the direction of the lines of force, then a decrease in the flux enclosed by the coil induces an e.m.f. in a clockwise direction."

past, and leaves the coil, an alternating e.m.f. is induced in the coil.

751. Where a High E.m.f. Is to Be Induced in the Armature of a Revolving-field Alternator a Number of Inductor Turns Must Be Connected in Series.—That this should be true will be evident from a consideration of the statements of Art. 601, where it is explained that in direct-current generators a number of turns are connected in series where voltages sufficiently high to be used in practice are to be induced. It is obvious, therefore, that the rate of cutting flux (Art. 474), hence the induced voltage, obtainable with an arrangement like that of Fig. 432 would be very small. For the induction of usable voltages a number of armature turns of the generator must be connected in series as shown diagrammatically in Fig. 436, which illustrates a principle rather than actual construction. The greater the number of turns in series in the armature winding, the greater the rotating-

field flux, and the greater the rotational speed of the field, the greater will be the rate of cutting. Hence, the effective induced e.m.f. will be increased if any one of these factors is increased and it will be decreased if any one of them is decreased.

752. An alternation is completed when a field pole has been moved a distance equal to the distance between center lines of adjacent field poles, that is, the distance AB (Fig. 433). A study of illustrations Figs. 421 to 432 and the next accompanying them will demonstrate the truth of this statement.

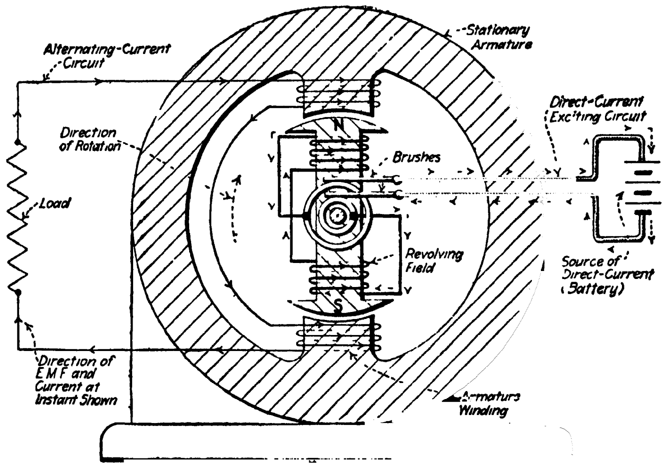


FIG. 436—Diagram of a two-pole, revolving-field generator (single phase). (Strictly, the pole faces indicated above should be no wider than the armature coils to insure correct generation of e.m.fs.)

Also, note that a cycle is completed when any field pole has been moved a distance AC (Fig. 433) equal to the distance between the center lines of alternate field poles—poles of like polarity.

753. To compute the frequency of the e.m.f. induced by any alternator (the following rule and formulas are true for revolving-armature or revolving-field and polyphase or single-phase alternators): Multiply the number of pairs of poles by the speed of the armature in revolutions per second. That is,

$$\text{Cycles} = \frac{\text{poles}}{2} \times \frac{\text{r.p.m.}}{60} = \frac{\text{poles} \times \text{r.p.m.}}{120} \tag{201}$$

$$\text{Poles} = \frac{120 \times \text{cycles}}{\text{r.p.m.}} \tag{202}$$

$$\text{R.p.m.} = \frac{120 \times \text{cycles}}{\text{poles}} \tag{203}$$

Wherein cycles = frequency of the machine in cycles per second.
r.p.m. = revolutions per minute of the rotor—either the field or the armature—of the machine.
poles = number of poles of the machine, either field or armature poles.

It also follows that

$$\text{Alternations per second} = \frac{\text{poles} \times \text{r.p.m.}}{60} \tag{204}$$

$$\text{Alternations per minute} = \text{poles} \times \text{r.p.m.} \tag{205}$$

Alternations are ordinarily expressed in alternations per minute and unless definitely specified in alternations per second, the term alternations is taken to designate alternations per minute. The term cycles is ordinarily understood to refer to cycles per second.

Explanation.—Obviously, from the explanation of Art. 752, each time the loop of an elementary bipolar alternator (Fig. 276) is rotated one complete revolution, the e.m.f. values pass through 1 cycle (Art. 721). If the speed is 1 revolution per sec., the frequency will evidently be 1 cycle per sec. Thus, with 1 revolution per sec. and one pair of poles, the frequency is 1 cycle per sec. Plainly, if the speed is increased, the frequency will be increased in proportion. In formula (201) above, r.p.m. is divided by 60 to get it into cycles per sec. *Poles* is divided by 2 to obtain number of pairs of poles.

A consideration of the facts will demonstrate that the e.m.f. induced by the two-pole revolving-field alternator of Fig. 436, which has one pair of poles, will pass through a cycle per revolution; with 2 revolutions per sec., the frequency will be 2 cycles per sec., etc. Now with the four-pole revolving-field alternator of Fig. 419, which has two pairs of poles, there will, manifestly, be twice as many cycles per revolution. Hence it is apparent that the frequency is also directly proportional to the number of poles.

See the table in the author's "American Electricians' Handbook" showing frequencies for different numbers of poles and speeds, and vice versa.

Example.—If a two-pole alternator (Fig. 436) is driven at 3,600 r.p.m., what will be the frequency? *Solution.*—Substitute in formula (201) cycles = poles × r.p.m. ÷ 120 = 2 × 3,600 ÷ 120 = 7,200 ÷ 120 = 60 cycles.

Example.—If the four-pole alternator of Fig. 419 is to operate at 60 cycles, at what speed must it be driven? *Solution* (formula 203).—r.p.m. = 120 × cycles ÷ poles = 120 × 60 ÷ 4 = 7,200 ÷ 4 = 1,800 r.p.m.

NOTE.—It is apparent from the foregoing examples that, for a frequency of 60 cycles, if 7,200 is divided by the number of poles, the r.p.m. will be the result. If 7,200 is divided by the speed the number of poles will be the result. For 25 cycles, the constant 3,000 may be similarly used. Thus, for a 60-cycle circuit, the 12-pole field of Fig. 416 would have to be rotated at 7,200 ÷ 12 = 600 r.p.m. For a 60-cycle circuit, the 100-pole field of Fig. 415 would have to turn at 7,200 ÷ 100 = 72 r.p.m.

754. Commercial Alternating-current Generators Develop E.m.fs. of Approximate Sine-wave Form.—Usually they are almost but not quite true sine waves. This matter has already been discussed in Art. 561. While the wave form of the e.m.f. induced in a single loop rotated at a uniform speed in a uniform magnetic field (as in Fig. 276) is a true sine curve, this wave form

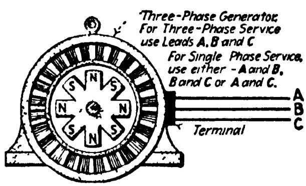


FIG. 437.—Use of a three-phase generator for single-phase service.

is not inherently characteristic of alternators. Practical alternators do not have single-loop inductors nor are their magnetic fields uniform. In fact, skillful designing is necessary in commercial machines to obtain the approximate sine-wave forms which these machines produce. The advantages of the sine-wave form are enumerated in Art. 561.

755. Single-phase generators are seldom manufactured now, first, because there is little demand for them and, secondly, because polyphase generators are more economical in the utilization of their constructive materials. Where it is necessary that a single-phase current be generated, it is the practice to use two phase wires of a three-phase machine (Fig. 437). The single-phase capacity of a three-phase machine is 70 per cent of its three-phase capacity. That is, a 100-kva., three-phase machine will deliver 70 kva., single phase.

SECTION 44

ALTERNATING-CURRENT VECTORS AND VECTOR DIAGRAMS

756. The words "lagging" or "lag" as they are used in alternating-current parlance really refer to time. If an alternating current (or an alternating voltage) lags behind another alternating current (or voltage) the current (or voltage) which lags, attains its maximum value in each alternation a certain interval of time later than does the current (or voltage) behind which it lags. Since time may be expressed in electrical degrees (Art. 565), it may be stated that a current—or voltage—lags behind another current—or voltage—by a certain number of degrees. With a frequency of 25 cycles (Fig. 77), 1 deg. is equivalent to $\frac{1}{25}$ sec. ÷ 360 deg. = $\frac{1}{25} \times \frac{1}{360}$ = 1/9,000 sec. With a frequency of 60 cycles (Fig. 79) 1 deg. is equivalent to $\frac{1}{60}$ sec. ÷ 360 deg. = $\frac{1}{60} \times \frac{1}{360}$ = 1/21,600 sec.

Example.—In Fig. 467 the current lags behind the impressed e.m.f. by 90 degrees. That is, in each alternation, the current attains its maximum value (at I_m), 90 deg. later than the impressed e.m.f. attains its maximum value (at A). If the current shown were a 25-cycle current, it would then with conditions as illustrated reach, in each alternation, its maximum value ($90 \times 1/9{,}000 = \frac{1}{100}$ sec.) later than the e.m.f. attained its maximum value With a 60-cycle current and the conditions as shown, the current would lag behind the voltage by $90 \times 1/21{,}600$ sec. = $\frac{1}{240}$ sec.

757. The words "leading" or "lead" as used in reference to alternating-current phenomena also refer to time. When a current (or voltage) leads another current (or voltage), it attains its maximum value in each alternation a certain interval of time before the voltage (or current) which it leads attains its maximum value. The amount of lead in any case may be expressed either in degrees or in seconds as may lag, as described above.

Example.—In Fig. 492 the current, A, leads the impressed e.m.f., B, by 90 degrees, as shown for instance at XY. In Fig. 467 the impressed e.m.f. leads the current by 90 deg., as shown for instance at MN, and the impressed e.m.f. leads the counter e.m.f. by 180 deg., as shown for instance at PQ.

758. There Are Two Graphic Methods of Representing Alternating E.m.f. or Current Values—by Vectors and by Sine Curves.—That this statement is true might be inferred from preceding statements (Art. 563). Each of the two methods has, as will be demonstrated, its applications. Furthermore, it is sometimes desirable to use the two in conjunction with one another.

759. A vector (review Art. 563) is a line representing by its length the intensity, and by an arrow the direction, of some certain alternating e.m.f. (voltage) or alternating current.

Example.—Refer to Fig. 438. The line *AB* represents, to scale, an e.m.f. of 10 volts in one, say the positive, direction (Art. 558). Then *DC* represents, to the same scale, an e.m.f. of 20 volts in the opposite or negative

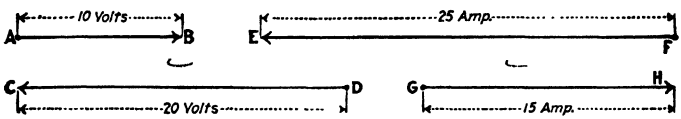


FIG. 438.—Vectors representing voltages and currents.

direction. Similarly, *FE* may represent to scale a current of 25 amp. in the negative direction while *GH* represents, to the same scale, a current of 15 amp. in the positive direction. Vectors may, obviously, be drawn to any convenient scale. Where two vectors are to be compared, both should be drawn to the same scale.

760. Vectors are sometimes assumed to rotate as in Figs. 292 and 439. When a vector rotates (Art. 563), the distance, at any given instant, of the end of the vector describing the circle, from the zero (usually taken as the horizontal) axis is proportional to the instantaneous value of the e.m.f. or current developed at the corresponding instant.

Examples.—The line *OA* in Fig. 439 is a rotating vector. The length of line *ED* is proportional to the instantaneous e.m.f. induced at the 45-deg. instant.

The distance around the circle of Fig. 439 is measured in degrees, and each degree represents a certain interval of time. (The actual length of the time interval in seconds can in any case, where necessary or desirable, be determined by considering the frequency as shown in Figs. 77 and 79.) Instantaneous values—for example, *ED*—determined as above described, may be plotted into a sine curve as shown in Fig. 439.

NOTE.—As ordinarily used a vector represents the magnitude of the *maximum* value of the alternating wave current or voltage, but it may, if drawn to proper scale, represent *effective* value. Figure 439 shows two methods of representing an alternating wave. The wave may be plotted on a time axis, XY, in which case the amplitude, BC, of the wave is the maximum value, and instantaneous values are ordinates (vertical distances between curve and base line) to the curve at the particular instants considered, for example DE is the instantaneous value at the 45-deg. instant.

Another way of plotting the wave is by a vector OA, the length of which is proportional to the maximum value of the wave and which is supposed to rotate counterclockwise at such a constant speed that it makes one complete revolution in the time required for the wave to complete one cycle. The instantaneous values of the wave are always equal to the projection of the vector at the given instant on a vertical line. Thus, it is shown in Fig. 439 that the ordinate ED of the curve at the 45-deg. instant is equal, by construction, to the projection P of the vector OA.

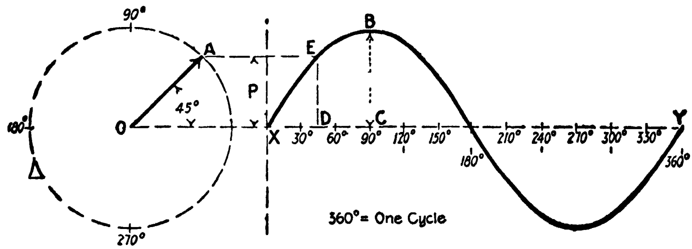


FIG. 439.—Explanation of vector conventions.

In the wave figure the difference of time phase is represented by the distance on the axis XY between the zero points of the waves. It is evident from the above that a rotating vector figure always represents *maximum* values. It is possible to represent *effective* values to a different scale by nonrotating vectors because they are equal to maximum values divided by the constant, 1.41 (or multiplied by 0.707, see formula 197). Instantaneous values can not be shown directly in a vector figure or phase diagram, but must be obtained by taking projections.

NOTE.—The lines of different kinds—open, black, dotted, dashed, etc.,—used in a number of the following illustrations, have—insofar as they are different kinds of lines—no special significance. The reason why the different kinds of lines have been used for representing different electrical quantities and values is merely so that the line representing one quantity may be readily distinguished from another line in the same diagram representing a different quantity. Plain black, thin lines could have been used for all of the lines in all of the diagrams—but the diagrams might then be much more difficult to follow.

761. Reference lines in phase or vector diagrams are often drawn before any of the vectors are plotted so that when the

vectors are plotted they can be located in some definite position. Thus Fig. 440,I shows how such reference lines AA and BB, are usually drawn at right angles to one another. Obviously these reference lines may be of an indefinite—any convenient—length. At II is shown a vector $O'E'$ plotted in the 45-deg.

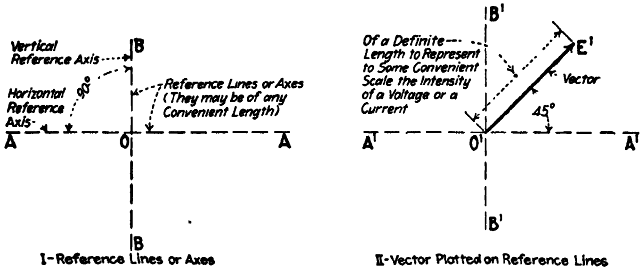


FIG. 440.—Showing reference lines or axes for vector diagrams and their application.

phase position—assuming counterclockwise rotation. Usually vectors, in phase diagrams, representing current values or values in phase with current values are plotted along the horizontal axis or reference line, as shown in Fig. 441. Also vectors representing the energy e.m.f.—to be discussed later, Art. 784—are usually plotted on the horizontal axis. The vector OA in Fig. 441 represents to scale, as shown, an alternating current of 80 amp., which may be either a maximum or an effective value.

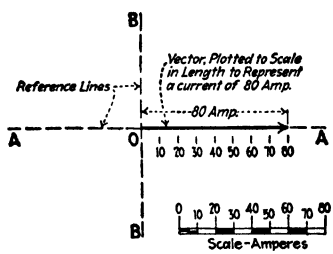


FIG. 441.—A vector representing a current of 80 amp. plotted on reference lines.

762. Phase diagrams and vector diagrams are shown in the accompanying illustrations. A phase diagram represents the magnitude and relative phase positions of electric pressures (e.m.fs. or voltages) or currents. A vector diagram represents the magnitude of the pressures or currents but *may* not represent their phase relations. A vector diagram is always a polygon and is often a triangle. Both phase and vector diagrams are composed of vectors which may (except where the vector is rotating) represent either effective or maximum values. As suggested above, rotating vectors can represent only maximum values. Since an effective value is always equal to 0.707 times its

maximum value (Art. 737), the same vector (unless it is a rotating vector), if the proper scale is assumed, may represent either an effective or a maximum value.

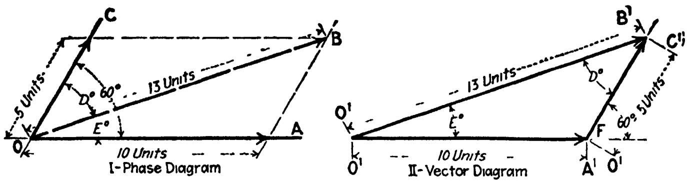


FIG. 442.—Phase and vector diagrams.

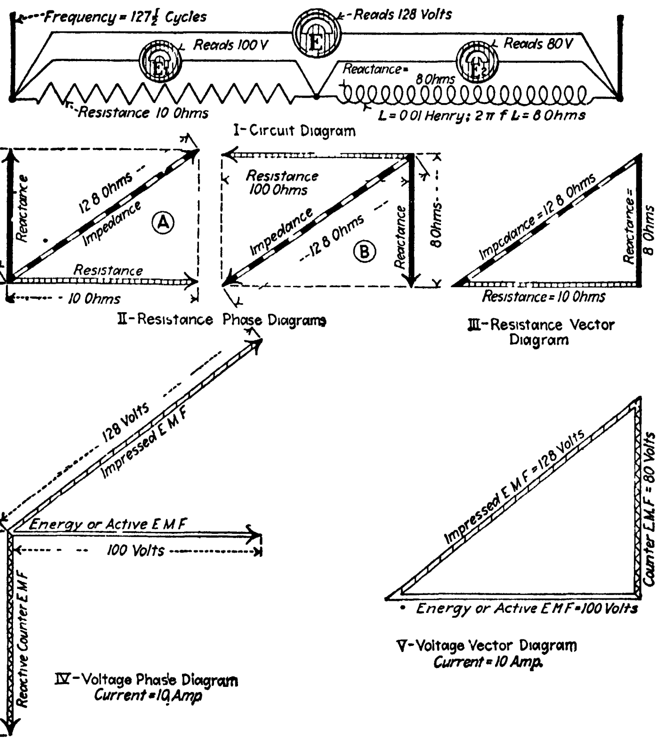


FIG. 443.—Examples of phase and vector diagrams.

Examples of phase and vector diagrams. For the principle of composition, first consider the phase diagram of Fig. 442,I. If an alternating e.m.f. of 5 volts be represented by *OC*, 5 units long, and another e.m.f. of 10 volts, of the same frequency but which lags 60 deg. behind the first, be represented by *OA*, 60 deg. away from *OC* and 10 units long, then *OB* (which

scales 13 units long) will represent their resultant or sum. In other words, the length of the line *OB* to scale is equal to the vector sum of the e.m.fs. of 5 volts and 10 volts which differ in phase by 60 deg. The line *OB* is obtained by completing a parallelogram, having *OC* and *OA* for two of its sides then the diagonal *OB* is the sum (vector sum) of *OC* and *OA*. The resultant e.m.f., 13 volts, will lag *D* degrees behind *OC* and will lead *OA* by *E* degrees.

The vector diagram, Fig. 442,II, which is equivalent to the phase diagram of I, is obtained by drawing lines equal and parallel to those representing the same values in the phase diagram. Note that in the vector diagram,

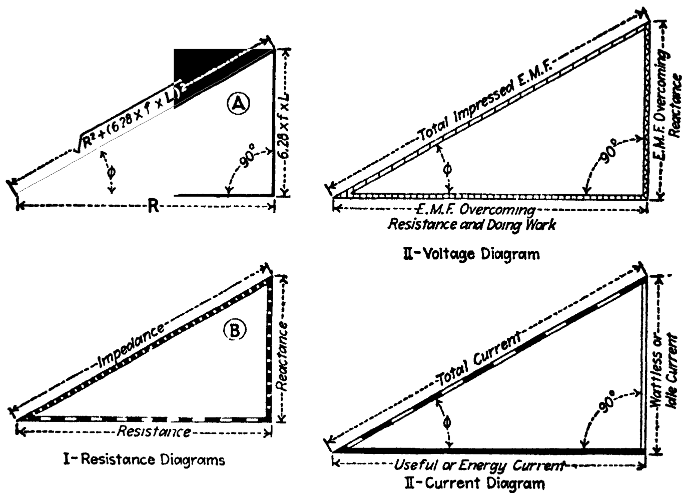


FIG. 444.—Typical vector diagrams.

the phase relations (angles) between *O'B'* and *O'A'* and between *O'B'* and *O'C'* are correct and correspond to those in I. But to obtain the true angle (60 *degrees*) between *O'C'* and *O'A'*, the angle *F* (in II) must be taken. The lines *OC*, *O'C'*, *OA* and *O'A'* are called *components; OB* and *O'B'* are respectively their *resultants.*

In Fig. 443 are shown the phase and vector diagrams of the resistance and voltage conditions obtaining in the circuit and with the frequency indicated. These diagrams and their significance will be further explained in following matter. Some other typical vector diagrams, which will also be further treated subsequently, are shown in Fig. 444. The symbol ϕ is a Greek letter pronounced phi; it is often used as a designation expression to indicate a lag angle. While the diagrams of Figs. 442, 443, and 444 show voltage values, similar diagrams may be plotted to represent current values

763. The resolution of an alternating e.m.f. into components is a process which is the inverse of combining two alternating e.m.fs. to ascertain their resultant e.m.f. or their sum as just

described in the example under Art. 762. Any alternating e.m.f. of a given frequency may be resolved into two e.m.fs. of the same frequency, which two e.m.fs. will differ in phase from one another by a selected phase angle. That is, the sum of the two e.m.fs. thus obtained will equal the resultant—or their combined—e.m.f. The process involved is similar to that used in mechanics for resolving a force into two components and may be best explained by a specific example:

Example.—Resolve an e.m.f. of 280 volts into two component e.m.fs. which differ in phase by 60 deg. so that one of these component e.m fs. will be 180 volts. It is understood that the component and resultant e m.fs. are of the same frequency *Solution*—Draw to scale (Fig. 445) a line, OM, 180 units long at an angle of 60 deg. from a base line OP of indefinite length This line OM represents the component e m.f. of 180 volts. Draw a line MT, of indefinite length, parallel to OP. With a compass, with O as a center and a radius 280 units long strike a part of a circle QR. From the point, V, where the circle cuts MT, draw VN parallel to MO. Then NO, which scales about 150 units long—or 150 volts—is the other component. Hence, an e.m.f. of 280 volts may be resolved into two component e m fs , differing in phase by 60 degrees, of 180 volts and 150 volts respectively. That is, the resultant sum of two e.m.fs. of 180 volts and 150 volts which differ in phase by 60 deg. is 280 volts. While the diagram of Fig. 445 shows voltage values, similar diagrams may be plotted to represent current values.

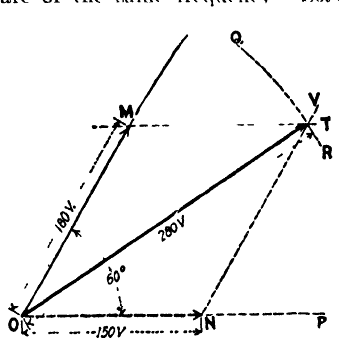


FIG 445—Illustrating the resolution of an alternating e.m f., OV, into two components.

764. An alternating e.m.f. may be resolved into an infinite number of pairs of components because it is obvious that for every different value of lag angle between components, the components will be of different values. Furthermore, for every change in value of one of the components there will be a corresponding change in value of the other component. This situation is illustrated in the following example, wherein e.m.f. values are resolved into components; current values may, by similar methods, be resolved into components.

Example.—Consider Fig. 446, wherein the e.m.f. OE of 150 volts, shown at I, resolved into component e.m.fs. of different values and differing in phase by different angles. In the phase diagram of II the e.m.f. of 150

volts, OE, is resolved into two components, differing in phase by 90 deg., of 130 volts and 75 volts respectively; at III is shown the corresponding vector diagram. In IV the e.m.f. of 150 volts is resolved into two component e.m.fs. of 100 volts and 110 volts which also differ in phase by 90 deg.; the equivalent vector diagram is plotted at V.

In VI, the 150-volt e.m.f. has been resolved into two components of 130 volts and 33 volts, which differ in phase by 60 deg., and in VII the same

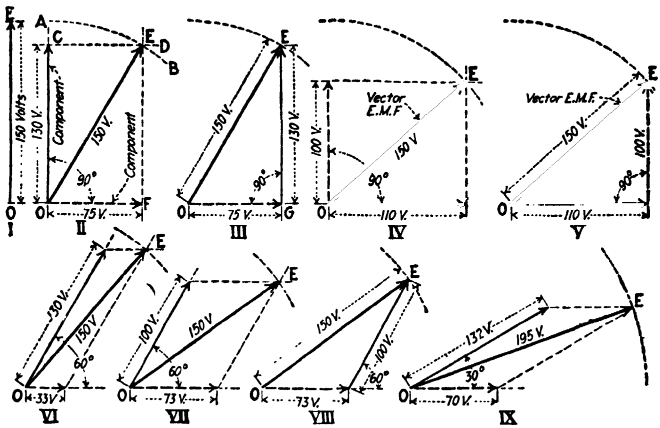


FIG. 446.—Examples of the resolution of an e.m.f., OE, into components.

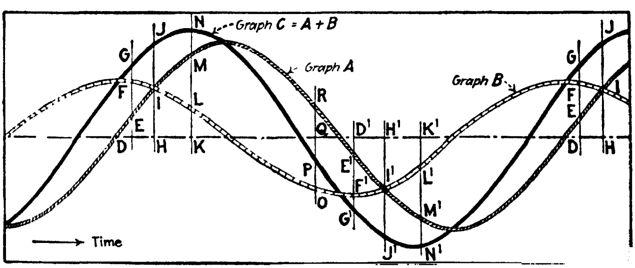


FIG. 447.—Illustrating the addition of sine curves.

150-volt pressure is again resolved into two e.m.fs. differing in phase by 60 deg., but in this example they are 100 volts and 73 volts; the equivalent vector diagram is shown in VIII. In IX an e.m.f. of 195 volts has been resolved into two components of 132 volts and 70 volts, respectively, which differ in phase by 30 deg.

765. Sine Curves Can Be Added and Subtracted (see Fig. 447).—If two sine curves (A and B, Fig. 447) each of the same frequency but differing in phase, are added together, a third sine curve (C) of the same frequency can be plotted which will represent their sum.

Example.—To obtain curve C (Fig. 447) the heights of corresponding points in A and B are added. Thus, $DG = DE + DF$, $HJ = HI + HI$, $KN = KL + KM$ and $PQ = QO - RQ$.

Similarly, sine curves of the same frequency can be subtracted from one another, by subtracting heights of points in one curve from the corresponding heights in another.

Example.—Curve $C = A + B$, $B = C - A$, and $A = C - B$, all from Fig. 447.

SECTION 45

THE ADDITION AND SUBTRACTION OF ALTERNATING-CURRENT VALUES

766. The addition and subtraction of alternating e.m.f. and current values can, where the values are represented by vectors, be readily effected by utilizing the principles of the composition (Art. 762) or resolution (Art. 763) which were briefly described hereinbefore. Where the e.m.f. or current values are represented by sine curves the addition or subtraction of these values can be made by utilizing the principles suggested for the addition and subtraction of sine curves previously outlined in Art. 765. Where the e.m.fs.—or currents—are to be added to or subtracted from

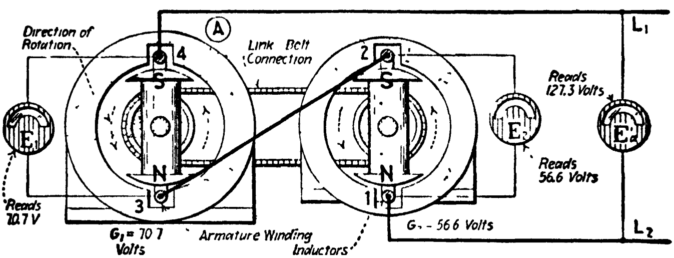


FIG. 448.—Two single-phase, alternating-current generators of the same frequency which are in phase connected in series so that their e.m.fs. augment or add up.

other e.m.fs.—or currents—with which they are in phase (Art. 725) the operation is, as will be shown, very simple in that it involves merely arithmetical addition or subtraction. But if the e.m.fs.—or currents—to be added are not in phase, then the process is somewhat more complicated. Where the values are not in phase, the addition or subtraction can most readily be made graphically by plotting suitable phase or vector diagrams. Also, it is frequently desirable, for the purposes of explanation, to plot sine curves in conjunction with the vector or phase diagrams, which curves may represent the same values as those presented by the vectors.

767. Four different cases in the addition and subtraction of e.m.f. values will be considered in some detail in Arts. 768 to 771. While these cases which will be cited refer specifically to e.m.f. values, precisely the same principles and process would be employed in adding or subtracting alternating-current values. The sources of e.m.f. in each of these four examples will (Fig. 448 and following illustrations) be represented by generators which are supposed to be rotating uniformly in a counterclockwise direction and developing the effective e.m.fs. indicated. All of the generators are of the same frequency. While generators are shown as sources in these examples, any other sources of alternating-current e.m.fs. could be used instead. Where the generators are in phase (Figs. 448 and 454) they are maintained in phase by the link-belt connection. Where the generators are not in phase (Figs. 451 and 457), the phase difference between them is maintained by the link-belt-drive connection.

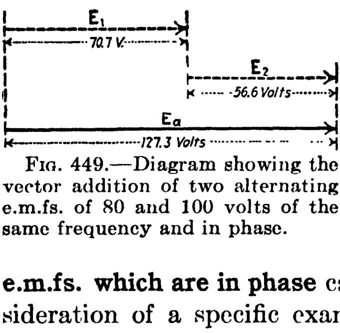


FIG. 449.—Diagram showing the vector addition of two alternating e.m.fs. of 80 and 100 volts of the same frequency and in phase.

768. The method of adding two e.m.fs. which are in phase can be best demonstrated by the consideration of a specific example. In this and in the examples which follow it is assumed that the conditions outlined in the preceding paragraph are satisfied:

Example.—What (Fig. 448) is the sum of an e.m.f. of 70.7 volts and one of 56.6 volts which are in phase? In other words, what e.m.f. will the two generators of Fig. 448, which are in phase and connected in series so that their e.m.fs. augment, impress upon the line at L_1 and L_2? *Solution.*—Since these two e.m.fs. are in phase they may be added directly to obtain their sum, or resultant. That is, 70.7 volts + 56.6 volts = 127.3 volts, which is the e.m.f. impressed across L_1 and L_2 as shown in Fig. 448. A graphic statement of this addition, wherein vectors are used to represent the e.m.f. values, is shown in Fig. 449. From this it is evident that the length of the vector E_1, which is equivalent to 70.7 volts + the length of the vector E_2, which is equivalent to 56.6 volts = the length of the vector E_a or 127.3 volts. Hence, E_a is the vector representing the magnitude, 127.3 volts, and direction of the e.m.f. impressed across L_1 and L_2 in Fig. 448.

Now refer to Fig. 450. At I the vector diagrams are repeated but in this illustration the lengths of the vectors are, as always must be the case with rotating vectors, proportional to the maximum values of the e.m.fs. (Maximum values may, Art. 737, be obtained by multiplying the corre-

sponding effective values by 1.41.) Since all three of these e.m.fs., E_{1m}, E_{2m} and E_{am} are in phase they lie directly over one another in the vector

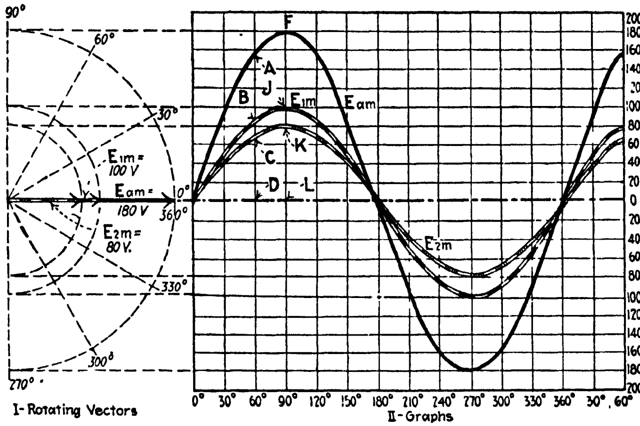


FIG. 450.—Rotating vectors and curves showing addition of e.m.fs. of 80 and 100 volts of the same frequency and in phase.

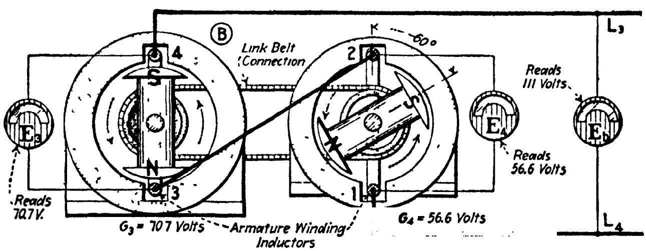


FIG. 451.—Two single-phase, alternating-current generators which are not in phase connected in series so that their e.m.fs. augment or add up.

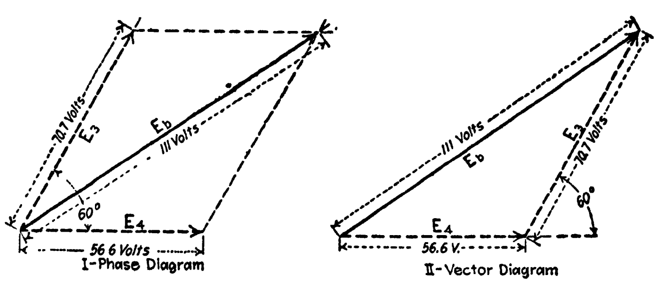


FIG. 452.—Diagrams showing the vector addition of two alternating e.m.fs. of 80 and 100 volts of the same frequency but differing in phase by 60 degrees.

diagram. The curves of II were plotted following the method hereinbefore described in Art. 760. It is evident that the sine curve E_{am} is equal to the

sum (Art. 765) of the sine curves E_{1m} and E_{2m} For example the distances $AD = BD + CD$ and $FL = JL + KL$.

769. The method of adding two e.m.fs. which are not in phase will be explained by considering Figs. 451, 452, and 453, and by means of a specific example.

Example.—What is the sum of an e.m.f. of 70.7 volts and one of 56.6 volts which differ in phase by 60 deg.? In other words, what e.m.f. will the generators G_3 and G_4 of Fig. 451, which are connected in series so that their e.m.fs. augment, impress across the line L_3 and L_4? *Solution.*—The phase diagram of Fig. 452,I shows a graphic solution of this problem, the vector sum of E_3 and E_4 being E_b which is equivalent to scale to 111 volts

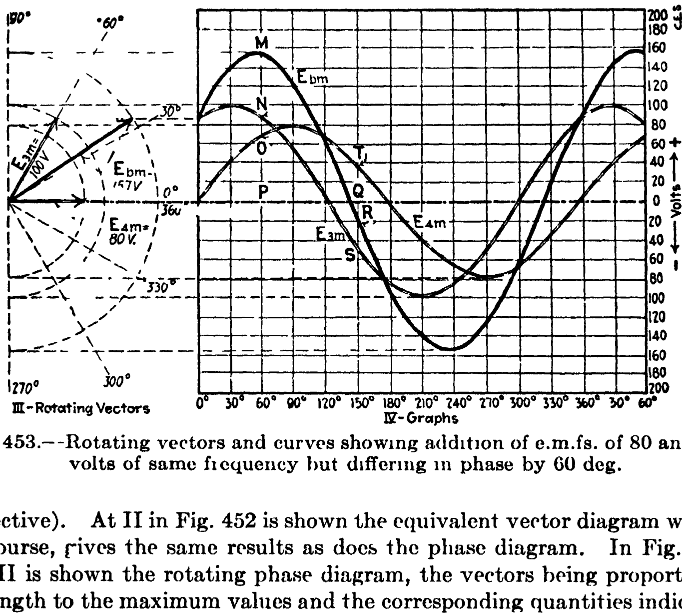


FIG. 453.—Rotating vectors and curves showing addition of e.m.fs. of 80 and 100 volts of same frequency but differing in phase by 60 deg.

(effective). At II in Fig. 452 is shown the equivalent vector diagram which, of course, gives the same results as does the phase diagram. In Fig. 453, at III is shown the rotating phase diagram, the vectors being proportional in length to the maximum values and the corresponding quantities indicated on the phase diagram of Fig. 452,I. By using the method hereinbefore described (Art. 762) the sine-curve representation of IV (Fig. 453), corresponding to the vector representation of III (Fig. 453), can be plotted. By inspection it will be evident that the sine curve E_{bm}. is equal to the sum (Art. 765) of the sine curves E_{3m}. and E_{4m}. For example, $MP = NP + OP$ and $QR = QS - QT$.

770. The method of obtaining resultant of two e.m.fs. which differ in phase by 180 deg. is illustrated in Figs. 454, 455, and 456 and is demonstrated in the following example:

Example.—The two generators of Fig. 454, which are developing, respectively, pressures of 70.7 and 56.6 volts, are connected in series, but, as shown, the connections are such that the e.m.fs. oppose or buck one another. What is the resultant e.m.f.? That is, what pressure is impressed across L_5 and L_6 on the line? *Solution.*—The vector solution of the problem is shown in Fig. 455, from which it is obvious that if the vector E_6, which is proportional in length to 56.6 volts, is subtracted from the vector E_5, which is proportional

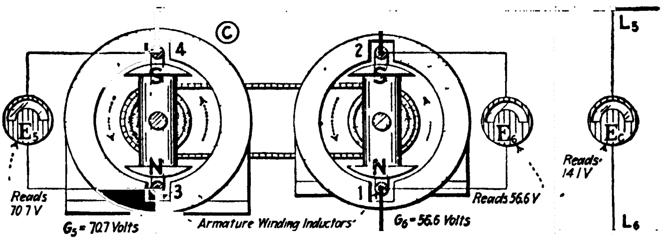


FIG. 454.—Two single-phase, alternating-current generators of the same frequency, which are in phase, connected in series so that their e.m.fs. oppose or buck.

to 70.7 volts, the difference is represented by the vector E_c, proportional in length to 14.1. (These vectors may thus be subtracted from one another because the e.m.fs. which they represent are all in phase.) Therefore, the e.m.f. across L_5 and L_6 is, as suggested in Fig. 454, 14.1 volts. In Fig. 456,V the rotating vectors (which are, of course, proportional in length to the maximum value of the e.m.fs. involved) are shown. From this rotating vector diagram the sine curves shown in Fig. 456,VI were plotted. A consideration of these curves will indicate that the curve E_c, which has a maximum value of 20 volts, is equal to the difference between (algebraic sum) the curves, E_5, maximum value 100 volts and E_6, maximum value 80 volts.

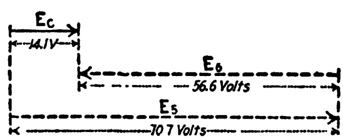


FIG. 455.—Diagram showing the vector subtraction of two alternating e.m.fs. of 56.6 and 70.7 volts of the same frequency and in phase.

771. The method of obtaining the difference of two e.m.fs. which are not in phase is outlined in Figs. 457, 458, and 459. The conditions specified in Art. 767 also apply for this problem which is solved below.

Example.—What will be the e.m.f. impressed across the line L_7 and L_8 by the two generators of Fig. 457. The generators are in series but they are so connected, as shown, that the e.m.fs. which they are developing oppose one another and, furthermore, differ in phase by 60 deg. In other words, what is the resultant e.m.f. produced by generators G_7 and G_8 combined, under the conditions outlined? *Solution.*—First consider the vector solution of the problem. When two e.m.fs. are in series so that they boost one another (Figs. 451 and 458,I), they can be added vectorially as shown in

the diagram at Fig. 458,III. But if one of the e.m.fs. is reversed, for example E_8 in II, then the e.m.f. values must be subtracted from one another, instead of added, to obtain their resultant. How this subtraction is effected vectorially is diagramed in IV. Since the e.m.f. E_8 is reversed in direction in II (Fig. 458), the vector representing it in the phase diagram must also

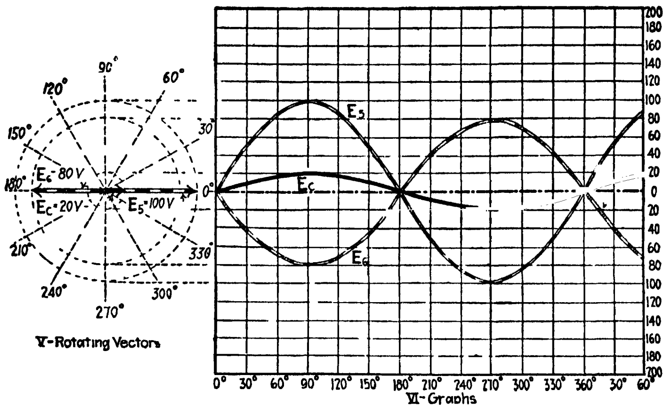


FIG. 456.—Rotating vectors and curves showing subtraction of e.m.f of 80 from 100 volts, both e.m.fs. are of the same frequency and are in phase.

be reversed in direction, as shown at OX. In other words, to perform the vector subtraction it must be plotted in the position OY. Then the vector OW is proportional in length to the difference between the e.m.fs. E_7 and E_8, reversed which differ in phase by 60 deg. That is, the actual difference in phase between E_7 and E_8 is 120 deg., as shown in IV. V (Fig. 458) is

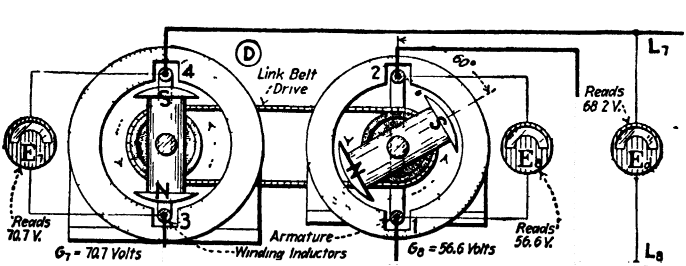


FIG. 457.—Two single-phase, alternating-current generators of the same frequency, which are not in phase, connected in series so that their e.m.fs. oppose each other.

merely an enlarged phase diagram of IV, and VI shows the corresponding vector diagram. Hence, it is apparent from these diagrams, if the length of E_d is measured, that the effective e.m.f., E_d across L_7 and L_8, is 68.2 volts.

If the e.m.fs. in the two coils differ in phase by a certain number of degrees and one of these coils is reversed, then, after the reversal the differ-

ence in phase between the two e.m.fs. will be 180 deg. minus the former phase difference in degrees. That is, in Fig. 458,I the difference in phase between the e.m.fs. of the two coils is, because of the conditions of the prob-

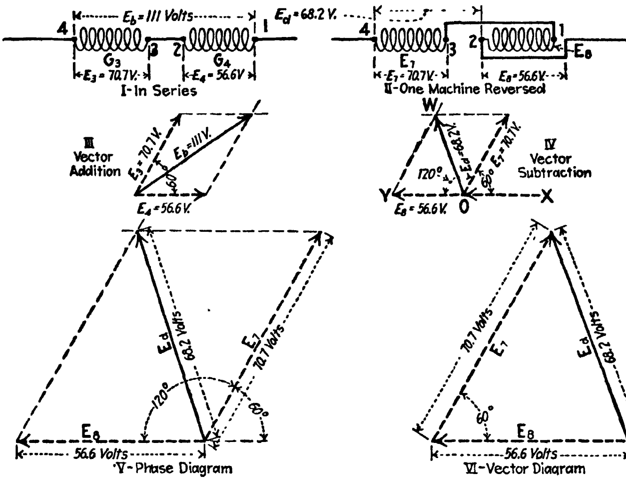


FIG. 458.—Illustrating vector addition and subtraction.

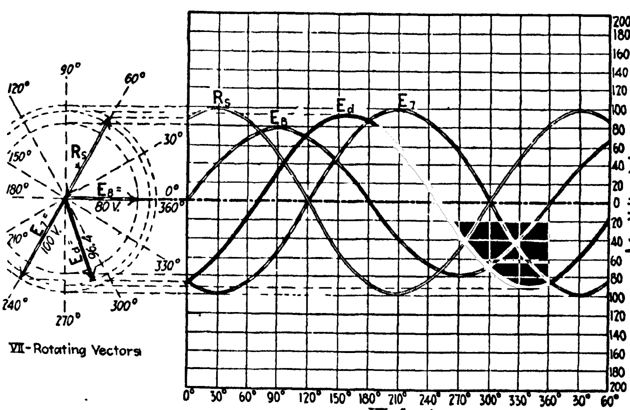


FIG. 459.—Rotating vectors and curves showing subtraction of e.m.fs. of 80 from 100 volts of same frequency but differing in phase by 60 deg.

lem, 60 deg. Then, if one of the coils is reversed, as shown at II, the difference in phase between the e.m.fs. of the two coils is 180 deg. − 60 deg. = 120 deg.

In Fig. 459 are shown the rotating vectors—maximum values—corresponding to the effective-value vectors of Fig. 458,V. In

the curve of VIII are shown the sine curves which may be plotted from these rotating vectors. Curve R, represents the variation of the e.m.f. OX (Fig. 458,IV). It will be noted that curve E_d = curve E_7 + curve E_8 and it is also evident that curve E_d = curve E_8 − curve R_s. This demonstrates the truth of the fact that the vector R_s, leading E_8 by 60 deg., is equivalent to the vector E_7 which lags behind E_8 by 120 deg., as diagramed in Fig. 458,IV and in Fig. 459,VII.

SECTION 46

EFFECTS OF RESISTANCE AND INDUCTANCE IN ALTERNATING-CURRENT CIRCUITS

772. The effect of resistance in alternating-current circuits which contain resistance only, that is, which contain no inductance (Art. 508) or permittance (capacitance, Art. 793), is the same as its effect in direct-current circuits. Resistance in alternating-current circuits limits the current therein and causes a loss or drop in voltage just as it does in direct-current circuits (Art. 140). With a given alternating e.m.f. impressed on a circuit, the current in it will vary inversely as its resistance. In an alternating-current circuit containing resistance only, the current is in phase with the impressed e.m.f. It follows that Ohm's law (Art. 151) is, without qualification, true for alternating-current circuits containing resistance only. Thus

$$I_E = \frac{E_F}{R} \text{ (amp.)} \tag{206}$$

and

$$E_E = I_E \times R \text{ (volts)} \tag{207}$$

or

$$R = \frac{E_E}{I_E} \text{ (ohms)} \tag{208}$$

Wherein I_E = effective current in the circuit, in amperes.
E_E = effective e.m.f. or voltage impressed across the circuit, in volts.
R = resistance of the circuit, in ohms.

Example.—What current will flow in the alternating-current circuit of Fig. 460 which contains resistance only. The impressed pressure (effective voltage) as indicated by the voltmeter is 110 volts, the resistance of each lead is 0.70 ohm and the resistance of the incandescent lamp is 200 ohms. The frequency of the current may be anything. *Solution.*—Substitute in the Ohm's law formula (206): $I_E = E_E \div R = 110 \div (0.7 + 0.7 + 200) = 110 \div 201.4 = 0.546$ amp.

Example.—What voltage must the alternating-current generator (it may develop any frequency) of Fig. 461,I develop to impel a current of 2 amp.

(effective) in the circuit shown which, it is assumed, has a total resistance of 50 ohms? *Solution.*—Substitute in equation (207) above. Thus $E_E = I_E \times R = 2 \times 50 = 100$ volts. Hence to circulate the current of 2 amp. (effective) the generator should produce 100 volts (effective) across its terminals A and B.

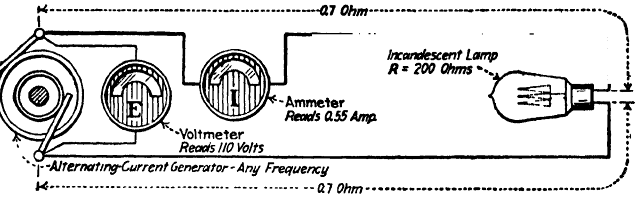


FIG. 460.—Illustrating the effect of resistance in an alternating-current circuit.

773. There is a loss of power when an alternating current flows through resistance just as there is when a direct current flows through resistance. The power loss, in watts, in any conductor conveying either an alternating or a direct current, always equals *the square of the current in amperes (effective current in alternating-current circuits) multiplied by the resistance of the conductor in ohms.* This rule is perfectly general and applies to

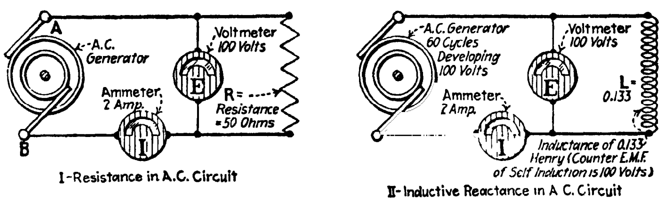


FIG. 461.—Examples of resistance and inductive reactance in alternating-current circuits.

alternating currents of all ordinary voltages and frequencies even if they contain inductance (Art. 508) or permittance (capacitance, Art. 793). There is no power loss in inductance or permittance. The electrical power thus lost reappears as heat and raises the temperature of the conductor. Expressed as a formula

$$P = I^2_E \times R \text{ (watts)} \tag{209}$$

or

$$I_E = \sqrt{\frac{P}{R}} \text{ (amp.)} \tag{210}$$

or

$$R = \frac{P}{I^2_E} \text{ (ohms)} \tag{211}$$

Where P = power lost in the conductor, in watts.
I_E = effective current, in amperes, in the conductor.
R = resistance of the conductor, in ohms.

Note that the above formulas are identical with those of Art. 189 for direct current, except that in this alternating-current formula, the current, I_E, is the effective current.

Example.—What is the power loss in the incandescent lamp in Fig. 460? *Solution.*—The lamp, as shown in the picture, has a resistance of 200 ohms and the current through it is 0.55 amp. Then, substituting in equation (209) $P = I^2_E \times R = (0.55 \times 0.55) \times 200 = 60.5$ watts.

Example.—What is the power loss in the inductive winding of Fig. 462 when an alternating current of 5 amp. flows in it? The winding has a resistance of 10 ohms. *Solution.*—Substitute in equation (209) $P = I^2_E R = (5 \times 5) \times 10 = 250$ watts. With a current of 5 amp., the power loss in the winding would be 250 watts regardless of the frequency and regardless of the fact that it is an inductive winding.

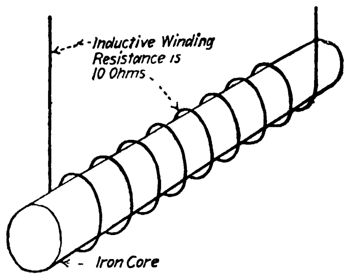


FIG. 462.—Illustrating power loss in an inductive winding.

774. In alternating-current circuits, containing resistance only, the current is in phase with the impressed voltage, as shown in Fig. 463,III. That is, in such circuits, the current attains its crest or maximum values at the same instants as those at which the impressed e.m.f. (which circulates the current in the circuit) reaches its crest or maximum values (see definition of "Phase" in Art. 725).

Example.—Fig. 463,I shows the diagram of an alternating-current circuit containing resistance only. The alternating e.m.f. (effective) impressed on the circuit is, as indicated by the voltmeter, 100 volts. The resistance of the circuit is 2 ohms. Then, since the circuit contains resistance only, by Ohm's law the effective alternating current in the circuit is $I_E = E_E \div R = 100 \div 2 = 50$ amp. The phase diagram given at II shows the phase relation of this e.m.f. and the current which it propagates. They are in phase with each other.

The light line CB, the impressed-e.m.f. vector, is made 141 units long to represent a maximum of 141 volts, which is the maximum corresponding to an effective e.m.f. of 100 volts (Art. 737). The heavy black line CA is made 70.5 units long to represent 70.5 amp., which is the maximum current corresponding to an effective current of 50 amp. (Art. 737). At III the curves of the e.m.f. and current are plotted, and they indicate that the e.m.f. and current are in phase, as defined in Art. 725.

775. The effects of inductance in alternating-current circuits are, in general, much more noticeable than those which it produces in direct-current circuits. Why this is the case will be explained. In Art. 508 and following articles (which should be carefully reviewed) the phenomenon of inductance was discussed. It was there shown that whenever the current, in a circuit having inductance, changes in intensity, a counter e.m.f. of self-inductance (Art. 516) is induced in the circuit. With direct currents, the effects of inductance are most noticeable: (*a*) just after a circuit

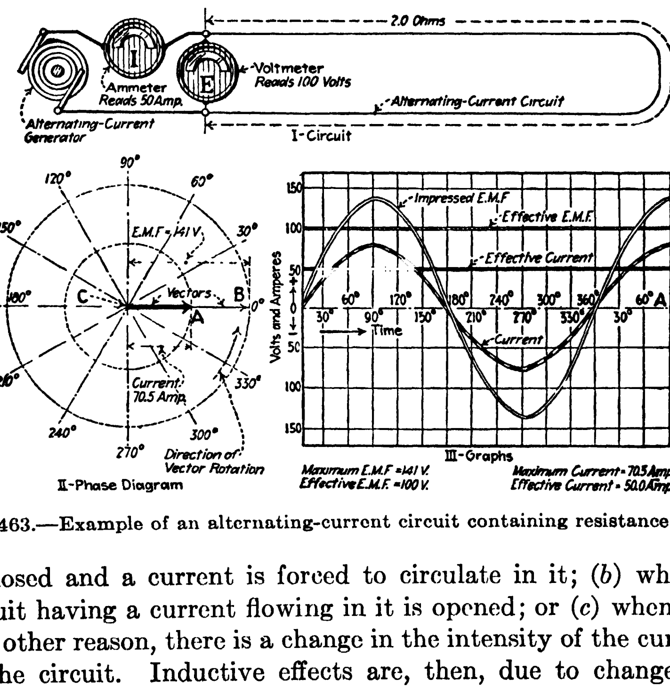


FIG. 463.—Example of an alternating-current circuit containing resistance only.

is closed and a current is forced to circulate in it; (*b*) when a circuit having a current flowing in it is opened; or (*c*) when, for any other reason, there is a change in the intensity of the current in the circuit. Inductive effects are, then, due to changes in current intensity.

NOTE.—Now alternating currents reverse in direction at regular intervals (Art. 127) and are constantly changing in intensity (see Fig. 463). Therefore, since in these circuits the current is constantly changing in intensity, the result is that a counter e.m.f. of self-induction is, if they contain inductance, being generated constantly—all the time—in them when current is flowing. The counter e.m.f. of self-induction is always in such a direction as to oppose any change in the intensity of the then existing current (Art. 470). This counter e.m.f. of self-induction varies in intensity just as does any alternating e.m.f., but it is not in phase with the e.m.f. or current

which produces it, as will be shown. Its variation with the time follows a sine-wave form—if the current which induces it has a sine-wave form.

In order to predetermine the intensity of the current—the number of amperes—which will flow in an actual alternating-current circuit containing inductance, it is first necessary to compute—either directly or indirectly—the value of the counter e.m.f. of self-induction of the circuit and then to subtract it (as will be outlined in Art. 786) from the impressed e.m.f. which it opposes. The difference, the e.m.f. which really circulates the current in the circuit, may be called the available or energy e.m.f. (The terms "net," "active," "impelling," or "actuating" e.m.f. are all used synonymously with available or energy e.m.f. However, in the text which follows, available or energy e.m.f. will be used to designate the difference between the impressed e.m.f. and the counter e.m.f. of self-induction.) In practice this subtraction is most readily effected by introducing a new quantity which is called reactance which will be treated in following Art. 787. Note that in ideal circuits which contain only inductance or permittance, or both, but which contain no resistance, there is no available or energy e.m.f. But, for reasons which will be discussed elsewhere, an applied alternating e.m.f. would, nevertheless, propagate an alternating current in such a circuit—if such a circuit could exist. However, it is obvious that there can not be an electric circuit which does not have some resistance.

776. The counter e.m.f. of self-induction always lags 90 degrees behind the current, that is, the effect of inductance is 90 deg. behind, or at right angles to, the current. This is stated here merely as a fact; why it is true will (Art. 780) be shown later.

777. To compute the counter e.m.f. of self-induction in an alternating-current circuit—or in any circuit for that matter—the inductance of the circuit, L, is multiplied by the rate of change of current in the circuit, Art. 527. If the inductance, L, is expressed in henrys and multiplied by the average rate of change in current, the change of current per second in the circuit, the result will be the average counter e.m.f., in volts, induced in the circuit during that second; this follows from the definition of the henry (Art. 509). Now an alternating current is continually changing in intensity; hence it is continually—if there be inductance in the circuit—inducing a counter e.m.f. However, in dealing with alternating-current circuits, it is not the average counter e.m.f. which is of interest but it is the effective (Art. 731) counter e.m.f. It can be shown (see proof in the following note) that the effective rate of change of a sine-wave-form alternating current—that is, the rate of change in the effective current per sec. equals $6.28 \times f \times I$. Hence

$$E_c = 6.28 \times f \times L \times I \text{ (volts)} \qquad (212)$$

and

$$f = \frac{E_c}{6.28 \times L \times I} \text{ (cycles per sec.)} \tag{213}$$

and

$$L = \frac{E_c}{6.28 \times f \times I} \text{ (henry)} \tag{214}$$

and

$$I = \frac{E_c}{6.28 \times f \times L} \text{ (amp.)} \tag{215}$$

Wherein E_c = the counter c.m.f. (effective) of self-induction, in volts.
f = frequency of the current, in cycles per second.
L = inductance of the circuit or conductor, in henrys.
I = current (effective) in the circuit or conductor, in amperes.

NOTE.—The symbol π (a Greek letter pronounced "pie") has been universally adopted to stand for the number 3.1416. Now, $2 \times 3.1416 = 6.28$ Hence the above formula may also be written: $E_c = 2 \times \pi \times f \times L \times I$, which is the form very frequently adopted in textbooks.

Example.—What counter c.m.f. is generated in the coil of Fig. 464, which has an inductance of 0.2 henry, when an effective alternating current of 1.33 amp. flows in it, at a frequency of 60 cycles per sec.? *Solution.*—Substitute in the above formula (212) $E_c = 6.28 \times f \times L \times I = 6.28 \times 60 \times 0.2 \times 1.33 = 100$ volts. This means that the generator must impress 100 volts (effective) across the terminals of the coil to circulate a current of 1.33 amp. through the coil. It also means that when a current of 1.33 amp. at 60 cycles flows in the coil the counter c.m.f. of self-induction developed in the coil is 100 volts.

NOTE.—For proof that $6.28 \times f \times I$ = effective rate of change of a sine-wave-form alternating current. Consider the rate of change of current in an alternating-current circuit. Refer, for example, to Fig. 465, which shows the curve of a sine-wave-form alternating current having a crest or maximum value of 1 amp. Note that during 1 cycle ($OABCD$) the current: (1) increases, OA, from 0 to 1 amp., in the positive direction; (2) decreases, AB, from 1 to 0 amp., in the positive direction; (3) increases, BC, from 0 to 1 amp., in the negative direction; and (4) decreases, CD, from 1 to 0 amp., in the negative direction. Note then that the alternating current traced in Fig. 465 changes 1 amp. four times per cycle, twice from 0 to 1 amp. and twice from 1 to 0 amp.

It follows that any alternating current changes four times per cycle between zero intensity and maximum intensity; twice from maximum to zero and twice from zero to maximum. Let the symbol I_M stand for maximum current. Then the change of current per cycle $= 4 \times I_M$.

Now average rate of change of current = change in amperes ÷ seconds. With an alternating current: frequency or f = cycles per second. Hence the time in seconds required for the completion of 1 cycle = $1 \div f = 1/f$ (For example, with a 60-cycle current $\frac{1}{60}$ sec. is required for the completion of 1 cycle, Fig. 79.) The time, in seconds, required for each of the form changes from zero intensity to maximum intensity in a cycle is, obviously, one-fourth the time required for the completion of an entire cycle. Therefore, the time required for one of the changes is $\frac{1}{4} \times 1/f = \frac{1}{4} \times f$ sec That is, the current changes from 0 amp. to I_M amp. in $1 \div (4 \times f)$ sec.

The average rate of change of current as defined in the preceding paragraph—the change of current per second—must be: the change in current

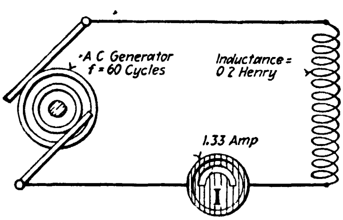


FIG 464.—Inductive winding in an alternating-current circuit

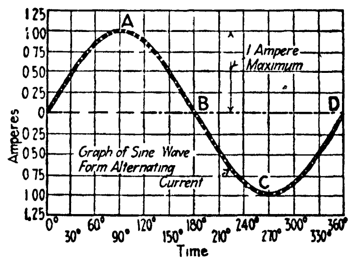


FIG 465.—Illustrating rate of change of an alternating current.

in amperes divided by the time in seconds elapsing while the change transpires; that is:

$$\text{Average rate of change of current} = \frac{I_M}{\frac{1}{4 \times f}} = 4 \times f \times I_M \qquad (216)$$

Now in practical work we deal with effective currents (Art. 731) rather than with maximum currents and hence I_M should be converted into its equivalent effective value. From equation (195) $I_M = 1\ 41 \times I_I$ or, to be more exact,

$$I_M = 1.4142 \times I_E \qquad (217)$$

Then, substituting (217) in (216),

$$\text{Average rate of change of current} = 4 \times f \times (1.4142 \times I_E) \qquad (218)$$

Or

$$\text{Average rate of change of current} = 5.6568 \times f \times I_E. \qquad (219)$$

If then this expression, $5.6568 \times f \times I_E$, which represents the average rate of change of current per second, were multiplied by the inductance, L, in henrys, of an alternating-current circuit, the product (Art. 527) would be the average (Art. 730) counter e.m.f. in volts, induced in the circuit. But we are usually interested in and deal with effective (Art. 731) counter e.m.fs., not average e.m.fs. Hence the formula (219) will now be so modified that it will give the effective rate of change of current. In equation (194) it was shown that Average values = 0.90 × effective values. Hence

Average rate of change of current = 0.90
$\times$ effective rate of change of current (220)

Now, substituting the expression of (220) in (219),

$$0.90 \times \text{effective rate of change of current} = 5.6568 \times f \times I_E \quad (221)$$

Then, simplifying,

$$\text{Effective rate of change of current} = \frac{5.6568 \times f \times I_E}{0.90} \quad (222)$$

or

$$\text{Effective rate of change of current} = 6.28 \times f \times I_E \quad (223)$$

NOTE.—Power is not, as will be shown (Art. 825), expended in overcoming a counter e.m.f. of self-induction. If there is resistance in the circuit—and there always is some, though in many cases very little—power is expended in forcing the current to circulate through the resistance.

778. The method of computing the counter e.m.f. of self-induction of any coil in an alternating-current circuit follows from the principles just outlined. From Art. 529, formula (137), the inductance of any coil, $L_1 = \phi_a \times N \div 10^8$, wherein ϕ_a = the flux produced by the coil per ampere of current flowing in it and N = number of turns in the coil. Then if this quantity, which represents the inductance of the coil, be multiplied by the average rate of change of current—the change in current per second—the product will be the average counter e.m.f. of self-induction induced (Art. 777). It has been shown (equation 216) that the average rate of change of a sine-wave-form alternating current $= 4 \times f \times I_M$. Hence if E_{CA} be taken to represent the average counter e.m.f. induced by the coil and performing the multiplication just suggested,

$$E_{CA} = 4 \times f \times I_M \times \left(\frac{\phi_a \times N}{10^8}\right) = \frac{4 \times f \times I_M \times \phi_a \times N}{100{,}000{,}000} \quad (224)$$

But, obviously, maximum amperes $\times$ flux per ampere = maximum flux, that is,

$$I_M \times \phi_a = \phi_M \quad (225)$$

Then, substituting ϕ_M for "$I_M \times \phi_A$" in equation (224)

$$E_{CA} = \frac{4 \times f \times \phi_M \times N}{100{,}000{,}000} \quad (226)$$

But equation (194) average value = 0.90 $\times$ effective value; hence

$$E_{CA} = 0.90 \times E_{CE} \quad (227)$$

Then, substituting (227) in (226)

$$0.90 \times E_{CE} = \frac{4 \times f \times \phi_M \times N}{100{,}000{,}000} \tag{228}$$

$$E_{CE} = \frac{4 \times f \times \phi_M \times N}{0.90 \times 100{,}000{,}000} \tag{229}$$

$$E_{CE} = \frac{4.44 \times f \times \phi_M \times N}{100{,}000{,}000} \tag{230}$$

Wherein E_{CE} = volts, effective, counter e.m.f. induced in any coil by a sine-wave-form alternating current.
f = frequency of the circuit in cycles per sec.
ϕ_M = maximum total flux or lines of force produced by the coil.
N = number of turns in the coil.

Example.—A certain coil has 50 turns. With a 60-cycle alternating current of such value flowing through it that a flux of 300,000 lines maximum is developed, what is the counter e.m.f. then induced in the coil? *Solution.*—Substitute in formula (230):

$$E_{CE} = \frac{4.44 \times f \times \phi_M \times N}{100{,}000{,}000} = \frac{4.44 \times 60 \times 300{,}000 \times 50}{100{,}000{,}000} = 39.9 \text{ volts}$$

Hence the induced counter e.m.f. is 39.9 volts (effective).

Equation (230) is very important, inasmuch as it is utilized repeatedly in the design of alternating-current generators, transformers, and motors.

779. To compute the counter e.m.f. of a coil in an alternating-current circuit on the basis of the area of the core of the coil and the flux density, it is merely necessary to modify equation (230) accordingly, as will be shown. In an alternating-current magnetic circuit it is, obviously, the flux density at the instants when the current is a maximum, that is, B_M, which should determine the area of the magnetic circuit. If the magnetic circuit has sufficient area to carry effectively the flux due to the maximum instantaneous current it will, plainly, carry the flux due to currents less than the maximum. Now (Art. 268) $\phi = A \times B$; hence it is obvious that $\phi_M = A \times B_M$. Then, substituting this value for ϕ_M in equation (230), the following working formula is obtained:

$$E_{CE} = \frac{4.44 \times f \times A \times B_M \times N}{100{,}000{,}000} \text{ (volts)} \tag{231}$$

Wherein all symbols have the same meanings as under equation (230) except that (Art. 268) A = cross-sectional area of core, in square inches = area—where an iron core is used—of cross section of iron in the core on which the coil is wound. B_M = maximum flux density, in lines per square inch, that is, the flux density at the instants when the current in the coil is at its maximum instantaneous values. In design, B_M is usually taken at some value below the saturation point (Art. 281).

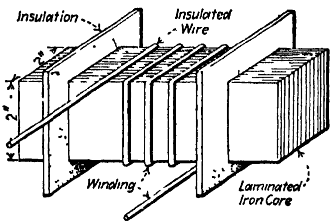


FIG. 466.—Laminated iron core with insulated-wire winding.

Example.—If an iron core similar to that of Fig. 466, except having an area of $1\frac{5}{8}$ sq. in., is wound with 700 turns of wire, what will be the effective counter e.m.f. which it develops in a 40-cycle circuit, assuming that the current through it is such that B_M = 70,000 lines per sq. in.? *Solution.*—From formula (231):

$$E_{CE} = \frac{4.44 \times f \times A \times B_M \times N}{100,000,000} = \frac{4.44 \times 40 \times 1.625 \times 70,000 \times 700}{100,000,000} = 141 \text{ volts}$$

Therefore, the counter e.m.f. developed under the above conditions (assuming that there is no magnetic leakage) is 141 volts.

Example.—It is desired to wind a coil on the laminated iron core of Fig. 466 which will develop a counter e.m.f. of 100 volts when connected in multiple on a 60-cycle circuit. Permissible maximum flux density is taken as 30,000 lines per sq. in. How many turns (assuming no magnetic leakage) will be required on the coil to produce this result? *Solution.*—Core is 2 in. $\times$ 2 in.; hence area = 4 sq. in. Solving equation (231) for N:

$$N = \frac{100,000,000 \times E_{CE}}{4.44 \times f \times A \times B_M} = \frac{100,000,000 \times 100}{4.44 \times 60 \times 4 \times 30,000} = 313 \text{ turns}$$

780. Phase relations of current and the counter e.m.f. due to inductance are shown in Fig. 467. The counter e.m.f. of self-induction in an alternating-current circuit is never in phase with the current which produces it. Nor is it in phase with the e.m.f. impressed on the circuit by the generator or other source. As has been previously stated (Art. 776) and as now will be shown, the counter e.m.f. of self-induction always lags 90 deg. behind the current. The reason for this 90-deg. lag—which represents a certain definite interval of time—is given in the following explanation.

Explanation.—The intensity of an e.m.f.—or a counter e.m.f.—induced in any conductor is determined by the rate at which the conductor cuts or is cut by flux (Art. 474). The greater the rate of cutting, the greater the induced voltage. The slower the rate, the lower the induced voltage. The counter e.m.f. of self-induction in any alternating-current circuit is

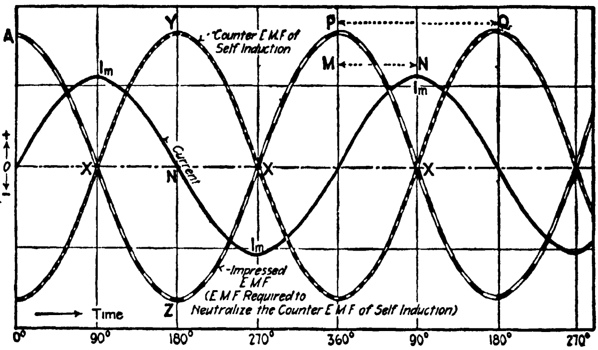


FIG. 467.—Curves of current and impressed e.m.fs. in an alternating-current circuit containing inductance only. (Actually there cannot be a circuit which does not have some resistance.)

induced by virtue of the movement of concentric lines of force (Art. 502) which emanate from the center of the conductor. These circular flux lines in expanding outward, as the current increases, or in contracting inward, as the current decreases (as the alternating current varies in intensity or changes in direction), cut the conductor (see Figs. 246 to 249) or the turns of the coil into which the conductor may be formed.

Obviously, the speed of travel and speed of cutting of these lines are proportional to the rate of change of current. It is evident from the values shown in Fig. 468 that with an alternating current the rate of change of current is constantly changing. During the time an alternating current is passing through its zero value its rate of change is greatest. For example, during the 10-deg. period *AB*, the current traced in Fig. 468 has changed (increased) in intensity by 17.4 amp. During the 10-deg. period *CD*, the rate of change is slower, viz., an increase of only 14.3 amp At about the time the current is passing through its maximum value, the rate of change is very slow indeed; during the 10-deg. period *EF* the decrease shown in the illustration is only 0.004 amp.

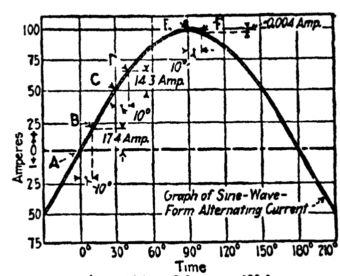


FIG. 468.—Showing how the rate of change of current varies at different periods in a cycle.

It can be shown that there is an instant, when the current is just at its maximum value, *E* (Fig. 468), during which the rate of change of current

intensity is zero. Therefore, since the intensity of a counter e.m.f. at any instant is determined by the rate of change of current at that instant, the counter e.m.f. of self-induction induced in an alternating-current circuit will be zero when the current is passing through its maximum value. The curves of Fig. 467 illustrate this condition. When the current is a maximum at Im, the counter e.m.f. is zero at X.

The rate of change of an alternating current is greatest when the current is passing through its zero value. Therefore, the counter e.m.f. of self-induction is a maximum when the current to which it is due is a minimum. In Fig. 467 at the instant when the current is zero, at N, the counter e.m.f. of self-induction is a maximum, at Y.

Since any induced counter e.m.f. is always in such a direction as to oppose any change in the current producing it (Art. 470), the curve of the counter e.m.f. must be in the relation to the curve of current that is shown in Fig. 467. At the instants when the current is increasing the counter e.m.f. is decreasing. At the instants when the current is decreasing the counter e.m.f. is increasing.

The counter e.m.f. of self-induction will lag behind the current by 90 deg., which means that the counter e.m.f. will reach its maximum intensities in each cycle, a certain interval of time (represented by 90 deg.; see example under Art. 565) later than the current reaches its maximum intensity.

781. The Current in an Alternating-current Circuit, Containing Only Inductance and Resistance, Always Lags Behind the E.m.f. Impressed by the Generator.—The amount of lag is proportional to the amount of inductance (or inductive reactance, Art. 787) in the circuit. If there is no inductance in the circuit, then there is no lag and the current will be in phase with the impressed e.m.f. as shown in Fig. 463. In a circuit consisting wholly of inductive reactance—such a circuit is, however, a physical impossibility—the current would lag exactly 90 deg. behind the impressed e.m.f., as shown in Fig. 467. With varying proportions of inductive reactance and resistance, the current—as will be shown later—will lag by some amount between 90 and 0 deg., behind the impressed e.m.f. With little inductance in the circuit there will be but little lag; with much inductance the lag may be almost 90 deg.

782. Skin effect is the name of that effect, in alternating-current conductors, whereby the current density (Art. 139) at their surfaces is greater than that along their axes. It amounts to a virtual increase in their resistances.

Explanation.—It has been shown that (see Fig. 246) current in a conductor sets up a field of circular lines of force about itself. This field is represented in Fig. 469. With an alternating current, at the instant when

the current is zero there is no field and, as the current increases during an alternation, the field of concentric circular lines forms and expands outwardly, like smoke rings from a locomotive stack, from the center of the conductor. When the current decreases to zero, during an alternation, the lines return to the center of the conductor and vanish. These circular lines of force, in expanding and returning, cut the conductor and produce self-induction and skin effect. More lines cut the metal at the axis of the conductor than cut the metal near its surface.

In Fig. 469, wherein a field is shown at its maximum value, all of the lines, represented by C, have cut the axis while only those represented by B have cut the surface. The lines represented by A do not, and can not, cut the surface. The result is that there is a greater counter e.m.f. of self-induction at the center of the conductor than at its surface. This tends to force most of the current toward the surface. With large conductors and high frequencies there is practically no current at the axis of the conductor. Most of the current is carried near the surface. A central core of some nonconducting material is often used in large stranded conductors so that the copper in the conductor will be worked at a good efficiency.

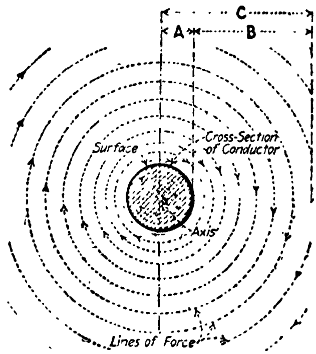


FIG. 469.—A graphic explanation of skin effect.

While the self-induction of a conductor, which causes voltage drop, and the skin effect in the conductor both originate from the same magnetic field, they are not otherwise related. They are two distinct effects and should always be considered separately. With small conductors, at commercial frequencies, the skin effect is negligible. Inasmuch as, because of skin effect, all of the sectional area of a large alternating-current conductor can not be effectively utilized, the result of skin effect amounts to an increase in the resistance of a conductor—since it increases the voltage drop and energy loss. Skin effect is considered as an increase in resistance. The following tables give constants by which the actual resistances of conductors must be multiplied to obtain their virtual resistances to alternating currents. The counter e.m.f. of self-induction of a conductor requires no energy to overcome it (see Art. 825), but energy is required to overcome the virtual increase of resistance due to skin effect.

783. Skin-effect Factors at 20°C. for Straight Cylindrical Conductors.—Values by means of which the skin effect of round conductors of copper, aluminum, iron, or steel can be obtained for any frequency:[1]

[1] "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc.

Product of circular mils times cycles per second	Skin-effect factor		Product of circular mils times cycles per second	Skin-effect factor	
	Copper	Aluminium		Iron	Steel
5,000,000	1.000	1.000	500,000	1.000	1.000
10,000,000	1.003	1.001	1,000,000	1.000	1.000
20,000,000	1.011	1.004	2,000,000	1.057	1.000
30,000,000	1.025	1.009	3,000,000	1.215	1.000
40,000,000	1.044	1.017	4,000,000	1.485	1.001
50,000,000	1.068	1.027	5,000,000	1.520	1.0015
60,000,000	1.095	1.038	6,000,000	1.640	1.002
70,000,000	1.125	1.052	7,000,000	1.750	1.003
80,000,000	1.156	1.067	8,000,000	1.850	1.004
90,000,000	1.188	1.077	9,000,000	1.950	1.005
100,000,000	1.222	1.101	10,000,000	2.040	1.006
125,000,000	1.340	1.148	12,500,000	2.250	1.009
150,000,000	1.440	1.200	15,000,000	2.440	1.013
175,000,000	1.530	1.260	17,500,000	2.620	1.018
200,000,000	1.620	1.330	20,000,000	2.780	1.023
250,000,000	1.780	1.460	25,000,000	3.080	1.036
300,000,000	1.920	1.570	30,000,000	3.340	1.051
350,000,000	2.060	1.680	35,000,000	3.600	1.069
400,000,000	2.180	1.770	40,000,000	3.820	1.088
450,000,000	2.300	1.870	45,000,000	4.040	1.107
500,000,000	2.410	1.950	50,000,000	4.240	1.131
550,000,000	2.520	2.040	55,000,000	4.440	1.155
600,000,000	2.620	2.110	60,000,000	4.620	1.177

Example.—What is the skin-effect factor with a frequency of 60 cycles for a round copper conductor of 500,000 cir. mil area? *Solution.*—(*Circular mils*) × (*cycles per second*) = 500,000 × 60 = 30,000,000. The factor corresponding to 30,000,000 for a copper conductor in the above table is 1.025. The conductor would have an apparent resistance for a 60-cycle current 1.025 times as great as for a direct current.

SECTION 47

REACTANCE AND IMPEDANCE

784. Ohm's law really applies to all alternating-current circuits as well as to all actual direct-current circuits. But the method of its application to some alternating-current circuits involves a somewhat tedious, though not complicated, process. First it will be shown, in a general way, why this is true; then a numerical example illustrating the truth will be recited:

With a direct-current circuit (Art. 151) $I = E \div R$, where I = current, in amperes, E = e.m.f., in volts, which impels I in the circuit, and R = resistance, in ohms, of the circuit. Note particularly that E = e.m.f., in volts, which impels I; this E is not necessarily the e.m.f. or voltage impressed on the direct-current circuit. If there is a source of counter e.m.f. in the circuit, the voltage E which impels the current will be impressed voltage minus counter voltage. That is, where there is a counter e.m.f. in the circuit, the available, net, or active, voltage is something less than the impressed voltage. The available voltage which actually forces current through a direct-current motor armature, which is turning, is the voltage impressed on the motor minus the counter e.m.f. induced in the motor armature because of its rotation. Excluding motor circuits, in many—in fact probably in a majority—of direct-current circuits, there is no source of counter voltage; hence for many (but not for all) direct-current circuits, the current equals the e.m.f. impressed on the circuit divided by the resistance of the circuit, because, in these cases where there is no counter e.m.f., the available or energy e.m.f. must necessarily be the impressed e.m.f.

Precisely the same general situation obtains in regard to the application of Ohm's law with alternating- as with direct-current circuits. The current in an alternating-current circuit equals the available (active or energy) e.m.f. divided by the resistance of the circuit. And to determine the available e.m.f. in any circuit, any counter e.m.f. developed in the circuit must be

subtracted from the e.m.f. impressed on the circuit—just as with direct-current circuits.

It was shown in Art. 772 that the current in an alternating-current circuit containing resistance only is equal to the e.m.f. impressed on the circuit divided by the resistance. The reason why this rule is a true one is that no counter e.m.f. is developed in an alternating-current circuit containing only resistance. Hence, in such a circuit the impressed e.m.f. is the available or energy e.m.f. But if there is permittance (capacitance) or inductance in an alternating-current circuit, each, when current flows, exerts a counter e.m.f. (Arts 775 and 793) which opposes the impressed e.m.f. Hence, to obtain the value of the available or energy e.m.f. (the e.m.f. which is actually serviceable in impelling current) in an alternating-current circuit, the counter e.m.fs., if there are such, developed due to inductance or to permittance or to both must be subtracted from the impressed e.m.f.

In direct-current circuits the counter e.m.fs.—if there are such —are in phase (Art. 725) with their impressed e.m.fs.; hence to obtain the net e.m.f. in a direct-current circuit the counter e.m.f. is subtracted arithmetically from the impressed e.m.f. (Art. 770). But in alternating-current circuits, the counter e.m.fs. of self-induction and of permittance are not in phase with the impressed e.m.f. (Arts. 776 and 803). That is, the counter e.m.fs. of inductance and permittance do not attain their maximum values at the same instants as those at which the impressed e.m.f. attains its maximum values. It follows that, to obtain the net —available—e.m.f. in an alternating-current circuit, the subtraction of the counter from the impressed e.m.f. can not be made arithmetically. It must be made vectorially and the phase relations between the e.m.fs. concerned must be given due consideration. Just how this may be done, and a verification of the fact that Ohm's law holds for alternating-current circuits, are shown in the illustrative example which follows. In the circuit of this example there are only resistance and inductance. Circuits containing permittance are treated in another section of the book.

Example.—In a certain coil in a 60-cycle, alternating-current circuit (Fig. 470), the current is 11 amp., with an impressed e.m.f. of 110 volts. The resistance of the coil is 5 ohms, and its inductance is 0.023 henry. Analyze this situation and show that the current—11 amp.—in the coil is

equal to the available or energy e.m.f. across the coil divided by its resistance —5 ohms. Also show that the available or energy e.m.f. is equal to the impressed e.m.f. minus the counter e.m.f. of self-induction developed in the coil.

Solution.—Now the resistance of the coil AB (Fig. 470) is 5 ohms. Hence, by Ohm's law (Art. 151) the available or energy e.m.f. which actually impels the 11-amp. current in it must be $E_e = R \times I = 5 \times 11 = 55$ volts.

The impressed e.m.f., E_i, is 110 volts. Since the coil has an inductance of 0.023 henry and the current in it is 11 amp., the counter e.m.f. of self-induction developed in the coil must be (Art. 777) $E_c = 6.28 \times f \times L \times I = 6.28 \times 60 \times 0.023 \times 11 = 95.3$ volts.

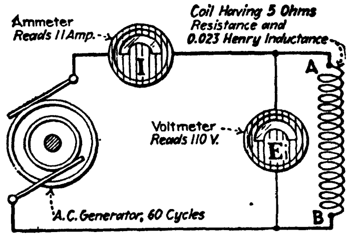


FIG. 470.—Current and voltage relations in a circuit having 5 ohms resistance and 0.023 henry inductance.

Now proceed with the solution of the problem to show that the energy e.m.f., 55 volts, is equal to the impressed e.m.f., 110 volts, minus the counter e.m.f. of self-induction developed in the coil, 95.3 volts. That is, show that 55 volts is the difference between an impressed e.m.f. of 110 volts and a counter e.m.f. of self-induction of 95.3 volts.

First draw reference lines (Art. 761) at right angles to one another as shown at I in Fig. 471. Now plot a vector, OE_e, as shown in II, to repre-

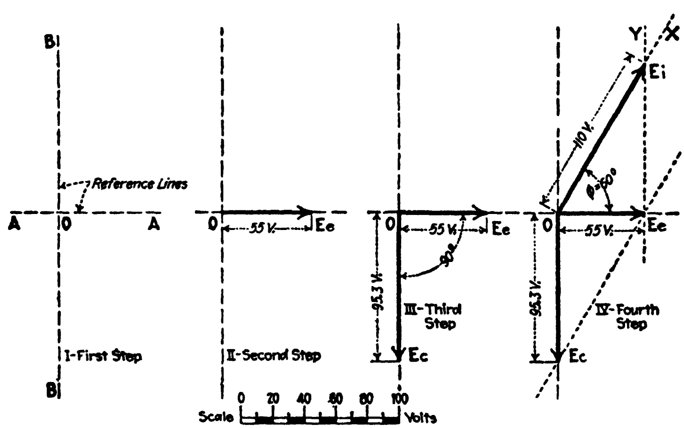


FIG. 471.—Illustrating the process of a vector solution of a problem.

sent in length to scale, the energy e.m.f. of 55 volts. This vector is plotted horizontally because the e.m.f. it represents is in phase with the current and (as suggested in Art. 761) it is usually most convenient to plot horizontally, vectors representing values which are in phase with the current.

Now plot the vector representing the counter e.m.f. of self-induction of 95.3 volts. Since (Art. 776) the counter e.m.f. of self-induction always lags

90 deg. behind the current—and consequently 90 deg. behind the energy e.m.f.—this vector plotted to scale 95.3 volts long will fall in the position OE_c, as shown at III.

There have now been plotted, in III, two vectors OE_e and OE_c representing, in length and phase relation, the energy e.m.f. of 55 volts and the counter e.m.f. of self-induction of 95.3 volts. It now remains to plot another vector which will be called OE_ι, which will represent in length and phase relation the impressed e.m.f. of 110 volts. If the vector OE_e (in III) represents the difference between two e.m.fs. and OE_c represents one of the two, it follows from the suggestions of Arts. 764 and 771 that OE_ι (in IV) must represent the resultant e.m.f. of the two. The vector OE_ι is plotted thus: Draw the construction line E_cE_e; then draw OX through O parallel to E_cE_e. Now draw construction line E_eY parallel to OE_c. Then the point, E_ι, of intersection of the OX and E_eY will determine the length of the vector OE_ι which is plotted along OX.

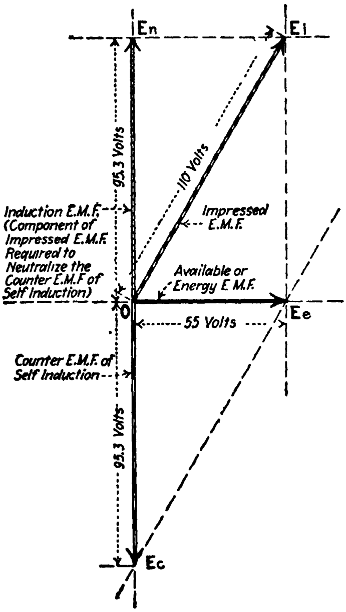


FIG. 472.—Phase diagram illustrating induction component of impressed e.m.f.

Now by scaling it will be found that the length of OE_ι equals 110 volts, which as stated in the specifications of the above example is the value of the impressed e.m.f. Thus it has been shown that the available or energy e.m.f. of 55 volts which impels the 11-amp current in the coil of Fig. 470 is equal to the difference between the impressed e.m.f. of 110 volts and the counter e.m.f. of self-induction of 95.3 volts.

By scaling with a protractor it is found that the angle ϕ, in Fig. 471, IV, is 60 deg. Hence, it follows that, in this particular circuit, the current and the energy e.m.f. lag 60 deg. behind the applied e.m.f.

785. The portion of component of impressed e.m.f. which is necessary to neutralize the counter e.m.f. of self-induction must be just equal in intensity and opposite in phase to the counter e.m.f. of self-induction. That this is true is almost self-evident. Any impressed e.m.f. may be considered as comprising two components (Art. 764) or parts at right angles to one another: an energy e.m.f. and an induction e.m.f. Consider Fig. 472 which has been plotted to the values of the example just above. Vector OE_ι (Fig. 472) represents the impressed e.m.f. It may be considered as comprising two parts OE_n and OE_e

differing in phase by 90 deg. The component OE_e represents the energy part of the impressed e.m.f. which is available for actually circulating the current in the circuit. The component OE_n represents the induction component or part of the impressed e.m.f. which is used up in neutralizing the counter e.m.f. of self-induction.

If the induction component of the impressed e.m.f. is to neutralize the counter e.m.f. of self-induction, the induction e.m.f. must be a maximum in the negative direction (Z, Fig. 467) at the instant (Y) the counter e.m.f. of self-induction is a maximum in the positive direction. That is, these two e.m.fs. should always differ in phase by 180 deg., as shown in the phase diagram of Fig. 472 and the curves of Fig. 467.

Example.—If a current of 2 amp. is forced through the inductance of 0.133 henry (no resistance or permittance in this imaginary circuit) of Fig. 461,II, the generator must impress across the inductance a pressure just equal to the counter e.m.f. induced by the 2-amp. current. As shown in the example relating to this Fig. 461, which is given under Art. 787, the counter e.m.f. developed by the 2-amp. current through the inductance of 0.133 henry is 100 volts. It follows, therefore, that the impressed e.m.f. necessary to impel this 2-amp. current must also be just 100 volts—since it is assumed that in this imaginary circuit there is no resistance. Furthermore, since it is assumed that there is no resistance in the circuit, there is no energy e.m.f. The current which circulates in this resistanceless circuit does no work and represents no energy (see "Power factor," Art. 823). The counter e.m.f. and the impressed e.m.f. in this imaginary circuit must differ in phase (see Fig. 467) by 180 deg.

786. It is not usual, in solving problems in alternating currents, first to determine the energy e.m.f. and then compute the current by dividing this energy or available e.m.f. value by the resistance, as was done in the solution of the problem relating to Fig. 470. It is, as will be shown, possible to treat the counter e.m.f. effects of inductance (and of permittance, Art. 793) as if these effects were a kind of resistance—which is called reactance. They —reactance effects—can be combined with the actual ohmic resistance of the circuit to determine the total opposition offered to flow of current in the circuit. Then, if the impressed-voltage value be divided by this composite quantity called impedance (Art. 788) which is expressed in ohms and represents the combined opposition due to actual resistance and inductance (and permittance if there is appreciable permittance

in the circuit) the value of current, in amperes, in the circuit will result. The situation is discussed and illustrative examples are given in following articles.

787. Inductive reactance is the name which has been given to the opposition offered to the flow of alternating currents due to the counter e.m.f. of self-inductance (Art. 516). It is numerically equal but opposite to the counter e.m.f. of self-induction in an alternating-current circuit. Reactance is expressed in ohms as is resistance. As an e.m.f. is necessary to impel current through resistance, an e.m.f. is also necessary to overcome or neutralize the counter e.m.f. induced in an alternating-current circuit which contains inductance—that is, inductive reactance. The formula whereby the inductive reactance in any alternating-current circuit may be computed is

$$X_l = 6.28 \times f \times L \text{ (ohms)} \qquad (232)$$

$$f = \frac{X_l}{6.28 \times L} \text{ (cycles per second)} \qquad (233)$$

$$L = \frac{X_l}{6.28 \times f} \text{ (henry)} \qquad (234)$$

Wherein X_l = inductive reactance of the circuit, in ohms.
f = frequency of the e.m.f. impressed on the circuit, in cycles per sec.
L = inductance in circuit, in henrys.

NOTE (Art. 777) that the counter e.m.f. of self-induction, $E_c = 6.28 \times f \times L \times I$ and that the portion "$6.28 \times f \times L$" of this quantity is called inductive reactance. When the reactance (X_l, in ohms) is multiplied by the current (I, in amperes), the portion or component of the impressed e.m.f. in volts which is necessary to neutralize (Art. 785) the counter e.m.f. of self-induction (so that there may be available an energy e.m.f. to force a current, I, to flow) is the result. The similarity between resistance and reactance is shown by the following examples:

Example.—In the circuit of Fig. 461,I, which contains only resistance, the e.m.f. necessary to produce a current of 2 amp. is, substituting in the Ohm's law formula (Art. 151) $E = R \times I = 50 \times 2 = 100$ volts. With an inductance of 0.133 henry in circuit, as in Fig. 461,II, the 60-cyclt-alternating e.m.f. necessary to force a current of 2 amp. through the reace, ance having an inductance of 0.133 henry is similarly obtained. Thus

$$E = X_l \times I$$
$$E = (6.28 \times f \times L) \times I$$
$$E = 6.28 \times 60 \times 0.133 \times 2 = 50 \times 2$$
$$E = 100 \text{ volts.}$$

The inductive reactance in this example is therefore 50 ohms.

For direct-current circuits and for alternating-current circuits having no inductance

$$\text{Current} = \frac{\text{e.m.f.}}{\text{resistance}} \text{ or } I = \frac{E}{R} \tag{235}$$

Similarly, for alternating-current circuits containing inductive reactance only

$$\text{Current} = \frac{\text{e.m.f.}}{\text{reactance}} \text{ or } I = \frac{E}{X_l} \tag{236}$$

and

$$E = I \times X_l \text{ (volts)} \tag{237}$$

or

$$X_l = \frac{E}{I} \text{ (ohms)} \tag{238}$$

Wherein E = the e.m.f., in volts (effective e.m.f. in alternating-current circuit), impressed on the circuit.
I = current in the circuit, in amperes.
X_l = reactance of the circuit, in ohms.

NOTE.—Reactance can not be added directly to resistance although their effects are similar in that they both tend to limit the current intensity in a circuit. How resistance and reactance can be added is shown in Art. 789. Reactance may also be defined as that quantity which, when multiplied by the current, gives that component or portion of the impressed e.m.f. which is at right angles to the current.

788. Impedance is the name given to that quantity which represents the combined resisting effects of: (1) actual (ohmic) resistance and (2) the apparent resistance (reactance) or the opposition due to counter e.m.fs. of self-induction and permittance. Impedance is expressed in ohms. If the impedance (ohms) of a circuit be multiplied by the current (amperes) in the circuit the resulting value will be the alternating e.m.f. in volts impressed on the circuit. It follows, then, that for alternating-current circuits

$$I = \frac{E}{Z} \text{ (amp.)} \tag{239}$$

hence

$$Z = \frac{E}{I} \text{ (ohms)} \tag{240}$$

and

$$E = I \times Z \text{ (volts)} \tag{241}$$

Wherein I = current (effective) in the circuit, in amperes.
E = e.m.f. (effective) impressed on the circuit, in volts.
Z = impedance of the circuit, in ohms.

789. To obtain the value of impedance, resistance must be combined with, that is, added to, reactance (Art. 787). However, since the effects of resistance and reactance differ in phase by 90 deg. they can not be added arithmetically. It can be shown that (Art. 790 and Fig. 475) the counter or opposing effect of inductive reactance (the $I \times X_l$ drop) leads the opposing

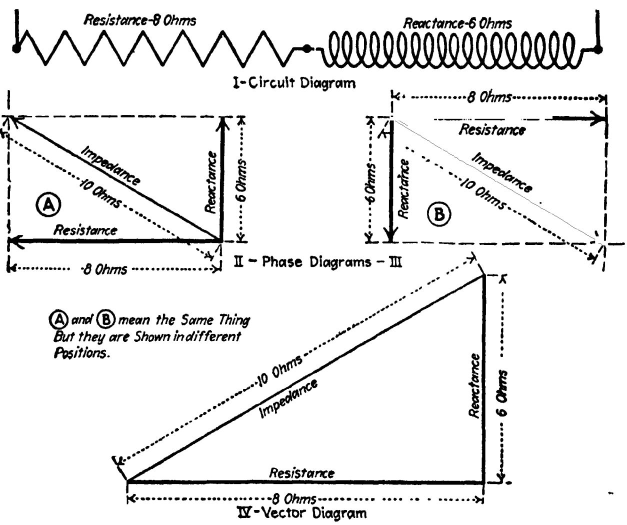


FIG. 473.—Impedance diagrams.

effect of resistance (the $I \times R$ drop) by 90 deg. They are at right angles to each other and can be represented by phase or vector diagrams as in Fig. 473.

Example.—Figure 473,I shows a circuit whose reactance (8 ohms) was computed from equation (232) of Art. 787. The impedance (the sum of the resistance and reactance) was obtained by drawing diagonals in the parallelograms of the phase diagrams and by drawing a hypotenuse in the vector diagram. The resulting value is the same in each case.

In a right-angled triangle

$$\text{Hypotenuse} = \sqrt{(\text{one side})^2 + (\text{other side})^2} \qquad (242)$$

It follows that

$$\text{Impedance} = \sqrt{(\text{resistance})^2 + (\text{reactance})^2} \qquad (243)$$

Now it was shown (Art. 787) that inductive reactance = $6.28 \times f \times L$. Therefore (for a circuit which contains only inductance and resistance—no permittance)

$$Z = \sqrt{R^2 + (6.28 \times f \times L)^2} \text{ (ohms)} \tag{244}$$

Wherein Z = impedance, in ohms.
R = resistance, in ohms.
f = frequency, in cycles per second.
L = inductance, in henrys.

Example.—What is the impedance of the coil of Fig. 474,I? It has an inductance of 0.03 henry and a resistance of 30 ohms. The frequency is 60 cycles. What will be the current with an impressed e.m.f. of 100 volts?

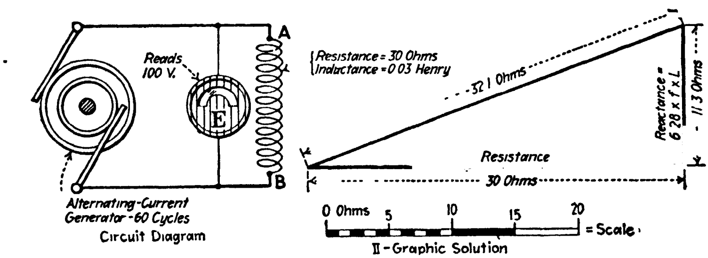


FIG. 474.—Example in computing impedance.

SOLUTION.—Substituting in the impedance formula,

$$Z = \sqrt{R^2+(6.28\times fL)^2} = \sqrt{(30\times 30)+(6.28\times 60\times 0.03)(6.28\times 60\times 0.03)}$$
$$= \sqrt{(30\times 30)+(11.3\times 11.3)} = \sqrt{900+127.7} = \sqrt{1{,}027.7} = 32.1 \text{ ohms}$$

The problem could be solved graphically, as suggested at II, by laying out the sides of a right-angled triangle to scale to represent the resistance and reactance and then scaling the hypotenuse to obtain the impedance. The current that would flow through the coil AB with a 60-cycle e.m.f. of 100 volts impressed across it would be (formula 239)

$$I = \frac{E}{Z} = \frac{100}{32.1} = 3.12 \text{ amp.}$$

790. The relations of the different e.m.fs. and the components thereof in alternating-current circuits containing resistance only, and containing resistance and inductance, are shown, respectively, in Fig. 475, at I and II. In a circuit containing resistance only, the e.m.f. $O'C'$ (as shown at I), impressed by the generator or other source of alternating e.m.f., is spent wholly in overcoming or neutralizing the resistance drop $O'F'$. The

current (I) in the circuit, (for which no vector is shown) is in phase with the impressed e.m.f. $O'C'$. In a circuit (II) containing resistance and inductance, there are two counter forces which

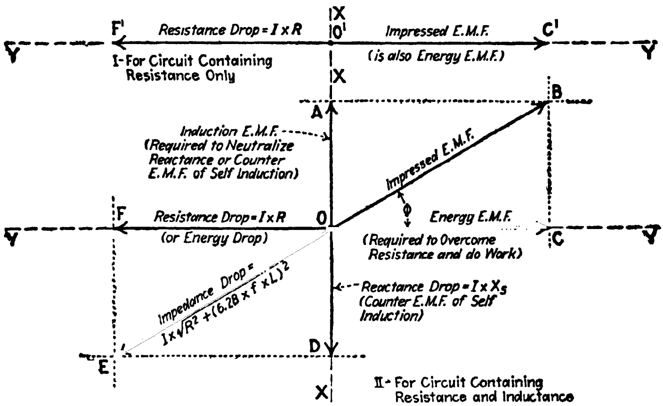


FIG. 475.—Graphic statement of e.m.f. relations in alternating-current circuit

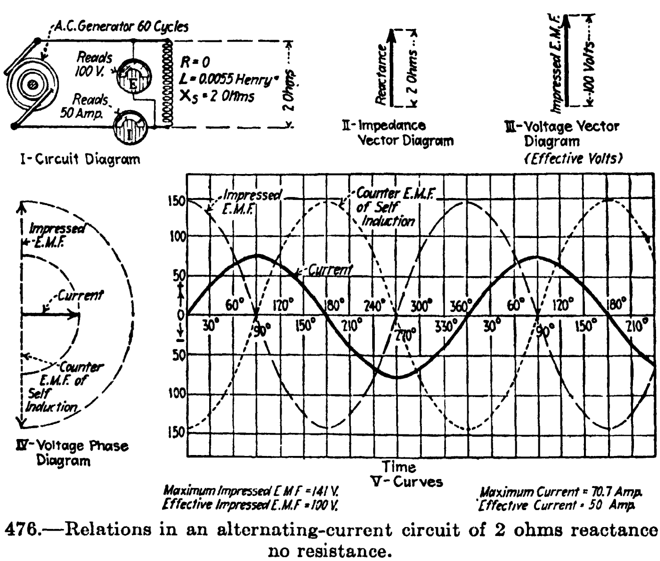


FIG. 476.—Relations in an alternating-current circuit of 2 ohms reactance and no resistance.

the impressed e.m.f. must overcome (1) the resistance drop OF. (2) the reactance drop OD—which is the counter e.m.f. of self-induction (Art. 787). The resultant or vector sum of OF and OD is the impedance drop OE. The e.m.f. OB impressed on

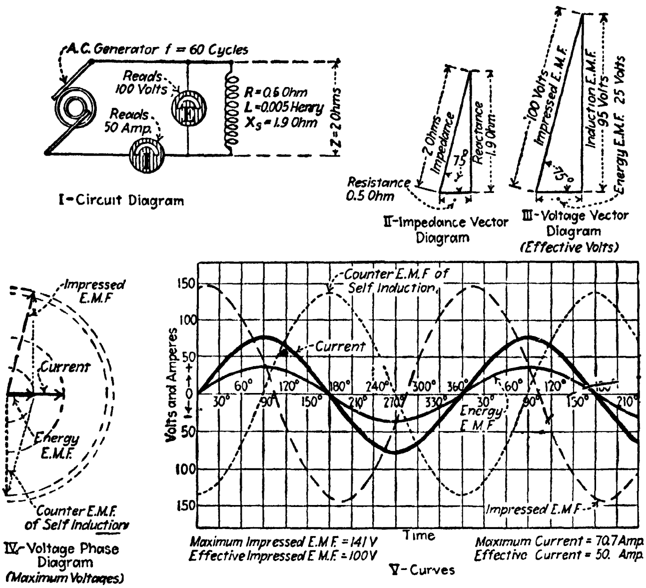


FIG. 477.—Relations in alternating-current circuit of 1.9 ohm reactance and 0.5 ohm resistance.

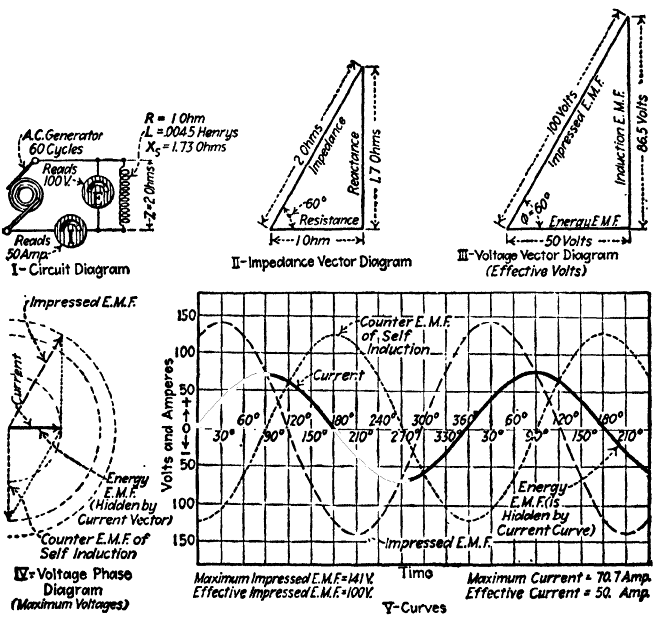


FIG. 478.—Relations in an alternating-current circuit of 1.7 ohm reactance and 1 ohm resistance.

the circuit under consideration must be equal and opposite to the impedance drop *OE*. This impressed e.m.f. *OB* may, as before outlined in Art. 785, be considered as being comprised of two components: (1) an induction component, *OA*, which neutralizes the counter e.m.f. of self-induction and which represents no real energy. (2) an energy component, *OC*, which actually impels the current which does work through the resistance and which represents real energy. Component *OA* is equal and opposite to *OD*.

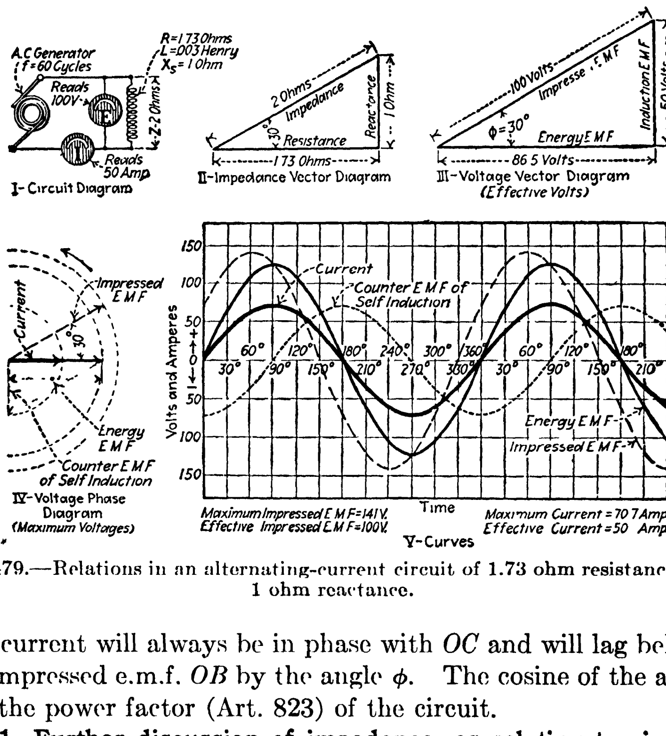


FIG. 479.—Relations in an alternating-current circuit of 1.73 ohm resistance and 1 ohm reactance.

The current will always be in phase with *OC* and will lag behind the impressed e.m.f. *OB* by the angle ϕ. The cosine of the angle ϕ is the power factor (Art. 823) of the circuit.

791. Further discussion of impedance, as relating to circuits containing permittance, and the solution of problems involving impedances in series and in multiple will be discussed in a following section of this book (Art. 813) after the phenomenon of permittance in alternating-current circuits has been treated.

792. Examples of voltage and current relations in circuits having different proportions of resistance and inductance are shown in Figs. 476, 477, 478, and 479. In all cases the current will be in phase with the energy or available e.m.f. (note under

Art. 775) but will, since there is inductance in these circuits, lag behind the impressed e.m.f. The greater the relative amount of inductance the greater will be this lag.

NOTE.—In each example the reactance was first computed as directed in Art. 787 and then the impedance and voltage triangles were drawn in accordance with the preceding suggestions of Art. 784. The angle, ϕ, between the hypotenuse and the base of each of the triangles is the angle by which the current lags behind the impressed e.m.f. Note that the lag angles in the triangles are the same as in the corresponding phase diagrams and curves. If a circuit having no resistance but some reactance could exist, its voltage and current relations would be as shown in Fig. 476; the current would lag exactly 90 deg. behind the impressed e.m.f. The condition shown in Fig. 476 can never be realized in practice because all conductors have some resistance; however, the condition suggested may be approximated. The relations in a circuit containing only resistance are shown in Fig. 463.

SECTION 48

PERMITTANCE OR CAPACITANCE IN ALTERNATING-CURRENT CIRCUITS

793. The action of inductance and permittance (capacitance) when the voltage of or the current in a circuit changes is this: The reaction of a permittor or condenser instead of tending—as does inductive counter e.m.f.—to prevent any change in current, tends to prevent any change in voltage. That is, it tends to keep the voltage constant.

Example.—If an e.m.f. is impressed across a dead circuit having inductance (Fig. 262), the inductance (Art. 508) tends to prevent (Fig. 265) a current from building up in the circuit. Now, with the current flowing, if the e.m.f. is discontinued and the circuit short-circuited (Fig. 262), the inductance will tend to prevent (Fig. 266) the decay of the current.

But if an e m.f. be impressed across a dead circuit containing permittance, the permittance does not oppose the building up but instead permits the displacement current to flow until the counter e.m.f. of the permittor due to its stressed dielectric, is equal to the impressed e.m.f. Then, with the dielectric charged, if the e.m.f. is discontinued and the circuit short-circuited, the permittance instead of retarding the decay of the current will accelerate it by virtue of the counter e.m.f. which it imposes. Obviously, then, inductance tends to prevent a current from building up in a dead circuit while permittance tends to permit it to build up. Furthermore, inductance tends to prevent the decay of current in a live circuit when the e.m.f. is discontinued while permittance tends to promote such decay.

NOTE.—The terms "capacity," "capacitance," and "permittance" are synonymous when used as relating to electrostatics. However, the term "capacity" may have two distinct meanings in electrical parlance. Hence, to eliminate misunderstandings, "capacity" should be used only in referring to "power capacity," "current-carrying capacity," or in similar senses. In referring to "electrostatic capacity," either "permittance" or "capacitance" should be used. "Capacitance" is the word recommended in the A. I. E. E. "Standardization Rules," but "permittance" (which was suggested by O. Heaviside) is preferred by the author because it is the more expressive of the actual property of the dielectric.

794. Inductance or permittance (capacitance) effects are produced respectively by changes of current or of voltage.—The

effects due to inductance occur in a circuit only when the current in the circuit changes. The effects due to permittance (or capacitance) in a circuit occur only when the voltage of the circuit changes.

Example.—If it were possible to change the current in a given circuit without altering the voltage conditions therein, then, in that case, only inductance effects would be observed—no capacity effects. If it were possible to change the voltage impressed on a circuit without changing the current intensity (amperes) in any part of the circuit, then only permittance effects would be observed. In actual circuits it is practically impossible to change the impressed voltage without changing the current, and vice versa. However, in practice, except where high voltages are involved or where the circuit is provided artificially with an unusual amount of permittance (with a permittor or condenser), the effects of permittance are of little consequence. The effects of inductance, since they are determined solely by the inductance of and the current in the circuit, may be and are frequently, very noticeable in low-voltage circuits—particularly in alternating-current circuits.

795. The effect of permittance (capacitance) in alternating-current circuits will now be considered. Just as the effects of inductance are much more noticeable in alternating-current circuits than in direct-current circuits, so are the effects of permittance much more pronounced in alternating than in direct-current circuits. Whenever there is a change in the voltage impressed across a permittor (condenser), electricity—that is, electrons—is displaced in the dielectric of the permittor and a displacement current flows.

In an alternating-current circuit the e.m.f. reverses in direction periodically and is constantly changing in value. From this it would be inferred that a displacement (charging) current must flow constantly in an alternating-current circuit containing permittance—and such is the case. Now note what occurs when an alternating e.m.f. is impressed across a permittor.

Example.—Figures 480 and 481 show an alternating-current generator connected across a permittor (condenser). If the alternator is rotated at a uniform speed, in the direction shown, a sine-wave-form e.m.f. will be induced. During the first 90 deg. (in Fig. 481) the e.m.f. will be in the direction *ABCD*. The portion *TR* of the e.m.f. sine curve of Fig. 482 shows how this induced voltage increases during the first 90 deg. During this period the permittor would be charged in the direction *BC* (Fig. 481) as indicated by *TJ* in the curve, Fig. 482. Also, during this period the dielectric would be stressed in the direction *BC* (Figs. 480 and 481) and a displacement current would flow around the circuit in the direction *ABCD*.

During the next quarter cycle (Fig. 481,II), the voltage RV (Fig 482) is in the same direction as before but it decreases to zero. During this 90-deg. period the elasticity of the dielectric asserts itself and the permittor dis-

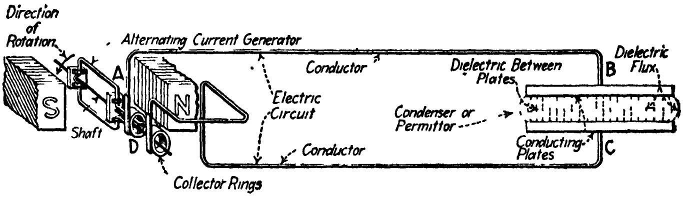


FIG. 480.—Condenser or permittor in an alternating-current circuit. (The hydraulic analogy of this illustration is shown in Fig. 484.)

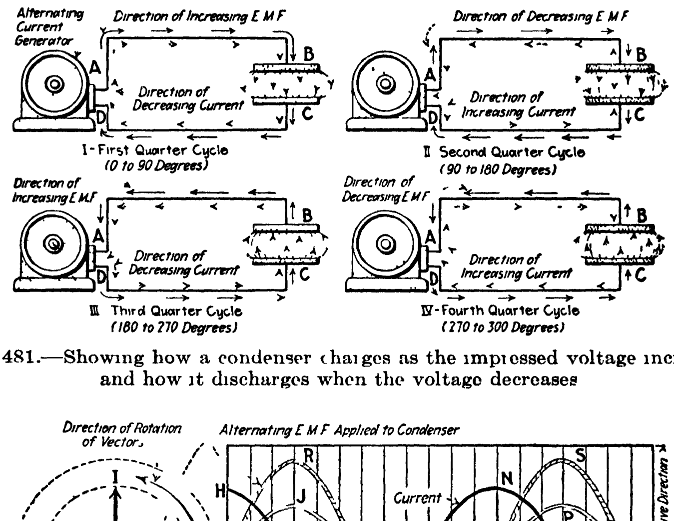


FIG. 481.—Showing how a condenser charges as the impressed voltage increases and how it discharges when the voltage decreases

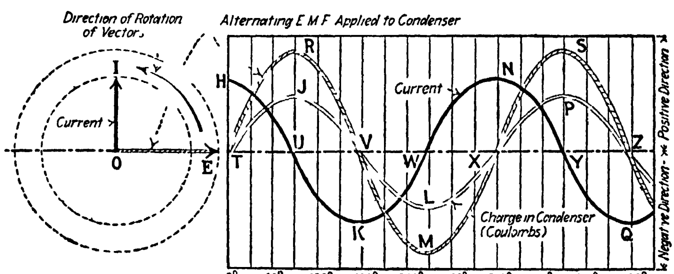


FIG. 482.—Relations in alternating-current circuit containing capacity only. The current leads the applied e.m.f. by 90 deg.

charges as the e.m f. decreases—and a displacement current is thereby forced around the circuit in the direction $DCBA$.

During the next quarter cycle the e.m f. increases from zero to a maximum (VM) but it is now in the opposite direction ($DCBA$), as shown in Fig. 481,III, from that obtaining in the preceding half cycle TRV. Now it again

charges the permittor but in the opposite direction. The displacement current continues to flow in the direction *DCBA*.

During the next quarter cycle (Fig 481,IV) the e m f decreases from *M* to *X* and the permittor discharges in the direction *ABCD*. Thus, an alternating displacement or charging current (of the frequency of the applied e m f) flows so long as the alternating e m f. is impressed across the permittor in spite of the fact that the circuit is open in the usual sense of the word. If incandescent lamps were connected, as shown in Fig. 483, in the circuit, they would be lighted by the alternating displacement current—provided the permittor had sufficient permittance (capacitance) to allow a current to flow of sufficient intensity to heat the filaments.

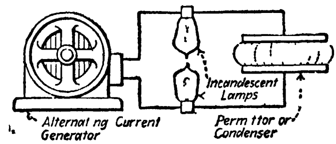


FIG 483 —Incandescent lamps in a circuit containing permittance.

796. Permittance (Electrostatic Capacity) Effects Are of Little Consequence in Low-voltage Circuits.—This follows from the fact that permittance effects occur only when there is a change in voltage. The greater the changes in voltage, the greater will be the permittance effects. Obviously, in low-voltage circuits the changes in voltage must be relatively small. The magnitude of the permittance effects—if the dielectric is not stretched beyond the rupturing point—is proportional to the applied voltage.

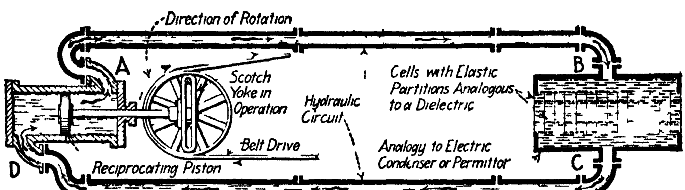


FIG 484 —Hydraulic analogy of an alternating-current circuit containing an alternating-current generator and a condenser or permittor (At this instant the permittor dielectric is unstressed The pressure or voltage is a minimum but the current is a maximum in the positive direction.)

On circuits operating at voltages lower than 6,000, it is seldom that the permittance effects—that is, the charging or displacement current effects—are particularly noticeable; they are, though possibly inconsequential, present nevertheless.

797. A hydraulic analogue of the action of permittance in an alternating-current circuit is outlined in Figs. 484 to 491. Figure 480 illustrates an electric circuit analogous to the fluid circuit of Fig. 484. Figure 482 is a graphic record of the phe-

nomena occurring in the circuit as the pump is operated. The reciprocating pump, driven at a uniform speed, is analogous to an alternating-current generator. Its piston, by virtue of the Scotch-yoke drive, has a harmonic longitudinal motion, the speed of which, as it varies with the time, could be represented by a sine curve. When the pump is driven, it impels in the fluid circuit an alternating current of the noncompressible fluid shown which is analogous to electricity.

The contrivance at the right of Fig. 484 (which is shown enlarged in the other illustrations) is analogous to a permittor (condenser). It consists of two chambers (condenser plates) with a cellular structure between them. The cellular structure has partitions of an imaginary semiporous elastic material and is analogous to a dielectric (Art. 101). The pipes and the cellular structure are all filled with the noncompressible fluid just as all matter is permeated with electricity. The pipes are frictionless (the circuit has no electrical resistance).

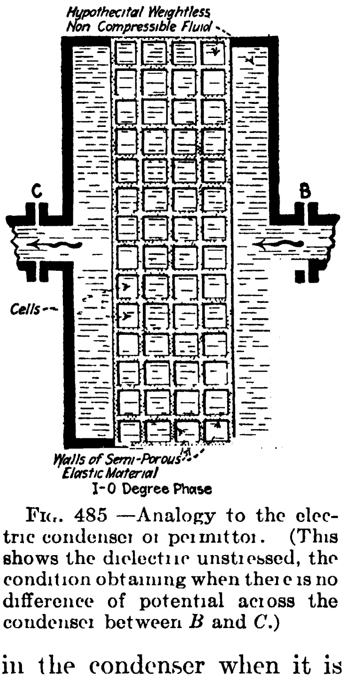


FIG. 485.—Analogy to the electric condenser or permittor. (This shows the dielectric unstressed, the condition obtaining when there is no difference of potential across the condenser between B and C.)

As the pump is driven, its piston displaces the fluid (electricity) in the circuit—first in one direction as the piston travels to the right, then in the other as it travels to the left. The fluid which resides in the condenser when it is in its normal unstressed condition (Fig. 484) has been rendered in darker lines—merely to illustrate how the displacement of the fluid in the circuit occurs. The force exerted by the piston on the fluid is analogous to an applied e.m.f. The current (rate of flow) of fluid is analogous to an electric displacement current. The quantity of fluid (gallons) displaced is analogous to coulombs. The reactive pressure exerted by the elastic walls of the "condenser" when they are stressed is analogous to the counter e.m.f. due to permittance—which will be further discussed in Art. 798.

Explanation.—Assume that the pump starts from the position of Fig. 484, in which the piston is at the center of the cylinder. Under these normal conditions the dielectric in the permittor is unstressed, as shown in the enlarged diagram of Fig. 485. As the piston is forced from this neutral position, it displaces fluid around the circuit and stretches the elastic-walled

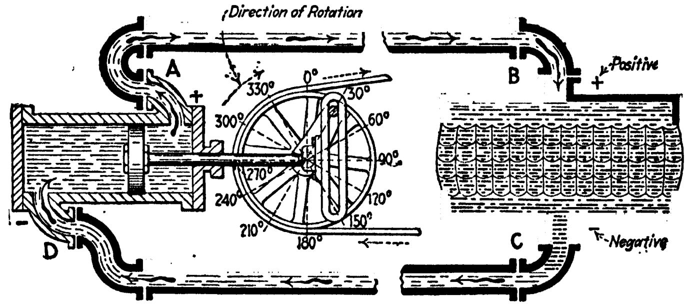


FIG. 486.—Pressure impressed by the pump is in the positive direction and is increasing and the dielectric strength is becoming stretched. The rate of flow is also in the positive direction and is decreasing.

cells (stresses the dielectric). Follow the cycle of events, referring at the same time to the curves of Fig. 482. Figure 486 pictures the situation after the crank has been rotated through 30 degrees.

At the instant at which the piston is at the left end of its stroke (Fig. 487,IV), the force which it is exerting must be a maximum because then

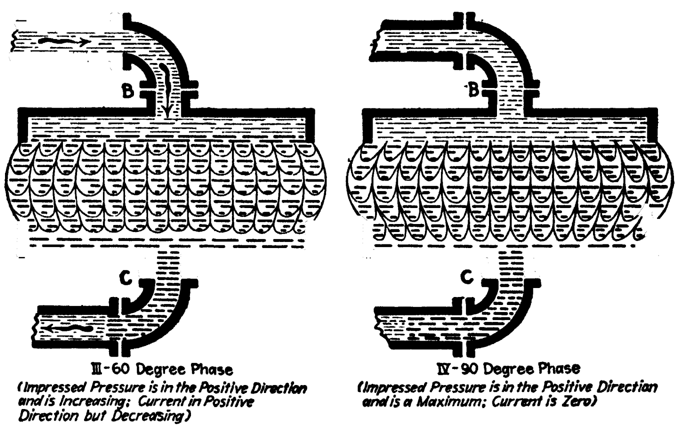
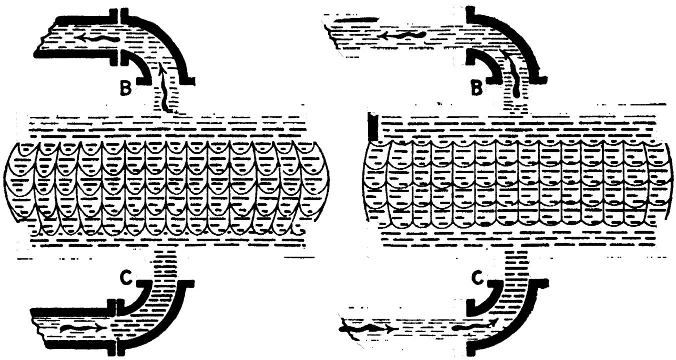


FIG. 487.—Showing conditions in the permittor at the 60- and 90-deg. instants.

the dielectric is stretched as much as is possible with the arrangement shown. Consequently, the dielectric is exerting its maximum elastic reaction (counter e.m.f.). During the next 90 deg. (Figs. 488,V to 489,VII), the pressure exerted by the piston is decreasing, and the permittor is discharg-

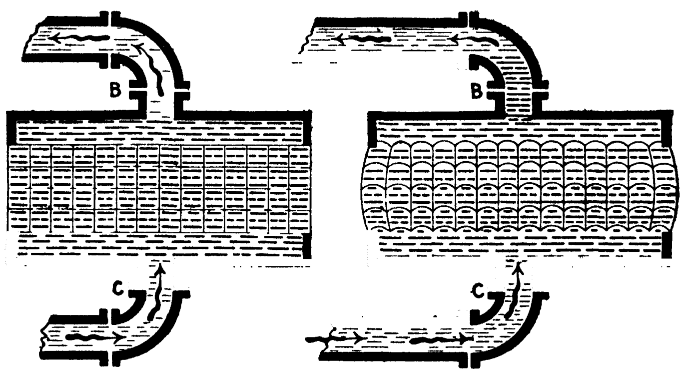
ing—the dielectric is returning to the unstressed condition. During the next 90-deg. period (Figs. 489,VIII to 490,X) the permittor is being charged in the opposite direction. During the following 90-deg. period (Figs. 491,XI to 491,XII) it discharges—in the opposite direction from the former discharge.



V-120 Degree Phase
(Impressed Pressure in the Positive Direction and Decreasing, Current in Negative Direction and Increasing)

VI-150 Degree Phase
(Impressed Pressure in Positive Direction and Decreasing, Current in the Negative Direction and Increasing)

FIG. 488 – Showing conditions in the permittor at the 120- and 150-deg. instants.



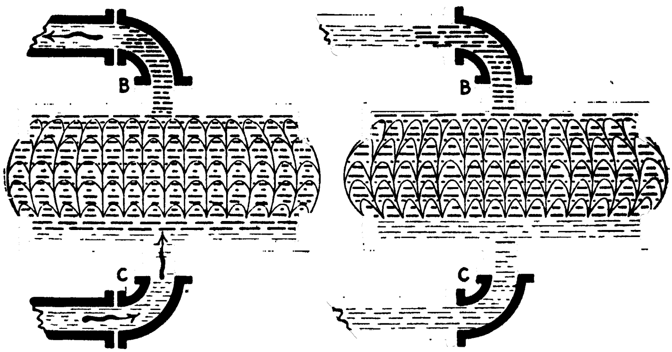
VII-180 Degree Phase
(Impressed Pressure is Zero, Current in the Negative Direction and a Maximum)

VIII-210 Degree Phase
(Impressed Pressure in the Negative Direction and Increasing Current in Negative Direction and Decreasing)

FIG. 489.—Showing conditions in the permittor at the 180- and 210-deg. instants.

Note that the phase relations in this fluid circuit are, as will be shown, the same as those in an electric circuit containing permittance only (Art. 803) on which an alternating e.m.f. is impressed. At the end of each stroke the force exerted by the piston is a maximum (Figs. 487,IV and 490,X), but at that instant the piston is reversing in direction and the fluid current—rate of flow—is zero. When the piston is at the center of each stroke,

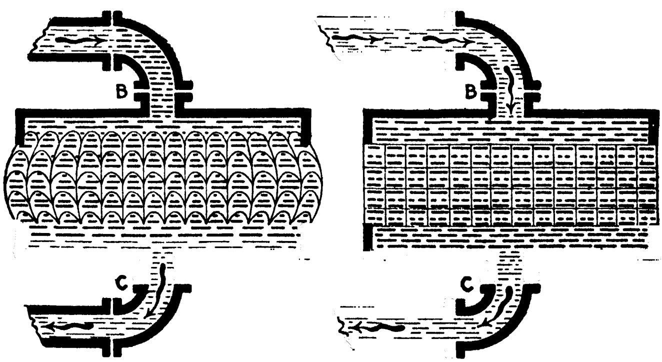
the force impressed by it is zero but the current is a maximum—it is flowing at the maximum rate because the piston is then moving at the maximum rate. Thus, the fluid current in this circuit leads the impelling force impressed by the piston by 90 deg.; Fig. 482 states the situation graphically.



IX - 240 Degree Phase
(Impressed Pressure in the Negative Direction and Increasing; Current in the Negative Direction and Decreasing)

X - 270 Degree Phase
(Impressed Pressure in the Negative Direction and is a Maximum; Current is Zero)

FIG. 490.—Showing conditions in the permittor at the 240- and 270-deg. instants.



XI - 300 Degree Phase
(Impressed Pressure in Negative Direction and Decreasing, Current in Positive Direction and Increasing)

XII - 360 Degree Phase
(Impressed Pressure is Zero; Current in the Positive Direction and a Maximum)

FIG. 491.—Showing conditions in the permittor at the 300- and 360-deg. instants.

798. A counter e.m.f. is exerted by permittance in an alternating-current circuit in a somewhat similar manner—but not in the same manner—as an inductance produces a counter e.m.f. in an alternating-current circuit. The dielectric of the permittor when it is being stressed during the charging period resists the stretching by virtue of its elastic properties. It therefore, in effect, exerts a counter e.m.f. However, the permittance counter

e.m.f. attains its maximum value, in each cycle, in the positive direction at the instant at which inductance counter e.m.f.

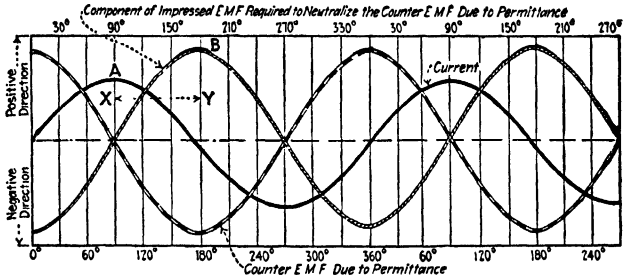


FIG. 492.—Showing how the counter e.m.f. due to permittance directly opposes the impressed e.m.f.

attains its maximum value in the negative direction. Also, the converse is true. A consideration of the hydraulic analogy of Figs. 484 to 491 in connection with the curves of Fig. 482 will verify the above statements.

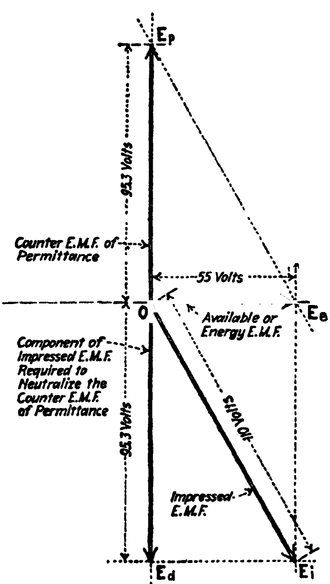


FIG. 493.—Showing relations of e.m.fs. in a circuit containing resistance and permittance.

799. The counter e.m.f. due to permittance in an alternating-current circuit is always directly opposite in phase and equal in value to the component of the impressed e.m.f. required to neutralize it, as shown in Fig. 492. That this must be true will be evident from a consideration of the foregoing statements of Arts. 795 and 797. The phase diagram of Fig. 493 indicates vectorially these relations. Hence there is always a difference in phase of 180 deg. between the counter e.m.f. of permittance and the component of the impressed e.m.f. required to neutralize it, the counter e.m.f. leading the component of the impressed e.m.f. In a circuit containing only permittance—if such a circuit could exist—the counter e.m.f. of permittance e.m.f.

would be directly opposite to—would lead by 180 deg.—and would be of the same value as the impressed e.m.f.

800. To compute the counter e.m.f. due to permittance in an alternating-current circuit, the e.m.f. impresses on it being of sine wave form, the value of the current in the circuit is multiplied by the quantity $1 \div 6.28 \times f \times C$. Hence it follows that

$$E_C = I \times \left(\frac{1}{6.28 \times f \times C}\right) \text{ (volts)} \qquad (245)$$

or, since $2 \times \pi = 6.28$, as it is frequently written,

$$E_C = \frac{I}{2 \times \pi \times f \times C} = \frac{I}{6.28 \times f \times C} \text{ (volts)} \qquad (246)$$

Wherein E_C = counter e.m.f. exerted by permittor, in volts.
I = effective current in the circuit, in amperes.
f = frequency of the circuit, in cycles per second.
C = permittance or electrostatic capacity of the circuit, in farads.

The proof of the above equation follows. Compare it with that of Art. 777 for the counter e.m.f. of inductance.

Proof.—The charge or displacement in a permittor on which an alternating e.m.f. is impressed changes from zero to a maximum during the time of ¼ cycle (see Fig. 482). That is, the charge changes $Q_m = C \times E_m$ (coulombs) in $1 \div 4 \times f$ sec. Now

$$\text{Charge} = \text{average current} \times \text{time} \qquad (247)$$

that is,

$$Q_m = C \times E_m = I_{av} \times \frac{1}{4 \times f} \text{ (coulombs)} \qquad (248)$$

or

$$C \times E_m = \frac{I_{av}}{4 \times f} \qquad (249)$$

Then from the above

$$I_{av.} = 4 \times f \times C \times E_m \text{ (amp.)} \qquad (250)$$

That is, the average charging current equals four times the product of the frequency, permittance and maximum e.m.f.

But (Art. 737)

$$I_m = I_{av.} \times 1.57 \qquad (251)$$

Then

$$I_m = 4 \times f \times C \times E_m \times 1.57 \qquad (252)$$

substituting

$$I_m = 6.28 \times f \times C \times E_m \text{ (amp. max.)} \qquad (253)$$

Then

$$I = 6.28 \times f \times C \times E \text{ (amp. effective)} \tag{254}$$

and

$$E = \frac{I}{6.28 \times f \times C} \text{ (volts effective)} \tag{255}$$

Since the counter e.m.f. exerted by a permittor must always equal the e.m.f. impressed, it follows that

$$E_c = \frac{I}{6.28 \times f \times C} \text{ (volts)} \tag{256}$$

801. Permittive reactance is the opposition offered by permittance to the flow of alternating current. It is similar to but not the same as inductive reactance (Art. 787). It is measured in ohms. The equation for permittive reactance is

$$X_c = \frac{1}{6.28 \times f \times C} \text{ (ohms)} \tag{257}$$

Wherein the symbols have the same significance as in the preceding article, except that X_c = permittive reactance, in ohms.

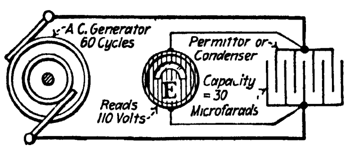


FIG. 494.—Illustrating permittive reactance.

Example.—What is the reactance, in ohms, of the permittor shown in Fig 494? What current will flow in the circuit (assuming that it has no resistance or inductance) there shown, when an effective alternating voltage of 110 is impressed across the permittor? *Solution.*—Substituting in formula (257) for reactance,

$$X_c = \frac{1}{6.28 \times f \times C} = \frac{1}{6.28 \times 60 \times 0.00003} = \frac{1}{0.011304} = 88 \text{ ohms}$$

Then the current, from the Ohm's law formula, would be

$$I = \frac{E}{X} = \frac{110}{88} = 1.25 \text{ amp.}$$

802. Permittance and Inductance Produce Precisely Opposite Effects in Alternating-current Circuits.—That is, their effects are 180 deg. apart in phase. Hence, one may partially or wholly neutralize the other. That this should be true follows from the statements of Art. 794. When inductance and permittance do neutralize in a circuit, then if the e.m.f. in volts, impressed on the circuit, be divided by the resistance, in ohms, of the circuit, the current, in amperes, which will flow in the circuit will

be the result. When inductance and permittance neutralize, the resulting condition is called "resonance" (Art. 810). If the permittive effect in a circuit is greater than the inductive effect, then the current in the circuit will lead (Art. 757) the applied e.m.f. But if the inductive effect is greater than the permittive, then the current will lag (Art. 756) behind the applied e.m.f.

803. The current, in a circuit containing permittance only, leads the impressed e.m.f. by 90 deg. as shown in Figs. 482 and 492. When the impressed e.m.f. is increasing in one direction the alternating displacement current is in the same direction—but it is decreasing in intensity. When the impressed e.m.f. is decreasing in one direction, the displacement current is increasing but is in the opposite direction. The displacement or charging current through permittance is greatest when the rate of change of the impressed e.m.f. is greatest—that is, at the instants at which the impressed e.m.f. is zero. At the instants at which the impressed e.m.f. is a maximum (when its rate of change is zero) the displacement current is zero. In any cycle the displacement current, in a circuit containing only permittance, attains its maximum intensity 90 deg. before the impressed e.m.f. reaches its maximum intensity; thus the current leads the impressed voltage by 90 degrees. The alternating displacement current lags 90 degrees behind the counter e.m.f. due to permittance as shown in Fig. 492.

804. The Current in an Alternating-current Circuit, Containing Only Permittance (Capacitance) and Resistance, Always Leads the E.m.f. Impressed by the Generator.—The amount of lead is proportional to the amount of permittance in the circuit. If there is no permittance—only resistance—in the circuit, then there is no lead, and the current will then be in phase with the impressed e.m.f. as shown in Fig. 463. In a circuit consisting wholly of permittance—such a circuit, however, is a physical impossibility—the current would lead the impressed e.m.f. by exactly 90 deg., as shown in Fig. 482. With varying proportions of permittance and resistance, the current will lead the impressed e.m.f. by some amount between 90 and 0 deg. With little permittance in the circuit there will be little lead; with much permittance in the circuit the lead may be almost 90 degrees.

805. Power Is Not Lost in Impelling an Alternating Current in a Circuit Containing Only Permittance.—The reason is this:

All (see note below) of the energy expended in displacing the electricity in one direction and in stressing the dielectric of the permittor, as the voltage increases during the first half alternation of a cycle, is returned to the circuit during the last half alternation (when the voltage is decreasing) as the dielectric exerts its elasticity and pulls back into an unstressed condition. Pressure must be exerted by the pump to stretch the dielectric cells and displace electricity through the permittor from the position of Fig. 484 to that of Fig. 487,IV. But as the pump pressure decreases (Figs. 488,V to 489,VII), the stressed dielectric returns to its normal condition forcing the electricity back again. However, power *is* required to overcome the frictional resistance (if there is any) that the pipe circuit offers to the current of water as it surges back and forth. Similarly, power (watts) is expended in overcoming the resistance (ohms) that any conductor in a permittor circuit offers to the current displaced in the circuit by virtue of the permittor. The power loss in any conductor in which *any* current flows is always $I^2 \times R$ (Art. 189).

NOTE.—It is not strictly true that *all* of the energy expended in displacing the electricity in charging a permittor is returned to the circuit when the permittor discharges. There are small energy losses, which occur when an alternating current flows through a permittance, called "dielectric hysteresis losses." These may be thought of as representing the energy wasted in a sort of intermolecular friction which occurs when the dielectric material is stretched and then returns to its original condition. The exact nature of these losses is not clearly understood. In any event, dielectric hysteresis losses are relatively very small and consequently are of little importance in ordinary practical work.

806. The Unit of Permittance Is Called the Farad.—A permittor (condenser) has a permittance of 1 farad when a difference of potential of 1 volt between the plates of the permittor will store up in it a charge of 1 coulomb (Arts. 11 and 137). See also formula (248), Art. 800. A farad is a very large unit, too large for general convenient use. A microfarad is one one-millionth (1/1,000,000) of a farad. That is, 1,000,000 microfarads = 1 farad. The permittors used in telephone instruments ordinarily have a permittance of something between 0.2 mf. and 2.0 mf. The telephone permittor in a wireless telephone receiving set ordinarily has a permittance of about 0.000,25 mf.

807. Elastance is the name which has been given to the reciprocal of permittance. Just as permittance may be measured

in farads, elastance may be measured in darafs. A permittor which has a permittance of 10 farads would have an elastance of $1 \div 10 = 0.1$ daraf. A permittor having a permittance of 0.5 farad would have an elastance of $1 \div 0.5 = 2$ darafs. The relation between permittance and elastance is analogous to that between conductance and resistance (Art. 147) and permeance and reluctance (Art. 260).

NOTE that "daraf" is "farad" spelled backward

SECTION 49

FIGURING ALTERNATING-CURRENT CIRCUITS

808. Susceptance is the name which has been given to the reciprocal of reactance. Just as the reactance of a circuit is a measure of the difficulty encountered in forcing an alternating current through the circuit, susceptance is a measure of the ease with which the current may be forced through the circuit. Reactance is a quantity similar to resistance (Art. 787). Susceptance is one similar to conductance (Art. 147) and it is likewise measured in mhos. A circuit having a reactance of 10 ohms has a susceptance of $1 \div 10 = 0.1$ mho. A circuit having a reactance of 0.5 ohm has a susceptance of $1 \div 0.5 = 2$ mhos.

809. The Net Reactance of Any Circuit Is Equal to the Sum of Its Permittive Reactance and Its Inductive Reactance.—Since these two reactances oppose one another the sum must be their algebraic sum. Inductance causes the current to lag behind the impressed e.m.f.; permittance causes the current to lead the impressed e.m.f., hence if there is more permittive reactance in a circuit than inductive reactance the current will lead, and vice versa. It follows that

$$X = X_l - X_c \text{ (ohms)} \qquad (258)$$

Wherein X = total reactance of the circuit, in ohms.
X_l = the inductive reactance of the circuit, in ohms.
X_c = the permittive reactance in the circuit, in ohms.

Note.—Inductive reactance is usually of most importance in circuits, except when the voltages involved are very high.

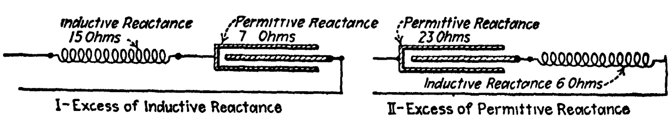


FIG. 495.—Illustrating addition of reactances in series.

Example.—The total reactance of the circuit (which contains only inductance and permittance) of Fig. 495,I $= X_l - X_c = 15 - 7 = 8$ ohms.

Example.—The reactance of the circuit of Fig. 495,II $= X_l - X_c = 6 - 23 = -17$ ohms. The minus sign preceding the value 17 indicates

that the resultant reactance is permittive, hence will cause the current in the circuit to lead the impressed e.m.f.

810. Resonance is that condition which occurs in an alternating-current circuit when the inductive reactance in the circuit is just equal to, and hence neutralizes, the permittive reactance. Under these conditions the current in the circuit is limited only by the resistance and may therefore attain enormous values. Obviously, total resonance occurs when

$$6.28 \times f \times L = \frac{1}{6.28 \times f \times C} \tag{259}$$

It follows that when resonance occurs

$$L \times C = \frac{1}{(6.28 \times f)^2} \tag{260}$$

or

$$f = \frac{1}{6.28 \times \sqrt{L \times C}} \text{ (frequency)} \tag{261}$$

Obviously, resonance can occur in a given circuit only when an e.m.f. of a certain frequency, f, above, is impressed on the circuit.

Example.—If an e.m.f. of such a frequency were impressed across the circuit of Fig. 496 so that the permittive reactance of X_c just equaled the

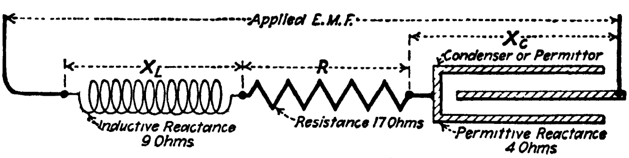


FIG. 496.—Inductive reactance, permittive reactance and resistance in series.

inductive reactance of X_l then the current through the circuit would be limited only by the resistance R.

811. Admittance is the name which has been given to the reciprocal of impedance (Art. 788) and it is expressed in mhos. The relation between impedance and admittance is similar to that between resistance and conductance. A circuit which has an impedance of 40 ohms has an admittance of $1 \div 40 = 1/40$ mho $= 0.025$ mho. A circuit which has an impedance of 0.2 ohm has an admittance of $1 \div 0.2 = 5$ mhos. In the problem

of Fig. 497, which will be discussed later, the admittance of the two impedances in parallel is 0.8 mho.

812. The impedance of a circuit containing only resistance and permittive reactance can be readily computed by drawing a vector diagram, as shown at Fig. 498,II. Although resistance and permittive reactance may be so associated in a circuit that

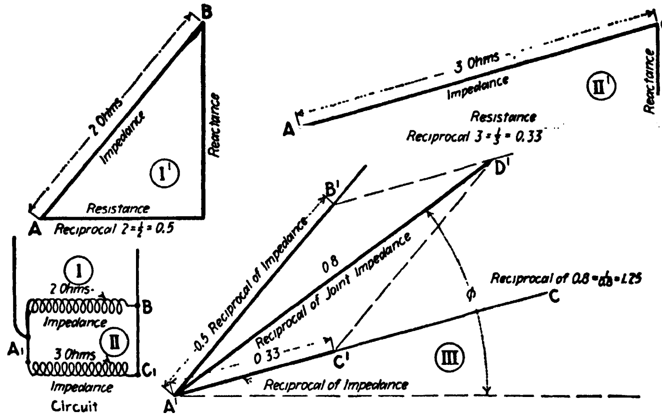


FIG. 497.—Graphic method of determining joint impedance of two impedances in parallel.

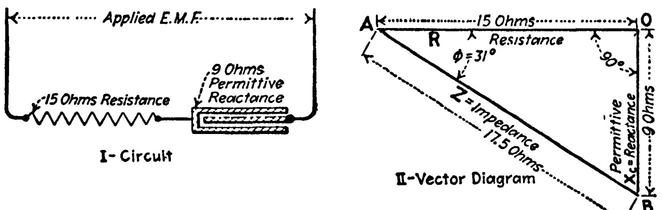


FIG. 498.—Indicating method of computing impedance of resistance and permittive reactance.

it is impossible to separate them physically, for the purposes of computation it may be assumed that they are separate properties, as suggested at I. On the basis of the explanation of permittive reactance which has been hereinbefore given (Art. 801) it can be shown that

$$Z = \sqrt{R^2 + X_c^2} \text{ (ohms)} \tag{262}$$

Wherein Z = impedance of the circuit, in ohms.
R = resistance of the circuit, in ohms.
X_c = permittive reactance of the circuit, in ohms.

Example.—To compute the impedance of the circuit of Fig. 498,I. The line OB is drawn vertically, proportional in length to 9 ohms, to represent the permittive reactance. It is drawn downward to indicate that permittive is the opposite of inductive reactance. Then the line OA, to represent the resistance, is laid off at right angles to OB. AB will then be proportional in length to the impedance of the circuit, which is 17.5 ohms. The angle ϕ, 31 deg. in this case, is the angle by which the current in the circuit will lead the impressed e.m.f.

813. The impedance of a circuit containing resistance, inductance, and permittance in series, may be readily computed. If one piece of apparatus or a circuit contains resistance, inductance and permittance, it can be assumed that each of these is a separate quantity as shown in Fig. 496. The permittive reactance, X_c, is first subtracted from the inductive reactance X_l which gives the reactance X. Then this reactance is combined with the resistance to obtain impedance; hence

$$Z = \sqrt{R^2 + (X_l - X_c)^2} \text{ (ohms)} \qquad (263)$$

or

$$Z = \sqrt{R^2 + X^2} \text{ (ohms)} \qquad (264)$$

Example.—What is the impedance of a piece of apparatus or a circuit (Fig. 496) which has a resistance of 17 ohms, an inductive reactance of 9 ohms, and a permittive reactance of 4 ohms? *Solution.*—It may be assumed that the reactances and resistances are separate quantities, as shown in Fig. 496. Then the vector diagram, Fig. 499, is constructed. Draw OR 17 units long to represent the resistance. Draw OL 9 units long to represent the inductive reactance. Draw OC vertically downward 4 units long to represent the permittive reactance. The total reactance is the difference between OL and OC, or OX, which is 5 units long. That is $X = 5$ ohms. Then RX is drawn and it will be proportional in length to the impedance of the circuit, 17.7 ohms in this case.

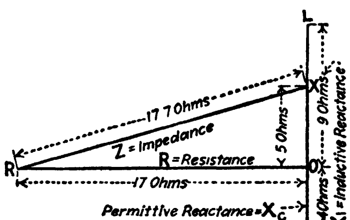


FIG. 499.—Solution of problem of FIG. 496.

NOTE.—For a circuit containing only resistance and permittance—no inductance—it follows, where $1 \div (6.28 \times f \times C)$ = permittive reactance, (Art. 801) that

$$Z = \sqrt{R^2 + \left(\frac{1}{6.28 \times f \times C}\right)^2} \qquad (265)$$

For a circuit containing resistance, inductance, and permittance.

$$Z = \sqrt{R^2 + \left[(6.28 \times f \times L) - \left(\frac{1}{6.28 \times f \times C}\right)\right]^2} \qquad (266)$$

814. The joint impedance of several impedances in series can be computed either graphically or arithmetically as shown in Fig. 500. It is necessary to know the resistance and reactance of each component to obtain their joint impedance because the arithmetical sum of the individual impedances of the components

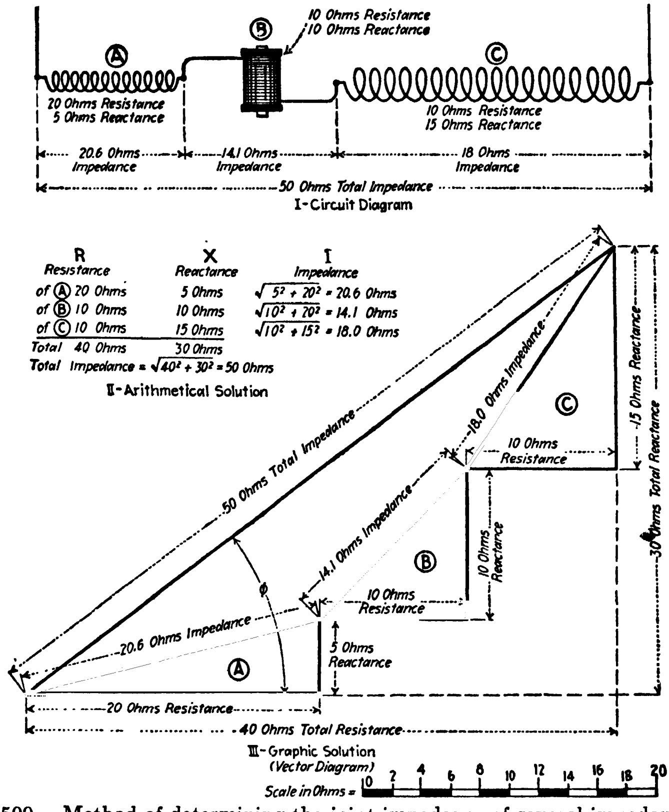


FIG. 500.—Method of determining the joint impedance of several impedances in series.

is not equal to their joint impedance. The angle ϕ is the angle of lag of the circuit—the angle by which the current will lag behind the impressed e.m.f. The cosine of this angle is, as will be shown later, the power factor (Art. 824) of the circuit.

815. The method of computing the joint impedance of a resistance and an impedance in parallel is illustrated in Fig. 501. The resistance, A_1B_1, shown at I is 10 ohms; the impedance,

A_1C_1, is 12.8 ohms. The impedance of 12.8 ohms was computed graphically as shown at III. The line $A'B'$ (IV) is drawn horizontally, and in length proportional (to any convenient scale) to the reciprocal—conductance (Art. 147)—of the resistor. Now $A'C'$ is drawn parallel to AC and proportional (to the same scale as that used for $A'B'$) to the reciprocal of the impedance AC. That is, $A'C'$ represents the admittance (Art. 811) of A_1C_1. The resultant $A'D'$ will be proportional in length to the joint admittance of A_1B_1 and A_1C_1 in parallel. That is, $A'D'$ will be proportional to the reciprocal of the joint impedance.

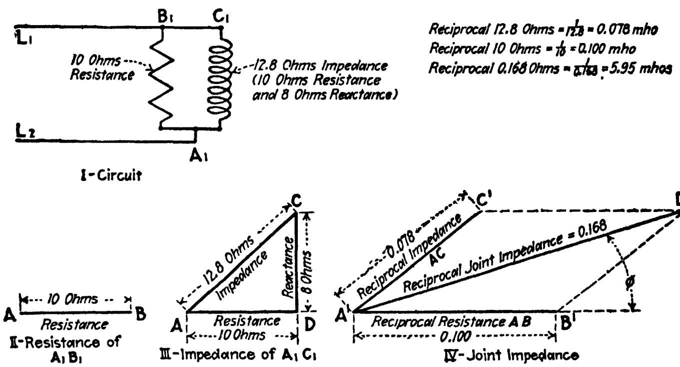


FIG. 501.—Graphic method of computing the joint impedance of resistance and impedance in parallel.

To obtain the joint impedance, compute the reciprocal of the value represented by $A'D'$, which, in the example illustrated is $1 \div 0.168 = 5.95$. Hence, the joint impedance of A_1B_1 and A_1C_1 is 5.95 ohms. The angle ϕ in IV is the angle by which a current in the circuit L_1L_2 would lag behind the impressed e.m.f.

816. To compute the joint impedance of two impedances in parallel, the method delineated in Fig. 497 may be used. First draw (as described in Art. 789), to some convenient scale, the impedance triangles I′ and II′ for impedances I and II. Then, draw $A'B'$, parallel to AB, proportional (to any scale) to the impedance A_1B_1, and draw $A'C'$, parallel to AC, proportional (to the same scale) to the impedance A_1C_1. The resultant $A'D'$ will be proportional to the reciprocal (admittance) of the joint impedance of A_1B_1 and A_1C_1. To obtain the joint impedance, compute the reciprocal of the value represented by $A'D'$, which in the example shown is $1 \div 0.8 = 1.25$. Therefore, the

joint impedance of the two impedances is 1.25 ohms. The angle ϕ is the angle by which the current in leads A_1 and B_1 lags behind the e.m.f. impressed by the generator.

817. Impedance of motors and transformers and energy resistance are phenomena which should be understood. In a direct-current circuit which does not contain a motor or other source of counter e.m.f., the current by Ohm's law (Art. 151) always equals the impressed e.m.f. divided by the resistance of the circuit. In alternating-current circuits, as has been shown, this relation seldom exists because the current in such circuits is limited not only by the resistance of the circuit but also by a counter e.m.f. of self-induction (Art. 516). In an alternating-

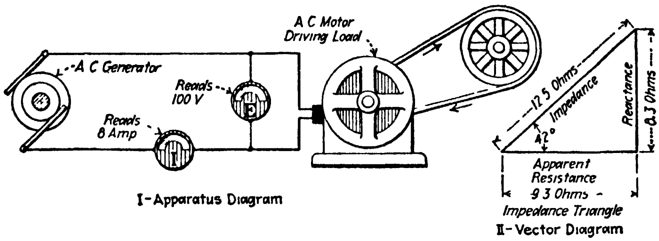


FIG. 502—Illustrating impedance of motor.

current circuit with which no iron (transformers or motor) is associated, the current with a given impressed e.m.f. is determined by the impedance (Art. 788) of the circuit. That is, the current is determined by the sum of the resistance and reactance.

However, in circuits in which are included transformers, motors, or other devices which convert electrical energy, the energy imparted to the circuit is obviously expended in other ways than by merely overcoming resistance. In such circuits the current is not equal to the impressed e.m.f. ÷ (resistance + reactance). The e.m.f. impressed on a motor or transformer may be considered as comprising two components: (1) an induction component which overcomes the counter e.m.f. of self-induction of the circuit. (2) An energy component which does actual work in making the motor or transformer deliver energy and in forcing current through the resistance of their windings.

Example.—Consider a motor (Fig. 502) which is driving a load. (Conditions similar to those to be described would obtain for a loaded transformer.) If the e.m.f. (100 volts) impressed on the motor be divided by the current at some certain load, the result will be the impedance, in ohms,

of the motor at that load. Assume 100 volts impressed and a current of 8 amp. Then the impedance of the motor will be 100 ÷ 8 = 12.5 ohms. Assume the angle of lag of the current to be 42 deg. (power factor = 74 per cent) and draw the impedance triangle as indicated at II. The resistance of the motor from this triangle would appear to be 9.3 ohms. Actually, the resistance of such a motor would be less than 2 ohms. Hence the balance of this apparent resistance of 9.3 ohms or at least 7.3 ohms would not represent real resistance but resistance to actual work being done by the motor in pulling the load.

This apparent resistance (9.3 ohms), representing partly real resistance and partly the doing of work, may be called energy resistance. Hence, in simple circuits without iron, such as overhead line and interior-wiring circuits, the components of the impedance of the circuit consist only of reactance and resistance. But if transformers or other devices containing iron are introduced, then the apparent impedance is composed of reactance and energy resistance.

NOTE.—Impedance of isolated circuits without iron, like overhead lines, is practically the same for all current strengths—but in circuits with iron, impedance may vary with the current. Voltage drop in an electrical energy transmission circuit can not be correctly estimated on the basis of the line impedance alone, since the drop may not correspond in phase with the total impedance of the circuit.

SECTION 50

POWER AND POWER FACTOR IN ALTERNATING-CURRENT CIRCUITS

818. Power in alternating-current circuits is determined by applying the same general laws (Art. 186) which relate to direct-current circuits. The power in any electric circuit (alternating-current or direct-current) is, at any instant, always equal, in watts, to the current (at that instant) multiplied by the voltage (at that instant) which impels the current. That is

$$P_i = I_i \times E_i \text{ (watts)} \tag{267}$$

Wherein P_i = instantaneous power of the circuit, in watts at a given instant.

I_i = instantaneous current in the circuit at the same instant, in amperes.

E_i = instantaneous volts in the circuit at the same instant, in volts.

(It being understood that this E_i represents the available or energy voltage which actually impels the current at the given instant.) In a direct-current circuit containing no source of counter e.m.f., the voltage impressed on the circuit (except during the transient intervals, Art. 518) is the voltage which impels the current. In an alternating-current circuit, the voltage impressed on the circuit may or may not be the one which impels the current: (1) If the circuit contains no inductance or permittance, that is, if the current is in phase with the impressed e.m.f. (Art. 772), the impressed voltage is the one which impels the current. (2) If the circuit contains inductance or permittance (capacity), or both, that is, if the current is not in phase (Art. 725) with the impressed e.m.f., some voltage other than the impressed voltage will impel the current.

819. Power taken by an alternating-current circuit in which the current is in phase with the impressed e.m.f. (this condition

obtains in a circuit containing resistance only) is equal to the product of volts × amperes. That is,

$$P = E_E \times I_E \text{ (watts)} \tag{268}$$

and

$$E_E = \frac{P}{I_E} \text{ (volts)} \tag{269}$$

also

$$I_E = \frac{P}{E_E} \text{ (amp.)} \tag{270}$$

Wherein P = power taken by the circuit, in watts.
E_E = e.m.f., effective, impressed on the circuit, in volts.
I_E = current, effective, in the circuit, in amperes.

Figure 503 illustrates conditions in such a circuit. If the current at any instant, for example OA, be multiplied by the

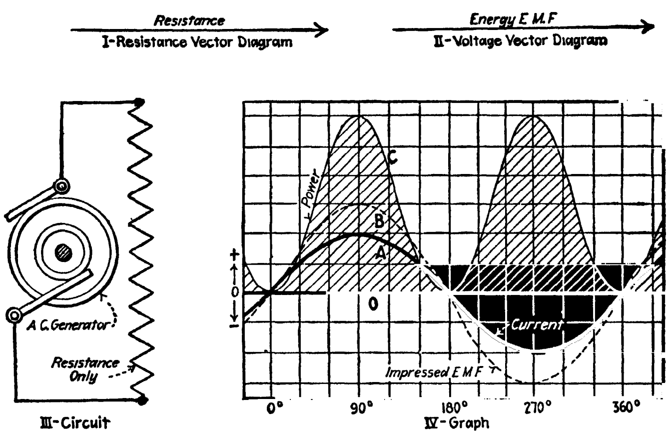


FIG. 503.—Illustrating conditions for 100 per cent power factor. Circuit composed wholly of resistance and current is in phase with impressed e.m.f.

impressed e.m.f. (OB) at the same instant, then product (OC) will be the power at that instant. By making this computation for a number of different instants in the cycle, a corresponding number of points in the power curve may be obtained. The power curve can then be plotted through them as shown. The total energy taken by the circuit during any alternation is proportional to the area (shaded in the illustration) within the power loop of that alternation.

NOTE.—All of each power loop will lie above the zero reference line—that is, the power will be positive—because when two + quantities or two − quantities are multiplied together the result will be a + quantity. By positive power is meant power which is delivered by the generator (or other source of e.m.f.) to the circuit. Sometimes a circuit may, as will be shown, deliver power back to the source of power which feeds it; such is called negative power.

Example.—What power is being taken by the incandescent-lamp load of Fig. 504,I (all incandescent-lamp loads are practically noninductive) which is being served by an alternating-current generator? The voltmeter reads 200 volts and the ammeter reads 24 amp. *Solution.*—From the

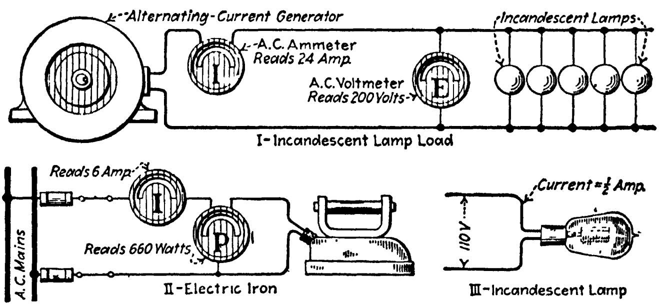


FIG. 504.—Examples in computing alternating-current power and power factor.

above, $P = E \times I$; hence $P = 200 \times 24 = 4{,}800$ watts or 4.8 kw. Therefore, the generator is delivering 4.8 kw.

Example.—The alternating-current wattmeter of Fig. 504,II indicates that the noninductive iron is taking 660 watts. The current, as shown by the ammeter, is 6 amp. What is the voltage impressed across the iron? *Solution.*—As outlined above, $E = P \div I$; hence $E = 660 \div 6 = 110$ volts, which is the pressure impelling the current in the iron.

Example.—The incandescent lamp of Fig. 504,III is taking 0.5 amp. at 110 volts. What is its power consumption? *Solution.*—$P = E \times I = 110 \times 0.5 = 55$ watts.

820. Power taken by an alternating-current circuit in which the current is not in phase with the impressed e.m.f. will not, as will be shown, be equal to the product of impressed volts × amperes. In any alternating-current circuit which contains inductance, and consequently reactance, the current lags (Art. 756) behind the impressed e.m.f. Consider Fig. 505 illustrating conditions in a circuit which has just enough reactance that the current lags behind the impressed e.m.f. by 30 deg. If the instantaneous currents at the different instants are multiplied by the instantaneous e.m.fs. at the same instants and a power

curve is plotted through points representing these products, it will have the form shown in IV. Note that a portion of the power curve N_1 and N_2 lies below the zero line, because when a + quantity is multiplied by a − quantity a − quantity results. The areas of these little loops (N_1 and N_2) represent negative energy or energy which is returned to the generator by the line and hence is not available for doing work in the circuit. Note that no power is required to overcome reactance. The vector diagrams I and II show the voltage and resistance relations, the

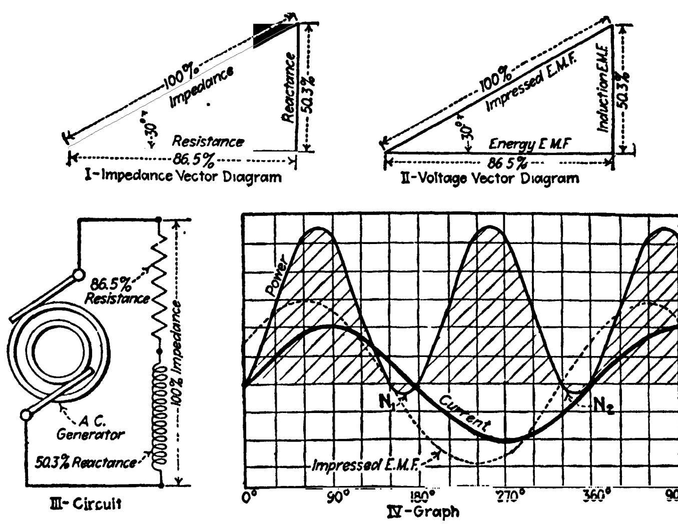


FIG. 505.—Illustrating conditions for 86.5 per cent power factor. Current lags 30 degrees behind impressed e.m.f.

angle of lag being the same in these as in the curve. It is evident then from a consideration of Fig. 505 that the power in an inductive alternating-current circuit is not equal to the product of effective volts × effective amperes because this product would not take into consideration the negative power (N_1 and N_2, Fig. 505) which is not available for doing work in the circuit.

821. The product of the available or energy e.m.f. times the current gives real power.—Thus, in Fig. 506, if the instantaneous energy or active e.m.f. at different instants be multiplied by the instantaneous currents at different instants, the products will be the instantaneous power at those instants. Points can thus be determined through which the power curve can be com-

pleted as shown at II. The shaded areas within loops represent, and are proportional to, the energy taken by the circuit because the current is always in phase with the energy or active e.m.f. The areas of the loops in Fig. 506,II are equal to the shaded areas above the reference line in Fig. 505.

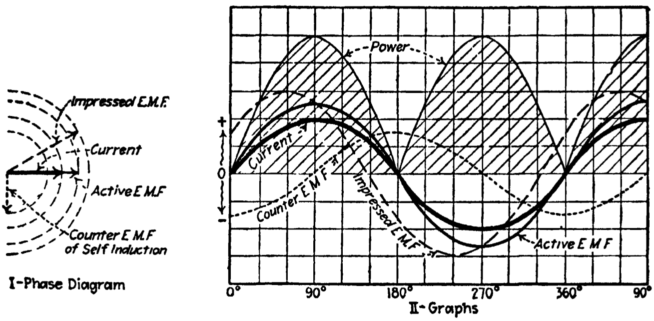


FIG. 506.—Showing how alternating-current power is proportional to the product of available, or energy, e.m.f. and current.

NOTE.—Ordinarily in alternating-current circuits the value of the energy e.m.f. is not known; only the values of the impressed e.m.f. and the current are known. But if these two values and the angle by which the current lags behind the impressed e.m.f. are known, the power taken by the circuit can be readily computed, as will be shown.

822. The computation of the power taken by an inductive circuit may be made by using a vector diagram or a phase diagram, Fig. 507. The e.m.f. impressed on any inductive circuit is used up in two ways (Art. 785): (1) to impel the current in the circuit through its resistance and to force the current to do work; this part of the impressed e.m.f. represents actual power (see also Fig. 472); (2) to overcome or neutralize the counter e.m.f. of induction in the circuit; this portion of the impressed e.m.f. does not represent actual power. Thus (Fig. 507) if E be laid out to scale to represent the effective impressed e.m.f., it may be resolved into two components: (1) an induction component E_1, which merely neutralizes the counter e.m.f. of self-induction and hence does no real work; and (2) the energy

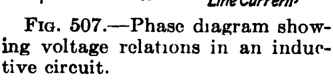


FIG. 507.—Phase diagram showing voltage relations in an inductive circuit.

component E_A, which impels current through resistance and also does work. The energy component must be in phase with the current, hence lags behind the impressed e.m.f. E by the angle ϕ. The vectors E_1 and E_A will be at right angles to one another, because the current always lags 90 deg. behind the component of the voltage which neutralizes the counter e.m.f. of self-induction. Since E_A is the only component available for doing work, it follows that the average power taken by the circuit is equal to the product of effective current $\times$ the available or energy component of the effective e.m.f. (of the component of the e.m.f. in phase with the current); hence

$$P = E_A \times I \text{ (watts)} \tag{271}$$

but

$$E_A = E \times \cos \phi \text{ (volts)} \tag{272}$$

hence

$$P = I \times E \times \cos \phi \text{ (watts)} \tag{273}$$

or

$$P = (I \times E) \times \cos \phi \text{ (watts)} \tag{274}$$

Wherein P = power taken by the circuit, in watts.
I = effective current in the circuit, in amperes.
E = effective e.m.f. impressed on the circuit, in volts.
E_A = energy component of the effective e.m.f.
$\cos \phi$ = cosine of the angle by which the current lags behind the impressed e.m.f.

NOTE.—Cos is the abbreviation for cosine. If the length of the hypotenuse of any right-angled triangle be multiplied by the value called the

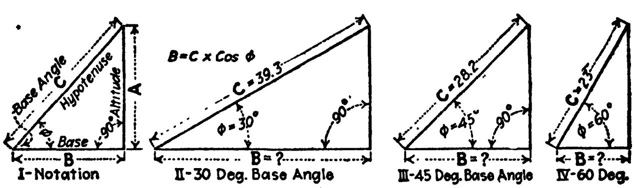


FIG. 508.—The cosine and problems involving its application.

cosine of the angle (base angle) the length of the base will be the result. That is (Fig. 508,I), $B = C \times \cos \phi$. Obviously, there is a different cosine value for every different angle. For example, cos 30 deg. = 0.866, cos 45 deg. = 0.707, cos 60 deg. = 0.500, etc. Complete tables of cosine values are given in the Appendix of this volume.

Example.—What is the length of base of the triangle of Fig. .508,II? *Solution.*—From a table of trigonometric functions it is found that cos 30 deg. = 0.866; hence $B = C \times \cos \phi = 39.3 \times 0.866 = 34$, which is the length of base.

Example.—The cos of 45 deg. = 0.707, hence the length of base of III = $28.2 \times 0.707 = 20$.

Example.—The cos of 60 deg. = 0.500. Hence, in IV, $B = C \times \cos \phi = 23 \times 0.5 = 11.5$.

Example.—In the circuit shown in Fig. 479 the relation of reactance to resistance is such that the energy e.m.f. as shown at III lags 30 deg. (as may be determined by constructing the impedance or voltage-vector diagrams, Art. 789) behind the impressed e.m.f. Now cos 30 deg. = 0.866. Hence $P = E \times I \times \cos \phi = 100 \times 50 \times 0.866 = 4{,}330$ watts = 4.3 kw., which is the power being taken by the circuit shown.

Example.—Referring to Fig. 478 the lag angle $\phi = 60$ deg. Cos 60 deg. = 0.50. Therefore, the power being taken by this circuit $= P = E \times I \times \cos \phi = 100 \times 50 \times 0.50 = 2{,}500$ watts = 2.5 kw.

Example.—Similarly, in the circuit of Fig. 477, $\phi = 75$ deg.; cos 75 deg. = 0.259. Hence, in this circuit, $P = E \times I \times \cos \phi = 100 \times 50 \times 0.259 = 1{,}295$ watts = 1.3 kw.

NOTE.—In the last two examples the current and the impressed e.m.f. are taken the same as in the first example, but the power taken by the circuits decreases as their inductances—or reactances—increase.

823. Power factor is the name given to that quantity by which the product of effective volts and effective amperes in a circuit must be multiplied to obtain the true power in watts taken by the circuit. From equation (273) above, it is evident that (for a circuit containing inductance) the power factor is equal to the cos of the angle by which the current lags behind the impressed e.m.f. That is, power factor = $\cos \phi$. Hence, the greater the amount of inductance in a circuit, the lower will be the power factor. Power factor is expressed as a percentage. Power factor is also defined[1] as the ratio of the true power to the apparent power. This definition is general, since it applies also to non-sinusoidal currents and voltages.

NOTE.—Obviously the nature of the load on a circuit determines its power factor. In practice the average values of power factors for circuits with different kinds of loads will be approximately as follows:

Incandescent lighting—no motors	95 per cent
Lighting and motors	85 per cent
Motors only	80 per cent

[1] V. Karapetoff, "The Electric Circuit," McGraw-Hill Book Company, Inc.

If a circuit contains resistance only, its power factor will be 100 per cent, and in such a circuit volts $\times$ amperes = true power (Art. 772). Power factor can not be greater than 100 per cent. See the author's "American Electricians' Handbook" for a much more complete table of power factors of circuits having loads of different characteristics.

824. The formulas for power factor are for single-phase circuits (since $\cos \phi$ = power factor which can be expressed merely as $p.f.$) these:

$$P = I \times E \times p.f. \text{ (watts)} \tag{275}$$

$$I = \frac{P}{E \times p.f.} \text{ (amp.)} \tag{276}$$

$$E = \frac{P}{I \times p.f.} \text{ (volts)} \tag{277}$$

$$p.f. = \frac{P}{I \times E} \text{ (power factor)} \tag{278}$$

Wherein all of the symbols have the meanings given above.

Examples of circuits of 100 per cent power factor are given in Fig. 504.

Example.—The alternating e.m.f. impressed on the alternating-current circuit of Fig. 509,I is 200 volts. It is known that the circuit is taking

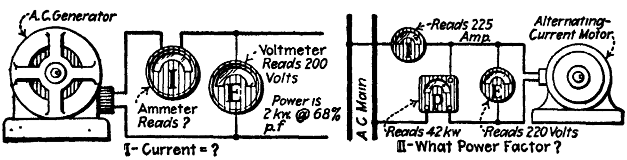


FIG. 509.—Problems involving power factor.

2 kw. at 68 per cent power factor. What is the current? *Solution.*—2 kw. = 2,000 watts. Then, substituting in formula (276), $I = P \div E \times p.f. = 2{,}000 \div (200 \times 0.68) = 14.7$ amp.

Example.—In an alternating-current circuit feeding a motor (Fig. 509,II) the following readings are taken: Power = 42 kw. E.m.f. = 220 volts. Current = 225 amp. What is the power factor? *Solution.*—42 kw. = 42,000 watts. Substitute in formula (278) $p.f. = P \div (I \times E) = 42{,}000 \div (225 \times 220) = 0.85$ = 85 per cent power factor.

825. The power factor of a wholly inductive circuit would, if such a circuit could exist, be zero. Figure 510 illustrates graphically the conditions in such a circuit. Note that the area

of the positive power loops above the reference line is the same as that of the negative power loops below the reference line; hence the net power taken by the circuit is zero.

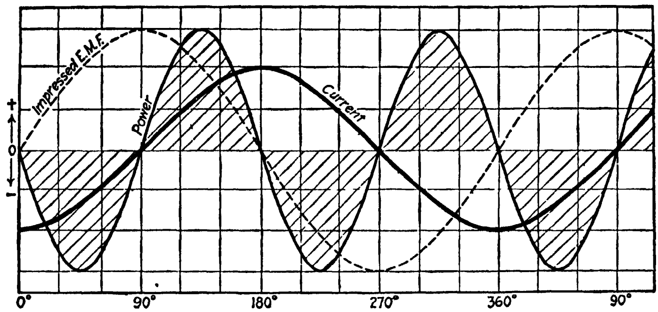


FIG. 510.—Illustrating conditions for zero (0) power factor. Circuit is wholly inductive and current lags 90 deg. behind impressed e.m.f.

NOTE.—Curve Fig. 510 is of theoretical interest only; it is apparent that no circuit can exist without some resistance. Some circuits, however, are almost wholly inductive. Therefore the conditions in them approximate those of Fig. 510. A circuit feeding the primary of a transformer, the secondary of which is open, is almost wholly inductive. The same is true of a circuit serving an induction motor operating at no load. The currents in such circuits are largely wattless (Art. 826).

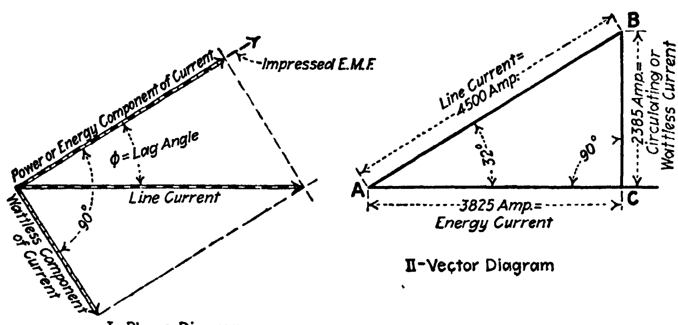


FIG. 511.—Illustrating the meaning of the term wattless current.

826. Wattless current is the name given to that part, portion, or component of an alternating current which may, when the situation is viewed in one way, be considered as being productive of no actual work. There is only one current—the line current —in a circuit, and it is always in phase with the available or

energy e.m.f. (Art. 775), but it is sometimes convenient to assume that this current comprises two portions or components: (1) the power or energy component, in amperes (Fig. 514), which is of such value that when it is multiplied by the impressed e.m.f., in volts, the product will be the actual watts power taken by the circuit; (2) the wattless component, in amperes, which, when multiplied by the impressed e.m.f. in volts, will give the circulating power or reactive volt amperes taken by the circuit. Thus, Fig. 511,I, shows how the line current may be resolved graphically into so-called energy and wattless components; which must, because of their definitions, differ in phase by 90 deg. Refer also to Fig. 512. Furthermore, the energy component, to satisfy its definition as given above, is in phase with the impressed e.m.f. The angle between the energy component of current and the line current is (because the line current is always in phase with the energy component of e.m.f.) the same as the angle by which the line current lags behind the impressed e.m.f. This angle may have any value between 0 deg. and 90 deg., depending on the ratio of resistance to reactance in the circuit. See Figs. 476 to 479; also Fig. 512.

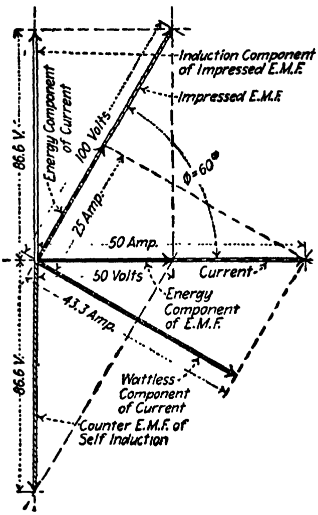


FIG. 512.—Vector relations, e.m.f., and current components of a circuit.

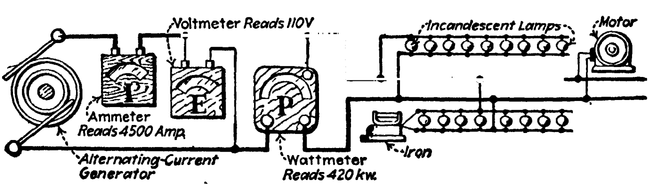


FIG. 513.—Determination of the power factor of an alternating-current circuit.

Example.—Figure 511,II is a vector diagram illustrating the relation between the line current, wattless current, and energy current for the problem of Fig. 513. The line-current vector AB was drawn proportional in length to 4,500 amp. AC was laid off 32 deg. (this being the lag angle) below AB and of indefinite length. BC was dropped from B perpendicular to AC.

Then the length AC, which scales 3,825 amp., represents the energy current and CB, scaling 2,385 amp., represents the wattless current. AC could have been obtained by multiplying AB by the cosine (note under Art. 822) of 32 deg.; BC, by multiplying AC by the sine (Art. 553) of 32 deg.

827. The effect of low power factor in a constant-potential circuit is to increase the current necessary for the transmission of a given amount of power over that current which would transmit the same power in a circuit of unity power factor (a noninductive circuit). That is, in circuits of low power factor the wattless current is considerable. This excess of current does not, in itself, represent an additional expenditure of energy, that is, it does not require more coal burned under the boilers. It does, however, involve slight additional energy expenditure because it increases the $I^2 \times R$ power loss in the conductors which it traverses. It has the further undesirable feature of decreasing, because of this same I^2R heating effect, the effective capacities of the generators. See the "American Electricians' Handbook."

828. Low power factor may be corrected by the installation of synchronous motors, synchronous condensers, or capacitors. Low power factor in practice is frequently due to underloaded induction motors.

829. The practical determination of the power factor of a circuit may be made with an ammeter, a voltmeter, and a wattmeter. Alternating-current wattmeters always at any given instant indicate the product of the average instantaneous current in a circuit × the average instantaneous energy e.m.f. at that instant, hence, they indicate true power. Ammeters and voltmeters indicate effective values and do not show anything in regard to phase relations. With 20 amp. in a circuit, the ammeter will read 20 amp. regardless of whether the current reaches its maximum and intermediate values at the same instant as does the voltage. That is, ammeters and voltmeters take no cognizance of phase relation. Hence, the product of volts and amperes as indicated by these instruments is (in circuits containing inductance or permittance) apparent power taken by the circuit. The true power may be determined from wattmeter readings. Then the power factor can be readily computed.

Example.—Figure 509,II and the accompanying solution give an example of the foregoing.

Example.—Figure 513 shows an alternating-current circuit serving a mixed motor and lighting load. The power as indicated by the wattmeter, taken by the circuit, is 420 kw. The impressed e.m.f. is 110 volts and the current is 4,500 amp. What is the power factor? *Solution.*—Kilovolt-amperes = (volts × amperes) ÷ 1,000 = (110 × 4,500) ÷ 1,000 = 495 kva. Now from equation (282) *p.f.* = kw. ÷ kva. = 420 ÷ 495 = 0.85 = 85 per cent power factor.

830. A vector diagram showing the relation of true power to apparent power may be plotted as suggested in Fig. 514. The general method involved is the same as that used in drawing impedance and voltage vector diagrams.

Example.—In the problem of Fig. 513 the apparent power was shown to be 495 kva. and the true power 420 kw. Hence if a right-angled triangle (Fig. 514) be drawn so that the base is proportional in length to the true power, 420 kw., and the hypotenuse so that it is proportional in length to the apparent power, 495 kva., the angle ϕ between them will be the angle by which the current lags behind the impressed e.m.f. The cosine of this angle will then be the power factor of the circuit. In the example illustrated the angle ϕ is found to be 32 deg., and cos 32 deg. is, by referring to a table, found to be 0.85. Hence the power factor of the circuit is 85 per cent.

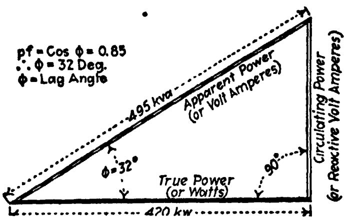


FIG. 514.—Vector triangle indicating relation of true to apparent power for problem of Fig. 513.

831. True watts and apparent watts and true power and apparent power are terms that are frequently used. The true watts or true power delivered to a circuit is the actual power P (as indicated by a wattmeter) which is being consumed in a circuit. True power may be expressed either in watts or in kilowatts. The apparent watts or power (which is always greater than the actual power in a circuit containing inductance or permittance is the product of the effective voltage impressed on the circuit × the effective current in the circuit. Apparent power may be expressed in volt-amperes or in kilovolt-amperes. Hence it follows from equation (275), since $I \times E$ = apparent watts and cos ϕ = power factor (the following applies specifically to single-phase circuits)

$$\text{True watts} = \text{apparent watts} \times \text{power factor.} \quad (279)$$
$$\text{Watts} = \text{volt-amperes} \times \text{power factor.} \quad (280)$$
$$\text{Kilowatts} = \text{kilovolt-amperes} \times \text{power factor.} \quad (281)$$

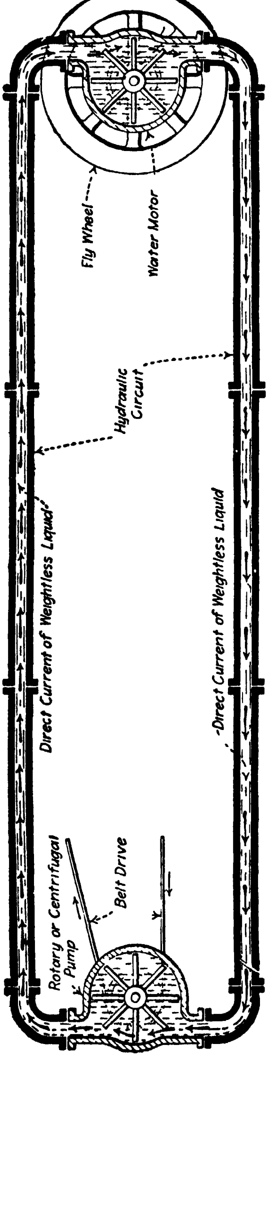


FIG 515.—Analogy to inductance in a direct-current circuit.

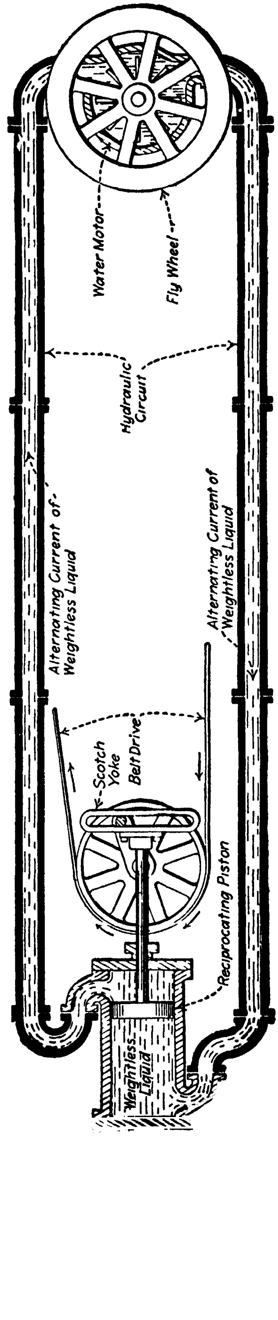


FIG. 516.—Analogy to inductance in an alternating-current circuit.

Therefore, power factor equals the ratio of true watts to apparent watts, that is,

$$\text{Power factor} = \frac{\text{true watts}}{\text{apparent watts}} = \frac{\text{watts}}{\text{volt-amperes}}$$
$$= \frac{\text{kilowatts}}{\text{kilovolt-amperes}} = \frac{\text{kw.}}{\text{kva.}} \qquad (282)$$

NOTE.—The above can be graphically expressed by a right-angled triangle, Fig. 514, in which the base may represent watts, true watts, kilowatts, or kw., and the hypotenuse may represent, respectively, apparent watts, volt-amperes, kilovolt-amperes, or kva. Then the base angle, ϕ, between them will be the lag angle. The cosine of this angle, ϕ, which is numerically equal to the ratio between the base and the hypotenuse, is the power factor.

832. Power factor due to combined inductance and permittance in a circuit can be computed on the basis of the method suggested in Fig. 499.

NOTE.—A hydraulic analogy to the effect of inductance in a direct-current circuit is suggested in Fig. 515, the inertia of the flywheel being analogous to the inductance of an electric circuit. The flywheel, by virtue of the blades of the water motor to which it is attached, tends to oppose the starting of a flow of current in the circuit. That is, it introduces a counter pressure for the period during which the flow of water is being started. After the flywheel has attained its speed, that is, when the flow of water has attained its normal value, the only counter flow then offered is due to the frictional resistance which is analogous to electrical resistance. Similarly, if the belt were thrown off the centrifugal pump, so that it no longer exerted an impelling force in the circuit, the inertia of the flywheel acting through the blades of the water motor would continue the flow of water in the circuit, after the driving power had been discontinued from the pump. Note that the inertia of this hydraulic circuit is not due to the current of liquid in it, because the liquid is supposed to be weightless.

NOTE.—A hydraulic analogy to inductance in an alternating-current circuit is illustrated in Fig. 516. The inertia of the flywheel acting through the blades of the water motor tends to oppose any change in the rate of flow of the weightless liquid, which is analogous to electricity. If the rate of flow tends to increase, the inertia of the flywheel opposes it. On the other hand, if the current tends to decrease or change in direction, the flywheel acting through the water-motor blades also opposes such changes.

833. Power factor due to permittance or capacitance is a quantity that applies to permittive circuits in a way similar to that in which power factor due to inductance applies to inductive circuits. However, since the charging current due to permit-

tance is leading, the power factor due to permittance is a leading power factor; that due to inductance is a lagging power factor.

Note.—Leading power factors of appreciable value are seldom encountered in circuits excepting these operating at relatively high voltages. Leading power factors can be produced in a circuit by operating an over-excited synchronous motor or condenser in the circuit. Hence with a synchronous motor or condenser lagging power factor in a circuit can be neutralized.

SECTION 51

POLYPHASE CIRCUITS AND SYSTEMS

834. A polyphase system is one of more than one phase (Art. 725). A polyphase circuit is one for transmitting electrical energy in a polyphase system. There are two important polyphase systems: (1) the two-phase and (2) the three-phase.

NOTE.—Polyphase systems are used for two reasons: (1) because they are more economical than the single-phase (Art. 727) and (2) because induction motors, which are of very simple, rugged, and reliable construction —they require no commutators—can be used with them.

835. Two-phase or quarter-phase is a term ("A.I.E.E. Standardization Rules") which characterizes the combination

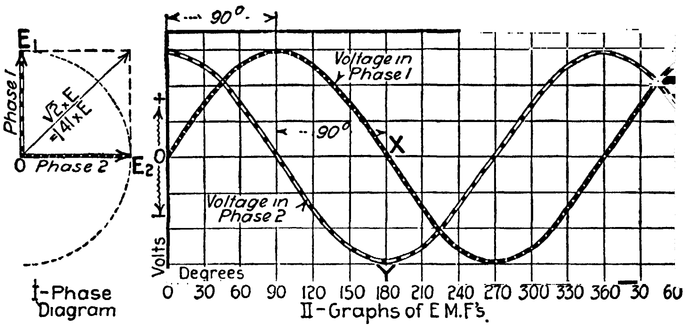


FIG. 517 Phase diagram and curves for the e.m.fs. produced by a two-phase alternating-current generator.

of two symmetrical circuits, each energized by alternating e.m.fs. which differ in phase by a quarter of a cycle, i.e., by 90 deg. (see Fig. 517). Hence, with a two-phase e.m.f. or two-phase current, there are at any instant two phases (Art. 725). Although two-phase systems are not used so frequently as formerly, an understanding of the principles underlying them is essential.

836. The production of a two-phase e.m.f. obviously consists in the generation of two alternating sine-wave e.m.fs., 90 deg. apart, as shown in Fig. 517. Each of these e.m.fs. may, as

shown in Fig. 518, be impressed on its own separate circuit. A single-phase e.m.f. (Art. 727) may be produced by rotating a suitably arranged magnetic field within a properly disposed core on which is wound the alternating-current armature winding. In Fig. 519 this construction is diagrammatically indicated.

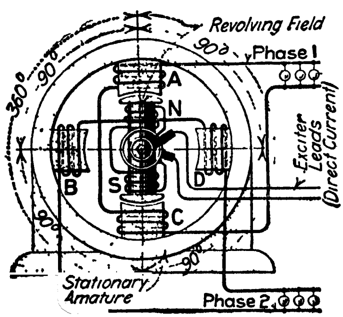


FIG. 518.—Elementary two-pole, two-phase, revolving-field alternator.

Two single-phase alternating-current generators could be made to produce a two-phase e.m.f. if their rotating parts were firmly keyed to the same shaft so that the 90-deg. phase relation would be preserved. However, it is more economical to arrange both of the phase windings on one frame, that is, to incorporate them in one machine, rather than to use two frames. With two-phase, as with single-phase, alternating-current generators there are two general types: revolving-armature generators (Fig. 520), and revolving-field generators (Fig. 518).

837. Really Two Separate Single-phase E.m.fs. Are Produced by a Two-phase Generator.—These may, as above noted, be used to impel two different currents in two distinct circuits.

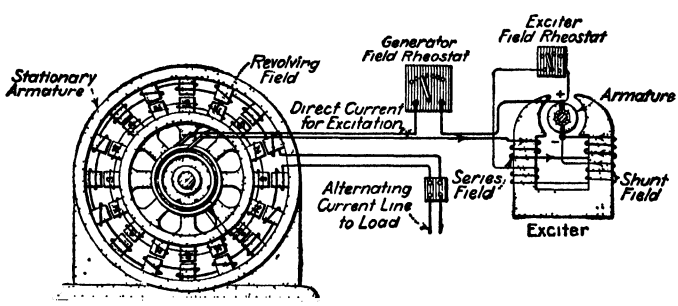


FIG. 519.—A sixteen-pole, single-phase, revolving-field alternator and its exciter.

However, it is advisable sometimes to combine or interconnect the two circuits in a manner which will be described later.

838. A revolving-armature, two-phase generator is shown diagrammatically in Fig. 520. If the armature winding of any direct-current generator is tapped at four points 90 electrical

deg. apart, as shown in Fig. 520,I, and each of these taps is connected to a collector ring, an e.m.f. will be impressed across each of the pairs of rings, which will differ in phase by 90 deg. from the e.m.f. impressed on the other pair of rings. This follows from the principles outlined in Arts. 742 and 743. Hence

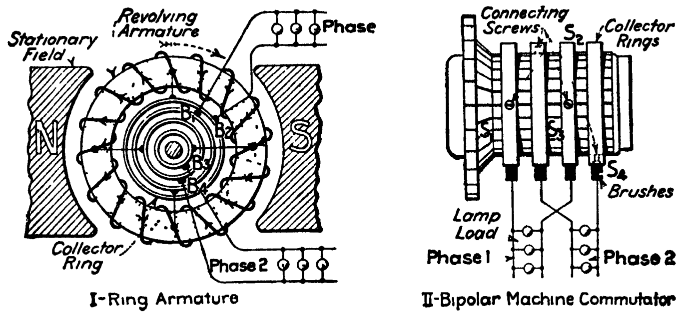


FIG 520 Illustrating methods of obtaining two-phase currents from a bipolar, revolving-armature generator.

a bipolar direct-current generator can be converted into a two-phase alternating-current generator by arranging on, but insulating from, its commutator four collector rings. Each of these collector rings is connected to a commutator bar by a screw passing through the ring and into the bar. However, the four bars must be 90 deg. apart in the commutator, as shown in Fig. 520,II. Revolving-armature, alternating-current generators are seldom used except for machines of very small capacity. However, the principle involved is frequently utilized for rotary converters.

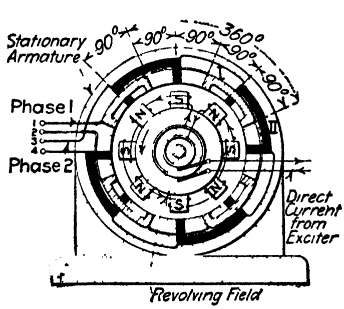


FIG. 521 —Diagrammatic representation of an eight-pole, two-phase revolving-field alternator.

839. Revolving-field, two-phase generators are the most frequently used. Figure 518 illustrates the principle. It is obvious that if the field core, *NS*, which is excited with direct current, is rotated at a uniform speed, it will induce in the set of coils *AC* an e.m.f. which will differ in phase by 90 deg. from that induced in the set *BD*. Note that to satisfy these conditions the armature coils must be located 90 electrical deg. apart. The arrangement of a practical two-phase, revolving-field

generator is shown in Fig. 521. Note that the distance between the centers of similar field poles, for example S and S, constitutes 360 electrical deg. and that the sides of the armature coils are on this basis located 90 deg. apart. Consideration will show that the principle of operation of the generator of Fig. 521 is the same as that outlined diagrammatically in Fig. 518 and that the e.m.f. induced in phase 1 (the black set of coils) will differ in phase by 90 deg. from that induced in phase 2 (the cross-hatched set).

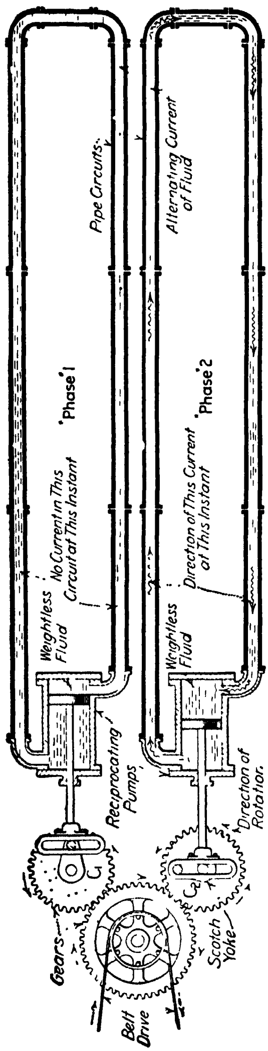


FIG. 522.—Hydraulic analogy to a two-phase, four-wire, alternating-current generator and circuit.

NOTE.—A hydraulic analogy of a two-phase, four-wire alternating-current generator and circuit is shown in Fig. 522. The two reciprocating pumps are analogous to two single-phase, alternating-current generators or, with their gear drive, to one two-phase, alternating-current generator, and the two pipe circuits are analogous to the two distinct electrical circuits of a two-phase, four-wire system. Note that the circuits and generators are precisely identical except that there is a definitely fixed difference of 90 deg. in the locations of the two cranks, C_1 and C_2, which drive the two pumps. Study Fig. 522 in connection with Fig. 517 and note that at the instant pictured in Fig. 522 when the current of fluid in phase 2 is being impelled at maximum velocity, that is, the current intensity is a maximum (piston head passing through center of stroke) in the direction shown by the arrows, the fluid in phase 1 is at rest, that is, at this instant the current is zero in phase 1. This corresponds with the 180-deg. instant in Fig. 517, where the voltage in phase 1 is at zero (X), and the voltage in phase 2 is a maximum (Y), in the negative direction. If the analogy be followed further, it will be found that the fluid currents in the two circuits of Fig. 521 will increase and decrease in intensity and change in direction precisely as is traced in Fig. 517,II, which, though it shows curves of the e.m.fs.. may also be taken as representing curves of a two-phase current.

840. There are two commercial methods of connecting the phase windings of two-phase generators. In the two-phase, four-wire system of Fig. 523,II, each of the two groups of armature coils is connected to its own distinct external circuit. The hydraulic analogy of this arrangement is shown in Fig. 522

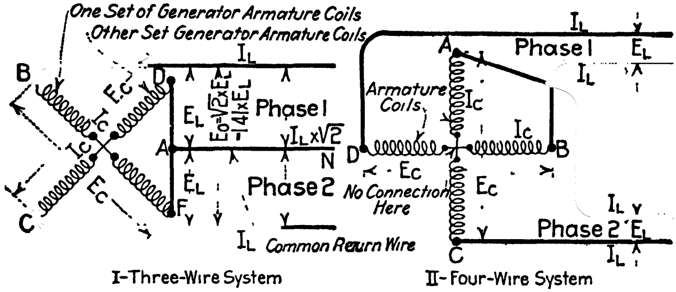


FIG 523.—Two methods of connecting two-phase-generator armature windings.

In the two-phase, three-wire system of Fig. 523,I, the two groups of armature coils are connected in series within the generator as shown. Connections to the external circuit are made from the junction point A of the two groups of coils and from the unjoined ends B and C of the two groups. Figure 524 delineates a hydraulic analogy of a two-phase, three-wire circuit.

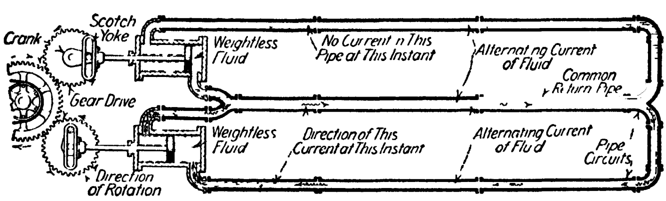


FIG 524.—Hydraulic analogy to a two-phase, three-wire, alternating-current generator and circuit.

841. The voltage and current relations in a two-phase, four-wire system may be determined by inspecting Fig. 523,II, and by noting that each phase or circuit may be treated as if it were a separate circuit as shown in Fig. 525. Then, assuming that the load is equally balanced on each of the two phases,

$$E_C = E_L \text{ (volts)} \tag{283}$$

and

$$I_C = I_L \text{ (amp.)} \tag{283a}$$

Wherein E_L = the e.m.f. impressed across each pair of the line wires.

E_C = the e.m.f. induced in each set of armature coils.

I_L = the current in each line wire.

I_C = the current in each set of armature coils.

842. The voltage and current relations in a two-phase, three-wire system (Fig. 526) may be determined by an inspection of

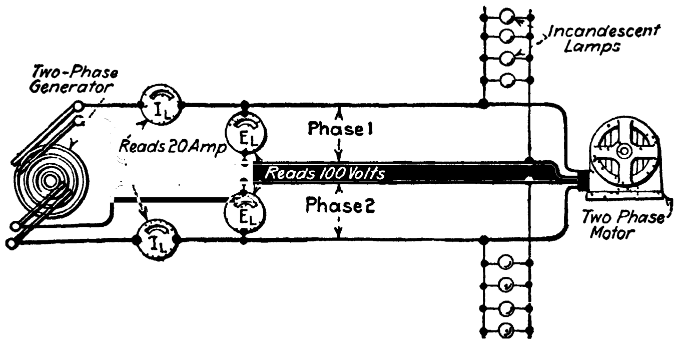


FIG 525.—Example of balanced two-phase, four-wire circuit

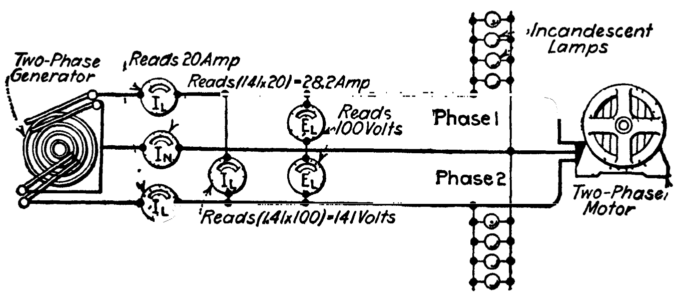


FIG 526.—Example of balanced two-phase three-wire circuit

Fig. 523,I. The voltage E_O between the two outside wires will equal the vector sum of the voltages $E_C + E_C$ (or $E_L + E_L$) induced in each set of armature coils. These two voltages (E_C and E_C) can not be added directly, since they differ in phase by 90 deg., but they can be added vectorially, as shown in Fig 517,I. Since the voltages are equal in intensity and differ in phase by 90 deg., their sum will be proportional to the length of the diagonal of a square. Now the diagonal of a square always equals 1.41 times the length of one of its sides.

Hence

$$E_O = 1.41 \times E_C = 1.41 \times E_L \text{ (volts)} \tag{283b}$$

The middle wire AN (Fig. 523,I) is similar to the neutral wire of a three-wire system in that it carries the currents of both of the phases. However, these currents differ in phase by 90 degrees and therefore can not be added directly. Obviously, their vector sum equals 1.41 times the current in the outside wires. Then, referring to the circuit diagrams of Figs. 523,I, and 526,

$$I_L = I_C \text{ (amp.)} \tag{284}$$

and

$$E_L = E_C \text{ (volts)} \tag{285}$$

$$I_N = 1.41 \times I_L = 1.41 \times I_C \text{ (amp.)} \tag{286}$$

843. The relations between power, current, voltage, and power factor for any balanced, two-phase circuit follow from the fact

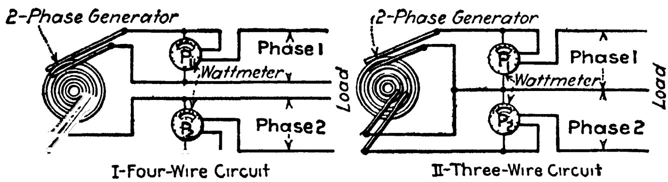


FIG. 527.—Wattmeters for measuring power in two-phase circuits.

that a two-phase circuit is merely two single-phase circuits. That is,

$$P = 2 \times I_L \times E_L \times p.f. \text{ (watts)} \tag{287}$$

and

$$I_L = \frac{0.50 \times P}{E_L \times p.f.} \text{ (amp.)} \tag{288}$$

hence

$$E_L = \frac{0.50 \times P}{I_L \times p.f.} \text{ (volts)} \tag{289}$$

therefore

$$p.f. = \frac{P}{2 \times I_L \times E_L} \text{ (power factor)} \tag{290}$$

Wherein P = power transmitted, in watts.
I_L = current, effective, in outside wires, in amperes.
E_L = the pressure between phases, in volts.
$p.f.$ = power factor.

Note that the total power is equal to the sum of the power in the two phases, so that in making power measurements where the load is unbalanced either a wattmeter must be used in each

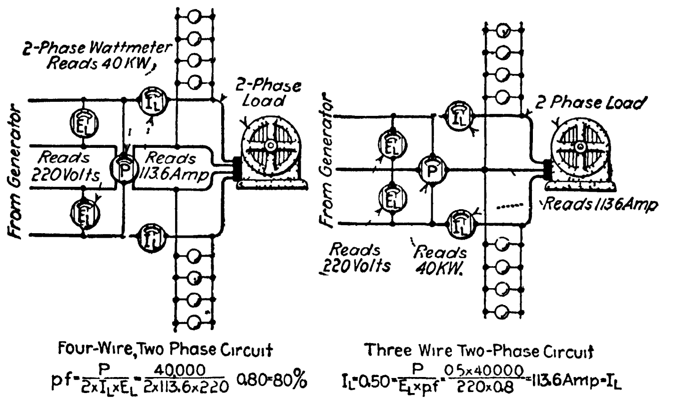


FIG. 528.—Illustrating e.m.f. and current relations in two-phase circuits.

phase, Fig. 527,I, or a two-phase wattmeter, which automatically adds the power of both phases, can be employed.

Example.—Figure 528 illustrates the application of the above equations from two problems.

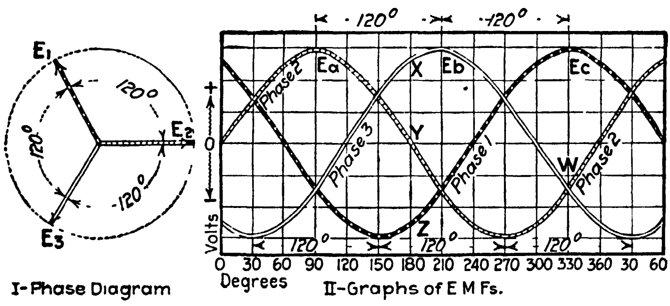


FIG. 529.—Phase diagram and curves for the e.m.fs. produced by a three-phase alternating-current generator.

844. Three-phase is a term ("A.I.E.E. Standardization Rules") characterizing the combination of three symmetrical circuits energized by alternating e.m.fs. which differ in phase by one-third of a cycle, i.e., 120 deg. That is, with a three-phase e.m.f. or current there are at any instant three phases.

The three-phase is the most widely applied of all polyphase systems because of its economy and adaptability (Art. 861).

845. To produce a three-phase e.m.f. (Fig. 529) it obviously follows from the definition given above that it is only necessary

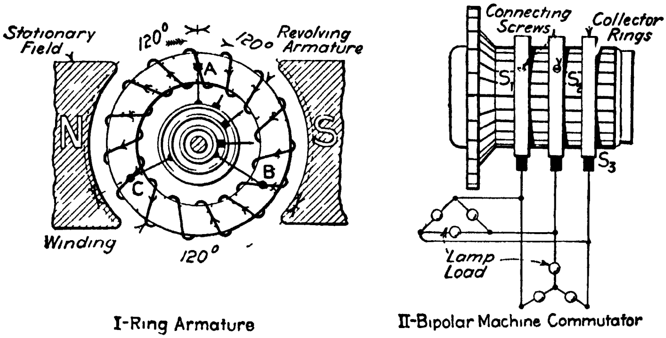


FIG. 530.—Showing methods of obtaining three-phase currents from a bipolar revolving-armature generator, delta connection.

to generate three sine-wave-form e.m.fs. which differ in phase by 120 deg. As with single-phase and two-phase systems, the generators for developing these three-phase e.m.fs. may be either of the revolving-armature (Figs. 530 and 531), or of the revolving-field (Figs. 532 and 533), types.

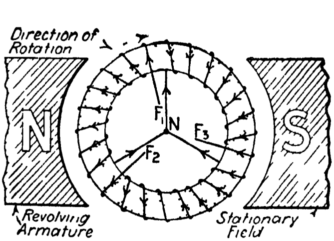


FIG. 531.—Bipolar-machine, ring-wound armature connected in Y

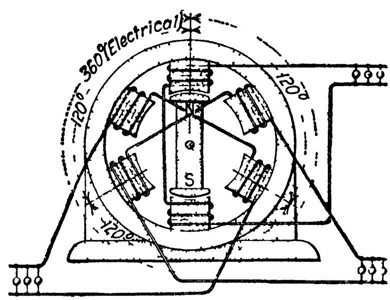


FIG. 532.—Elementary, two-pole, three-phase revolving-field alternator.

846. A Three-phase E.m.f. Really Comprises Three Separate E.m.fs.—Likewise, a three-phase current comprises the three separate currents impelled by these three e.m.fs. Hence, it is perfectly feasible to develop a three-phase e.m.f. with three single-phase generators, the shafts of which are rigidly coupled together so that the 120-deg. phase relation between the e.m.fs.

developed by each of the three machines would be preserved. However, it is much more economical of material to combine the three sets of armature coils for the development of these three e.m.fs. into one machine. Therefore, this is the procedure always followed in practice.

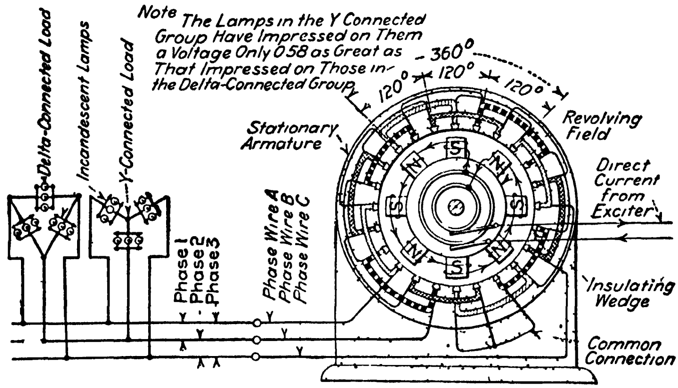


FIG 533.—Diagrammatic representation of an eight-pole, three-phase, Y- or star-connected, alternating-current, revolving-field generator.

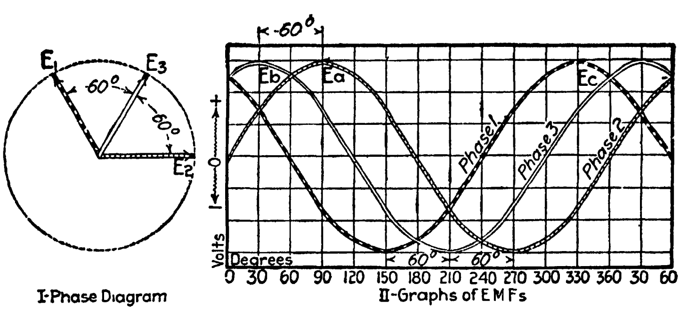


FIG. 534.—Phase diagram and curves produced by a three-phase alternating-current generator before the winding of phase 3 is reversed.

847. The principle of the three-phase generator is suggested in Fig. 532. If three different sets of armature coils be mounted in a frame 120 electrical deg. apart and a magnetic field arranged as shown (*NS*, Fig. 532) is caused to rotate within them, it is obvious from a consideration of the principles outlined in Art. 748 that three e.m.fs. will be induced, one in each set of coils, which will differ in phase by 120 deg., as shown in Fig. 529. Furthermore, if three sets of overlapping armature coils be

arranged with their sides 60 electrical degrees apart, as shown in Fig. 533, three e.m.fs. which differ in phase by 60 deg. (Fig. 534) will be produced by rotating a properly arranged field magnet within the structure. If the terminals of the coil group of phase 3 be reversed then 3 e.m.fs., differing in phase by 120 deg. (Fig. 529), will be impressed on the external circuit.

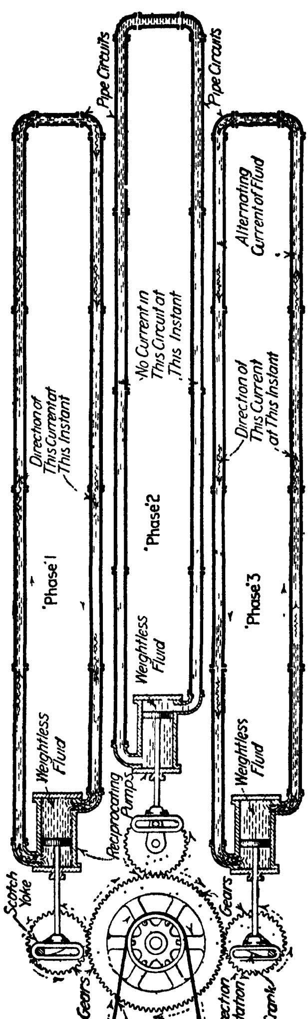


FIG. 535.—Hydraulic analogy to a three-phase, six-wire, alternating-current circuit and generator.

848. Revolving-armature, three-phase generators are now seldom used except for small machines, possibly those of capacities of 25 kva. or less. It follows from a consideration of the principles of electromagnetic induction (Arts. 742 and 743) that, if a direct-current-generator armature winding be tapped at points 120 electrical degrees apart, as shown in Fig. 530, and each of these tap wires be connected to a collector ring, a three-phase e.m.f. will be impressed on these rings. This can be accomplished with a bipolar machine as shown in Fig. 530-II, by using three collector rings tapped to three equidistantly spaced commutator bars after the manner hereinbefore described. Figure 530 shows the delta method of connection which will be described later. The *Y*-method of connection, also described later, is shown in Fig. 531.

849. A revolving-field, three-phase generator is illustrated diagrammatically in Fig. 533. The armature winding is stationary, and the field structure which is excited from a direct-current source—usually a small direct-current generator exciter—is rotated within the armature. Note

that the distance between adjacent similar poles, that is from N to N or S to S, constitutes 360 electrical degrees. Besides, the armature coils are 60 deg. apart, and the distance between similar sides of the different coils is 120 deg.

NOTE.—A hydraulic analogy of a three-phase generator and circuit is shown in Fig. 535. Each of the three reciprocating pumps is analogous to a single-phase generator and the combination of the three and their gear drive is analogous to a three-phase generator. The three hydraulic circuits are analogous to a three-phase, six-wire circuit. The only difference between each of the three generators and its circuits is that the cranks driving the three pumps are so geared that there is always a phase difference of 120 deg. between them. The instant pictured in Fig. 535 corresponds to the 180-deg. phase of Fig. 529. That is, at this instant the piston of phase 2 is stationary and there is no current of water in its pipe circuit. This corresponds to point Y in Fig. 529. The fluid current in phase 1 is decreasing in the negative direction, Z (Fig. 529), and the fluid current in phase 3 is increasing, X (Fig. 529), in the positive direction. If the analogy is followed further it will be found that the curves of Fig. 529,II indicate just how the fluid currents in the three different pipe circuits would increase and decrease in intensity and change in direction individually and in relation to each other as the three pumps are driven by the large gear.

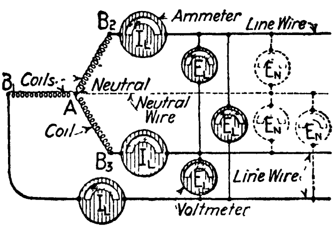


FIG. 536.—Ammeters and voltmeters in a three-phase, Y-connected circuit.

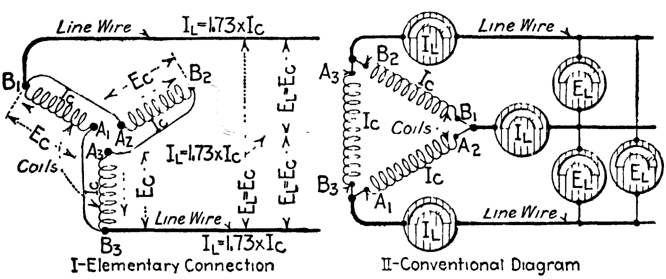


FIG. 537.—Illustrating the delta (Δ) connection of generator or receiver coils.

850. There are two methods of connecting the windings of three-phase generators, and, in general, there are two methods of connecting devices of any sort to a three-phase circuit. They are (1) the Y connection (Fig. 536) and (2) the delta connection[1] (Fig. 537,II). Most generators have their coils Y-con-

[1] Also sometimes called star and mesh connections, respectively.

nected, but it is ordinarily impossible to determine from an external inspection of a machine whether it is Y- or delta-connected. Machines connected in either way can be made to provide the same performance, but for a given voltage impressed on the line the armature coils for a Y-connected machine must be different from those for a delta-connected machine, as will be shown.

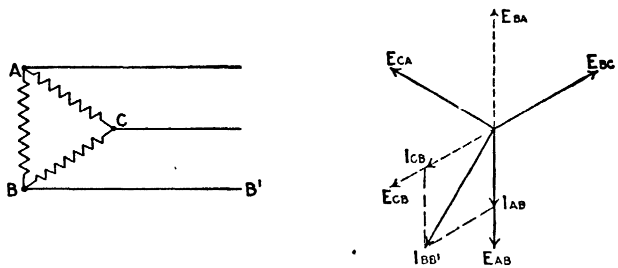


FIG. 538.—Delta connection and corresponding phase diagram.

NOTE.—Three-phase Vectors Convention.[1]—A feature which sometimes gives some difficulty is the selection of positive directions. Any selection will do as long as it is consistently adhered to. Thus, it is convenient to consider the counterclockwise direction around a delta and the direction out from the neutral point of a Y as positive. Confusion can be avoided by following a simple convention. Draw a diagram of the circuit and letter all intersections. It is helpful to arrange these diagrams to correspond to

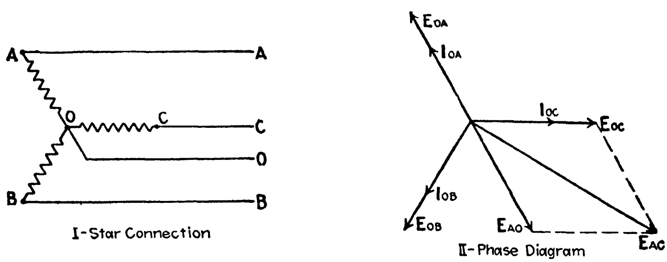


FIG. 539.—Star or Y connection and phase diagram.

the shape of the vector figure. When a vector is drawn and lettered, use two subscripts denoting the points between which the current flows or voltage exists. Thus, in Fig. 538, E_{AB} represents the difference of potential between A and B, A being the higher. E_{BA} represents the difference of potential between B and A which is equal and opposite to E_{AB}. In other words, $E_{AB} = -E_{BA}$. The same is true of currents. If the current in a line is desired, say BB', add I_{AB} and I_{CB} since both these currents feed into B and

[1] R. H. Willard, in *Electric Journal*.

$I_{BB'}$ feeds out. If the currents are in phase with the voltages, the vector $I_{BB'}$, the sum of I_{AB} and I_{CB} (Fig. 538), will represent the current in BB'.

Figure 539 shows a Y-connected generator. If voltage E_{AC} is desired, add E_{AO} and E_{OC}. If the current in the neutral of a balanced star is desired it will equal $I_{OB} + I_{OC} + I_{OA}$. Using vector addition, this gives zero as the current in the neutral, if the currents in the phases are equal and at the same power factor.

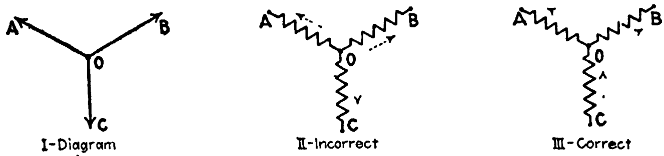


FIG. 540.—Vector diagram of currents and correct and incorrect methods of showing the instantaneous currents in a Y connection.

Care must be taken not to confuse vectors with instantaneous currents or voltages in the windings. In Fig. 540,I represents by vectors the currents in a Y, but at no instant is the current flowing as in II, for the current flowing in from C to O at a given instant flows out from O to B and O to A as shown at III. That is, for *instantaneous* currents, Kirchhoff's law holds: the algebraic sum of all currents flowing to a point is zero.

851. The Y connection of phase windings or coils is indicated diagrammatically in Fig. 536. A three-phase generator or

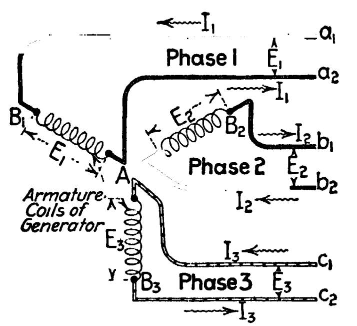


FIG. 541.—Armature coils of a three-phase generator. Coils not interconnected.

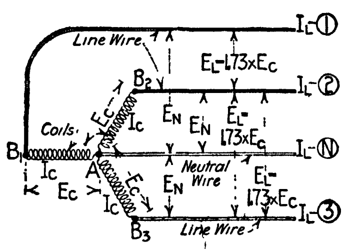


FIG. 542.—Coils of a three-phase generator connected in Y or in star.

device may be represented by a diagram like that of Fig. 541, in which each of the three sets of coils AB_1, AB_2 and AB_3 represents one of the sets of armature windings in which an e.m.f. is induced. However, the number of line wires may be reduced to four as shown in Fig. 542, by using a common return wire AI_L for all three of the circuits, and by connecting it to a junction, A, of

the three sets of coils. However, if the three circuits are balanced, that is, equally loaded, as they usually are in practice—approximately at least—then no current would flow in the neutral wire, AI_L; hence it can be omitted as diagramed in Fig. 536.

852. Why the neutral wire of a Y-connected circuit may be omitted is suggested in Fig. 543, which indicates the conditions

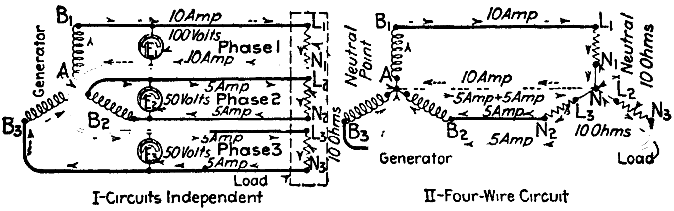


FIG. 543.—Instantaneous currents at a certain instant in three-phase Y-connected circuits.

existing in a three-phase circuit at a certain instant. It is assumed that the e.m.f. induced in each of the three sets of armature coils AB_1, AB_2, and AB_3 has a maximum instantaneous value of 100 volts. Each of the three load components, L_1–N_1, L_2–N_2, and L_3–N_3, has a resistance of 10 ohms. At the instant

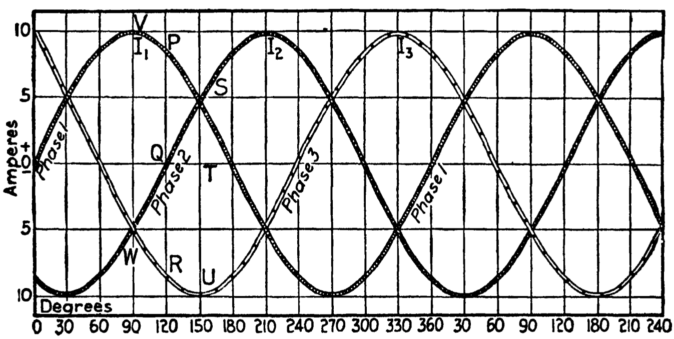


FIG. 544.—Curves of three currents in a three-phase circuit. Maximum e.m.f. = 100 volts, resistance = 10 ohms. See Fig. 543 for circuit diagrams.

shown in Fig. 543, which corresponds to the 330-deg. phase of Fig. 529, the e.m.f. impressed on phase 1 is a maximum (E_C) or 100 volts. The e.m.fs. impressed on phases 2 and 3 are then each one-half maximum (W) or 50 volts but are in opposite direction from that of phase 1. Since the resistance of the load in each of the phases is 100 ohms, the currents of the instant

pictured will be 10 amp., 5 amp., and 5 amp., respectively, as shown. Therefore, Fig. 544 indicates a curve of the three-phase current in this circuit, the conditions of Fig. 543,I corresponding to the 90-deg. phase (VW) of Fig. 544. Now, if a common return wire, AN, is provided as shown in Fig. 543,II, there would be a tendency for a current of 10 amp. to flow in it toward the left, and also for a current of 5 amp. + 5 amp., = 10 amp., toward the right. The consequence is that these tendencies neutralize and that there would be no current in the neutral wire, therefore it is unnecessary. Now, although Fig. 543 pictures conditions for only one instant, it is evident from a study of the curve of Fig. 544 that in a three-phase circuit the current in one of the phases is always equal and opposite to the sum of the currents in the two other phases, which indicates why there is no occasion for a return wire. For example (Fig. 544), at the 120-deg. phase, the current represented by the ordinate PQ (8.5 amp.), is equal and opposite to the current represented by the ordinate QR. Also, at the 150-deg. phase, $ST + ST$ is equal but opposite to TU.

853. The voltage and current relations in a Y- or star-connected three-phase circuit are indicated in Fig. 542. It is evident that the current in each one of the three coils AB_1, AB_2, and AB_3 must be the same as the current in the line wire attached to it. The voltage impressed on any pair of line wires, for instance, on line wires 1 and 2, must be equal to the sum of the voltages induced in the two coils which are connected in series between these two wires. For example, the voltage impressed across B_1–B_2 (Fig. 542) is equal to the sum of the e.m.f. induced in AB_1 plus that induced in AB_2. However, since these voltages differ in phase by 120 deg. they can not be added arithmetically. They can, however, be added vectorially and then, as is shown in the following paragraph, their resultant = 1.73 times the voltage generated in either coil. That is, for a Y-connected circuit, the following equations must hold:

$$I_C = I_L \text{ (amp.)} \tag{291}$$

$$E_L = 1.73 \times E_C \text{ (volts)} \tag{292}$$

Therefore

$$E_C = \frac{E_L}{1.73} = 0.58 \times E_L \text{ (volts)} \tag{293}$$

It is evident that the voltage from either one of the three line wires to the neutral wire, if one is provided, must equal the voltage induced in each coil, that is,

$$E_N = E_C \text{ (volts)} \qquad (294)$$

Wherein, assuming balanced load and effective values of current and e.m.f.,

I_L = the current in any one of the three line wires.

I_C = the current in any one of the three-phase windings or coils which are connected in Y.

E_L = the e.m.f. impressed between any pair of the three line wires.

E_C = the e.m.f. induced in or impressed on any one of the three-phase windings or coils.

E_N = the voltage between any one of the three line wires and the neutral point, or between any one of the line wires and the neutral wire, if such is used. Figure 536 indicates the reference letters above used, on ammeters and voltmeters properly arranged in a three-phase Y-connected circuit.

854. Why the line e.m.f. in a Y-connected circuit equals 1.73 times the coil e.m.f. follows from a consideration of Fig. 545. AB and CD represent two different coils in each of which are induced equal voltages. However, the voltage E_1 in AB lags 120 deg. behind E_2 in CD. If the coils are connected as shown at I, then the vector sum of the voltages E_1 and E_2 will be, as is evident from the vector diagram of II, proportional to the length of the resultant vector OE_4. Now OE_4 (since it is the resultant of two equal vectors 120 deg. apart) is equal to OE_2 and to OE_1. Hence the sum of the e.m.fs. E_1 and E_2, with the coils connected as shown at I, is the same as their e.m.fs. taken singly.

If, however, one of the coils, DC, be reversed as at III, the e.m.f. in CD being equal to that in AB, but leading it by 120 deg., this is equivalent to reversing the e.m.f. (Reversing the coil which generates an e.m.f. changes its phase relation to the two other coils by 180 deg.) Then E_5 will be the vector difference of E_1 and E_2 rather than their vector sum. The vector diagram for the sum of E_1 and E_2 then becomes that shown in IV, where the vector OE_5 is proportional in length to the difference of OE_2 and OE_1. Now, the angle A is obviously one-half of 60 deg.

(30 deg) and ON equals, then, cos 30 deg $\times E_1$, and $OE_5 = 2 \times$ cos 30 deg $\times E_1$ Cos 30 deg is 0 866, therefore OE_5 or $E_5 =$ 1 73 $\times E_1 =$ 1 73 $\times E_2$ Therefore, since in a Y-connected three-phase generator any of the groups of two coils which are in

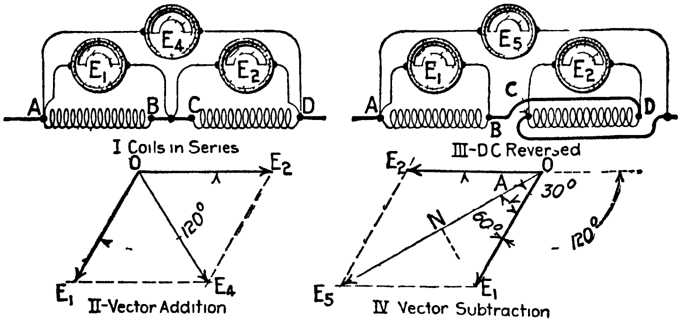


FIG 545 —Method of determining voltage across two coils in series the e m fs in which differ in phase by 120 deg

series between the line wires are reversed relative to one another as at III, the e m f impressed on the line wires of such a machine equals 1 73 $\times$ the e m f induced in each of the sets of coils The relations just suggested can be conveniently shown by an equilateral triangle (Fig 546), the sides of which are proportional in length to the voltage between the line wires.

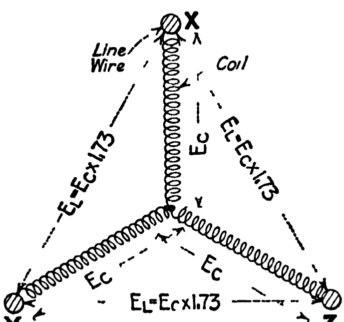


FIG 546 —Equilateral triangle indicating voltage relations in a three-phase, Y-connected circuit

Example —With a group of three sets of four incandescent lamps each, connected in Y across a 110-volt circuit, as in Fig 547,I, what e m f will be impressed upon each set? *Solution* — Substitute in equation (293) $E_C =$ 0 58 $\times E_L =$ 0 58 $\times$ 110 = 63 8 volts

855. The power in a Y-connected circuit may be determined as follows It is evident from the power relations in a single-phase alternating-current circuit that the power P_p, developed in any one-phase winding of a three-phase generator, is

$$P_p = E_C \times I_C \times pf \text{ (watts)} \qquad (295)$$

But from equation (293) $E_C = E_L \div$ 1 73 and from (291)

$I_C = I_L$. Substituting these values in (295), the power developed in any one coil is

$$P_p = \frac{E_L}{1.73} \times I_L \times p.f. \text{ (watts)} \qquad (296)$$

Since, with a balanced load, an equal amount of power is developed in each of the three-phase windings, the total power

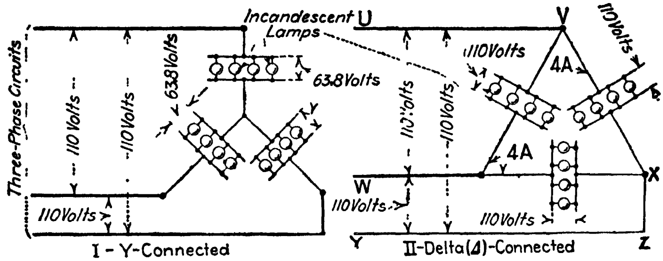


FIG. 547—Incandescent lamps connected in delta and in Y.

will be three times that developed in one winding, or

$$P = 3 \times \left(\frac{E_L \times I_L \times p.f.}{1.73}\right) = \frac{3}{1.73} \times E_L \times I_L \times p.f.$$
$$= 1.73 \times E_L \times I_L \times p.f. \text{ (watts)} \qquad (297)$$

856. The delta connection of phase windings or coils is indicated at Figure 537, where I shows the elementary diagram and II the simplified arrangement. It might be assumed that since the three windings form a closed circuit, current would circle around in it, but such is not the case because the resultant sum of the voltages, which differ in phase by 120 deg. in the three windings, is at every instant zero, as will be evident from a consideration of the curve of Fig. 529. That is, the voltage in one phase is at any instant always equal and opposite to the sum of the voltages in the two other phases. If, however, three line wires are connected to these phase windings, one to each of the junction points between windings as shown in Fig. 537, II, then alternating e.m.fs. will be impressed on the external circuit and a three-phase current will be forced through the circuit if it is closed.

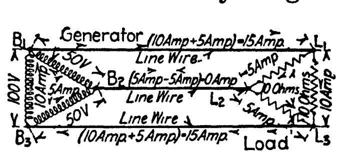


FIG. 548.—Instantaneous currents, at a certain instant, in a delta (Δ)-connected three-phase circuit.

Example.—Figure 548 shows the instantaneous currents in the circuit shown at the instant at which the e.m.f. in coil B_1B_3 is a maximum. Note that at this particular instant the current in B_1L_1 is 15 amp., but that it is opposite to the current of 15 amp. in B_3L_3. At this instant there is no current in B_2L_2. This example verifies the statement previously made that, in a three-phase circuit, the current in some one of the line wires is at any instant always equal and opposite to that in the two others.

857. Voltage and current relations in a delta-connected circuit are shown in Fig. 537. It is evident that the voltage impressed between each of the two line wires is equal to the voltage induced in each phase winding because the line wires are connected directly across the phase windings. It is also evident that the current in each one of the line wires is the sum of the currents in two of the phase windings. However, these currents, since they differ in phase by 120 deg., cannot be added directly but must be added vectorially, somewhat as suggested in Fig. 545,IV. Hence it follows that for a three-phase, delta-connected circuit

$$E_C = E_L \text{ (volts)} \tag{298}$$

$$I_L = I_C \times 1.73 \text{ (amp.)} \tag{299}$$

$$I_C = \frac{I_L}{1.73} = 0.58 \times I_L \text{ (amp.)} \tag{300}$$

Wherein, assuming balanced load and effective values of current and e.m.f., all of the symbols have the same meanings as used in the Y-connection demonstrations.

Example.—In the circuit of Fig. 547,II, each of the three groups of 110-volt incandescent lamps takes 4 amp. What is the current in the line wires UV, WX, and YZ? *Solution.*—Substitute in formula (299) $I_L = I_C \times 1.73 = 4 \times 1.73 = 6.92$ amp.

858. The power developed in a delta-connected circuit can be determined on the basis of the explanation hereinbefore given for the determination of power in a Y-connected circuit. Obviously in a delta-connected circuit the power developed by each phase winding is (Art. 824)

$$P_p = E_C \times I_C \times p.f. \text{ (watts)} \tag{301}$$

But, equation (298), $E_C = E_L$ and, equation (300), $I_C = I_L \div 1.73$; hence, substituting these values in (301),

$$P_p = E_L \times \frac{I_L}{1.73} \times p.f. \text{ (watts)} \tag{302}$$

Then the total power, P, for the three windings must be

$$P = 3 \times E_L \times \frac{I_L}{1.73} \times p.f. = 1.73 \times E_L \times I_L \times p.f. \text{ (watts)} \quad (303)$$

859. The three-phase, four-wire system (Fig. 549) is now being used to a considerable extent, principally for the distribution of electrical energy for light and power in cities. Such a

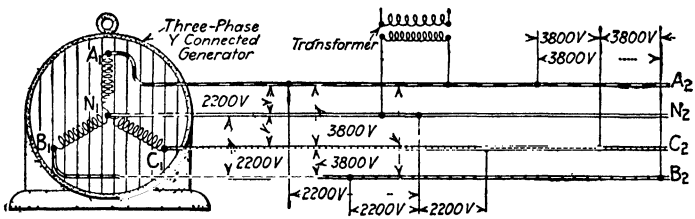


FIG. 549.—Illustrating diagrammatically the three-phase, four-wire system

system is, as suggested, served by a three-phase, Y-connected generator and is precisely like any other Y-connected system except that the neutral wire N_1–N_2 is carried out from the machine and along the line with the three other wires. The system permits the use of a transmission voltage 1.73 times the voltage impressed on the transformers, a material economy in copper for the same power loss—or a material decrease in energy lost for the same amount of copper—resulting.

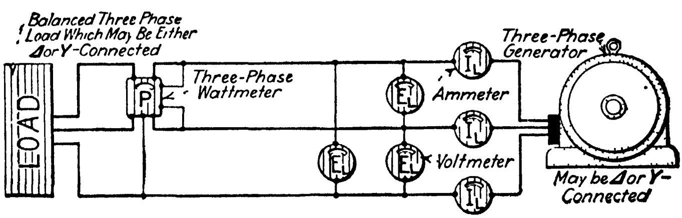


FIG. 550.—Typical three-phase circuit.

Example.—In the three-phase, four-wire system of Fig. 549, the transformers are connected between the outside wires and the neutral wire, the pressure on their primaries being 2,200 volts. However, the pressure between the phase wires, A, C, and B equals $1.73 \times 2{,}200$ volts, is 3,800 volts Since with a balanced load, and it is usually so—approximately at least—in practice, there is no current in the neutral wire; the energy is really, in the circuit shown in the illustration, transmitted at 3,800 volts.

860. Power and voltage relations for any three-phase circuit which may be either Y- or delta-connected are given in the

equations below, which refer to the symbols of Fig. 550. Note from the two articles preceding, which indicate how the power may be determined in delta- and in Y-connected circuits, that the final equation is the same for both of these circuits; hence it follows, for any three-phase circuit with a noninductive load, that

$$P = 1.73 \times E_L \times I_L \text{ (watts)} \tag{304}$$

For an inductive load, then, the following must be true:

$$P = 1.73 \times E_L \times I_L \times p.f. \text{ (watts)} \tag{305}$$

or

$$E_L = \frac{P}{1.73 \times I_L \times p.f.} = \frac{0.58 \times P}{p.f. \times I_L} \text{ (volts)} \tag{306}$$

$$I_L = \frac{P}{1.73 \times E_L \times p.f.} = \frac{0.58 \times P}{p.f. \times E_L} \text{ (amp.)} \tag{307}$$

$$p.f. = \frac{P}{I_L \times E_L \times 1.73} = \frac{0.58 \times P}{I_L \times E_L} \text{ (power factor)} \tag{308}$$

Wherein, assuming balanced load and effective values of current and e.m.f.,

P = the power, in watts, being transmitted in the circuit.
E_L = the e.m.f., in volts, between line or phase wires.
I_L = the current, in amperes, in each line or phase wire.
$p.f.$ = the power factor of the circuit.

Example.—A 220-volt, three-phase circuit feeds an induction motor, as shown in Fig. 551. The current in each line wire is 120 amp., and the power

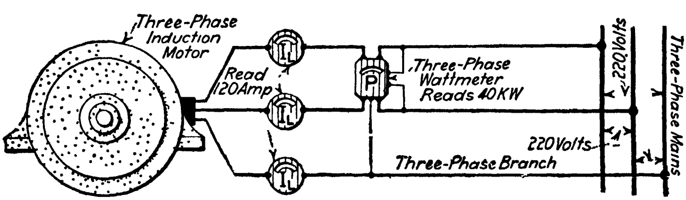


FIG. 551.—Three-phase load feeding from a three-phase main.

taken by the motor is 40 kw., as indicated by a wattmeter. What is the power factor of the circuit? *Solution.*—$p.f. = 0.58 \times P \div I_L \times E_L = 0.58 \times 40{,}000 \div 120 \times 220 = 0.878$. That is, the power factor of the circuit is, say, 88 per cent.

Example.—Figure 552 shows a three-phase branch circuit carrying energy from a distributing center to a load. The current in each of the line wires is 65 amp. The line e.m.f. is 220 volts, and the power factor of the load is

known to be 70 per cent. What power in kilowatts is being transmitted? What horsepower? *Solution.*—$P = 1.73 \times E_L \times I_L \times p.f. = 1.73 \times 220 \times 65 \times 0.70 = 17{,}300$ watts = 17 3 kw. 17.3 kw. ÷ 0.746 = 23.2 hp.

Example.—Figure 553 shows the different values of current and voltage in a certain group of receivers connected in Y and delta on a three-phase circuit.

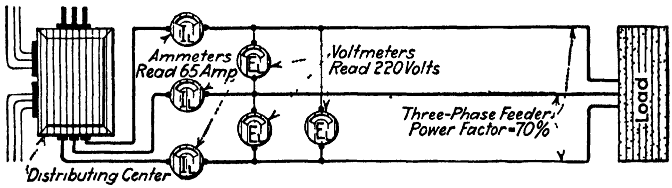


FIG. 552 —Three-phase load feeding from a distribution center.

A study of this illustration will be of great assistance in fixing in mind the voltage and current relations of three-phase circuits. The generator shown may be either Y- or delta-connected without affecting the external results.

861. The three-phase system is very widely applied and is now usually utilized in preference to any other system for the

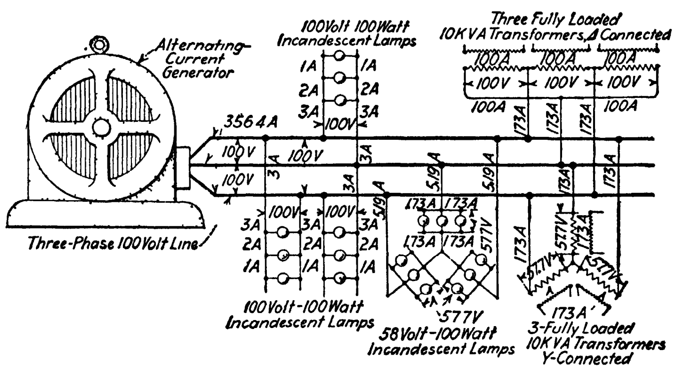


FIG. 553.—A 100-volt, three-phase line showing receivers (lamps and transformers) connected in delta and in Y.

transmission of electrical energy over a considerable distance. It is simpler than the two-phase system, is more economical of copper, and its operating performance in practice is quite as satisfactory.

SECTION 52

TRANSFORMERS, THEIR PRINCIPLES AND APPLICATIONS

862. The stationary transformer is a device (operating by virtue of the principle of mutual induction, Art. 491) whereby the energy of an alternating-current circuit may be received at one voltage and delivered at a higher or lower voltage. It is really an alternating-current induction coil. A transformer is one of the most important electrical devices—if not the most important. It is remarkably simple in elementary principle and in construction, in that it involves no moving parts. However, some of the reactions that occur within a transformer are extremely complicated and tedious of explanation. Fortunately, these complicated phenomena are of little importance to the practical man.

NOTE.—The essential features of the theory of transformer operation, an understanding of which will enable one to solve the most frequently encountered transformer problems, are relatively simple and readily understood. These will be explained in the following articles. The transformer will operate only with alternating current and will not work with direct current. There is no electrical connection between the primary and secondary windings. They are electrically independent (except in the balance coil) but are magnetically interconnected by the alternating flux.

863. Transformers Are Essential in the Transmission of Electrical Energy.—If electrical energy is to be transmitted with economy over any considerable distance, the transmission voltage must be high, so that the line losses will be a minimum. However, it is not feasible or desirable to utilize high-voltage electrical energy for electric lamps, motors, and other receiving appliances. Furthermore, it is not feasible to generate electrical energy at the high voltages that must be used for transmitting large amounts of energy over great distances. Therefore, where the energy is generated at a low voltage, a step-up transformer, *A* (Fig. 554), is used for raising the generator voltage to one suitable for energy transmission and then a step-down trans-

former, B, is applied for again decreasing the voltage to one suitable for utilization.

864. The three principal parts of a transformer are (Fig. 555): (1) a core, which provides a circuit of low reluctance for the magnetic flux; (2) the primary winding, that which receives the energy from the supply circuit; and (3) the secondary winding,

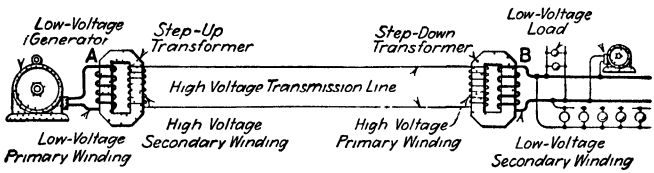


FIG. 554.—Illustrating the application of voltage transformers to electrical energy transmission.

that which receives energy by induction from the primary and delivers it to the secondary circuit.

NOTE.—Where possible or feasible, misunderstandings are eliminated if the windings are referred to, respectively, as the high-tension or high-voltage winding and the low-tension or low-voltage winding. Either the high or the low may (Fig. 554) constitute the primary or the secondary winding of a transformer, depending upon its arrangement in the circuit.

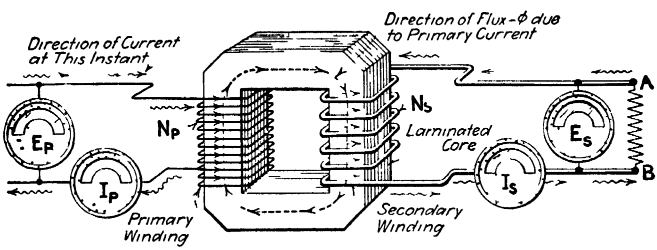


FIG. 555.—Showing the principle of the voltage transformer.

865. The cores of transformers are, in practice, to insure maximum economy, laminated to minimize eddy-current losses and are usually of specially prepared silicon steels because of the low hysteresis losses which occur in these special steels. Furthermore, these special steels can be used for long—probably indefinite—periods in transformer cores without increase of the losses in them, whereas if ordinary irons or steels are used, the losses in the cores increase as the term of service of the transformer lengthens, that is, such steels are subject to "aging."

866. Current transformers and voltage transformers (Fig. 556) constitute two classes into which these devices may in general be grouped. With voltage transformers, the primary windings are connected (Fig. 556,I) in multiple across the supply mains. With current transformers (Fig. 556,II) the primary windings are connected in series in the primary circuit. The characteristics of these two different types will be discussed in the articles that follow. Inasmuch as the voltage transformer is the more frequently used, it will be considered more in detail.

867. How a voltage, or potential, transformer transforms voltages can be understood from a consideration of the principles of mutual induction. If an alternating voltage, E_p (Fig. 555) is impressed on the primary winding of a transformer—assuming

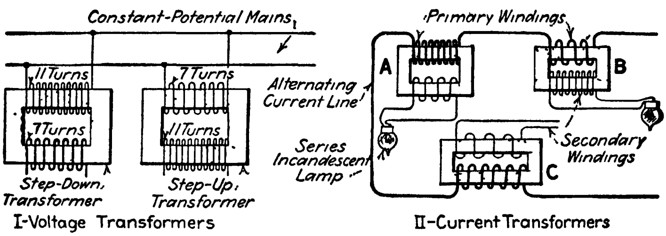


FIG. 556.—Illustrating the difference between voltage or potential transformers and current transformers.

that the secondary circuit is open—an alternating current, I_p, will be impelled in this winding and it will produce in the core of the transformer an alternating flux, ϕ. This flux (which in building up expands out of, and in dying down contracts back into, the primary turns, as the primary alternating current increases and decreases in intensity and changes in direction) will cut the turns of the secondary winding. Thereby an e.m.f., E_s, will be induced in this secondary winding. If the secondary circuit be closed as at AB, Fig. 555, then the secondary e.m.f., E_s, induced as above, will impel a current, I_s, in the secondary circuit. The intensity of this current will be directly proportional to the secondary e.m.f. induced as described, and inversely proportional to the impedance of the secondary circuit, that is: $I_s = E_s \div Z_s$, wherein Z_s is the impedance of the secondary circuit.

868. How the voltage transformer operates will now be explained by referring to Fig. 557. With all the switches,

K_1 to K_5 (Fig. 557) open, that is, with the secondary circuit open, the primary current I_p will be so small as to be relatively negligible. Hence, it may be stated that, when the secondary circuit of a transformer is open, there is practically no current in its primary circuit. Now if K_1 be closed connecting one lamp across the circuit, the secondary e.m.f. E_s induced as above described will impel a current I_s in the secondary circuit, and furthermore, the ammeter, A_p, will indicate current in the primary. But the primary current, I_p, will always, in a step-down transformer, be as many times smaller than the secondary current, I_s, as the number of primary turns is greater than the number of secondary turns, as explained below. In a step-up transformer, the primary current will be correspondingly greater than the secondary current.

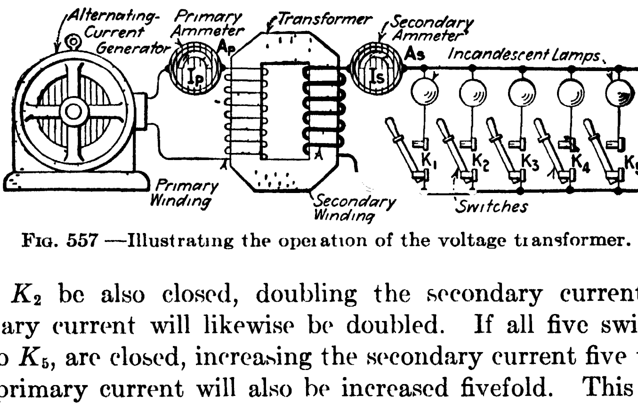


FIG. 557 —Illustrating the operation of the voltage transformer.

If K_2 be also closed, doubling the secondary current, the primary current will likewise be doubled. If all five switches, K_1 to K_5, are closed, increasing the secondary current five times, the primary current will also be increased fivefold. This automatic action of the primary may be likened to a reducing valve, in that it permits only just enough current to flow to supply the requirements of the secondary. Why this occurs is explained below.

869. As to the theory of transformer operation, it might appear that the primary winding, because it has relatively low resistance, would short-circuit the primary circuit, and so it would, if a direct e.m.f. were applied.

Explanation.—Although the resistance of the primary winding is low, its inductance, since it is a coil wound on an iron core, is high. Hence, when an alternating e.m.f. is applied across the primary winding it induces a counter e.m.f., practically equal in intensity to the applied e.m.f., E_p. This prevents current—except a very small magnetizing current—from flow-

ing in the primary. This magnetizing current is sufficient, however, to develop an alternating flux, ϕ, in the core, so that an e.m.f. is produced in the secondary by ϕ cutting the secondary turns and so that the counter e.m.f. of self-induction is maintained in the primary. The primary winding is so designed that it has a sufficient number of turns to make its counter e.m.f. of self-induction practically equal the impressed e.m.f., E_p. This is the reason why the current which flows in the primary winding of a transformer, the secondary of which is open, is of very small intensity. Furthermore, since the primary winding is largely inductive, this magnetizing current involves almost no energy.

However, when the secondary circuit is closed by connecting a load across it, as at AB, Fig. 555, the secondary e.m.f., E_s, induced as above suggested, in the secondary turns, N_s, impels the secondary current, I_s, in the secondary circuit. As this current, I_s, flows, it also tends to produce an alternating flux in the transformer core. But the flux due to I_s would be (Lenz's law, Art. 470) in a direction opposite to that of the flux, ϕ, produced by the primary winding. The result of this demagnetizing effect is that the total flux, ϕ, in the magnetic circuit is slightly decreased with every increase in secondary load, which decreases correspondingly the counter e.m.f. induced in the primary turns. Thereby a suitably greater primary current is permitted to flow. Thus, the primary current always is automatically maintained at such a value that the secondary circuit will be properly supplied.

870. The actual change in flux and in primary counter e.m.f. in a transformer is, between full load and no load, very small, inasmuch as the change seldom exceeds 1 per cent, so that for practical purposes it may be assumed that the flux in the transformer core is constant at all loads and that the primary counter e.m.f. is also constant. Furthermore, it may be assumed that, if the primary or impressed voltage remains constant, the secondary voltage of a well-designed transformer also remains practically constant at all loads. Actually there is a slight drop in the secondary voltage from no load to full load, possibly from 1 to 3 per cent, depending on the design and characteristics of the transformer under consideration.

871. The Ratio of the Number of Primary Turns to the Number of Secondary Turns Determines the Ratio of the Primary to the Secondary Voltage or E.m.f.—It follows from equation (333) given in Art. 875 for determining the counter e.m.f. of a coil, that for any given coil and with a given frequency, the counter e.m.f. induced in the coil is proportional to the product of flux $\times$ turns, that is, to $\phi \times N$. The counter e.m.f. of the primary of a transformer equals (practically) the e.m.f. impressed on it. Hence, the primary e.m.f. of a transformer, E_p, is proportional

to $\phi \times N_p$. Also, it follows that the e.m.f. induced in the secondary turns must be proportional to $\phi \times N_s$. Now, since the same flux, ϕ, which produces the counter e.m.f. in the primary also induces the e.m.f. in the secondary turns, it is obvious that the following proportion must hold:

$$\frac{\text{Primary voltage}}{\text{Secondary voltage}} = \frac{\text{primary turns}}{\text{secondary turns}} \tag{309}$$

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} \tag{310}$$

$$E_p = \frac{N_p \times E_s}{N_s} \text{ (volts)} \tag{311}$$

$$E_s = \frac{E_p \times N_s}{N_p} \text{ (volts)} \tag{312}$$

$$N_p = \frac{E_p \times N_s}{E_s} \text{ (turns)} \tag{313}$$

$$N_s = \frac{E_s \times N_p}{E_p} \text{ (turns)} \tag{314}$$

Wherein, E_p = e.m.f. impressed on primary, in volts, effective.
E_s = e.m.f. delivered by secondary, in volts, effective.
N_p = number of turns in the primary winding.
N_s = number of turns in the secondary winding.

Example.—The transformer of Fig. 558,II has 9 turns in its primary and 4 turns in its secondary winding. If an e.m.f. of 200 volts is impressed on the primary, what would be the secondary voltage? *Solution.*—Substituting in equation (312); $E_s = (E_p \times N_s) \div N_p = (200 \times 4) \div 9 =$ 89 volts, which will, then, be the secondary e.m.f.

Example.—A certain transformer is designed for a high-tension e.m.f. of 3,120 volts and has 1,400 turns on its high-tension winding. A low-tension voltage of 230 is required. How many turns must be in the secondary winding? *Solution.*—Substitute in equation (314) $N_s = (E_s \times N_p) \div E_p = (230 \times 1{,}400) \div 3{,}120 = 103$ turns in the low-tension winding.

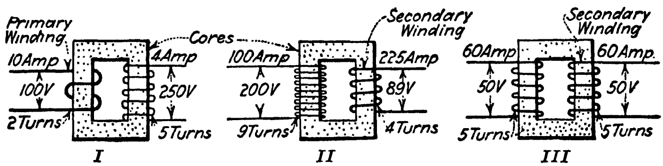
872. The ratio of a transformer or the "turn ratio" ("A.I.E.E. Standardization Rules") is the ratio of the number of turns in the high-voltage winding to the number of turns in the low-voltage winding. The voltage ratio is the ratio of the primary terminal effective voltage to the secondary terminal effective voltage. The current ratio is the ratio of the primary current (effective) to the secondary current (effective). That is,

$$\text{Turn ratio} = \frac{N_{HV}}{N_{LV}} \tag{315}$$

$$\text{Voltage ratio} = \frac{E_P}{E_S} \tag{316}$$

$$\text{Current ratio} = \frac{I_P}{I_S} \tag{317}$$

Examples illustrating the above ratio values are given in Fig. 558.



(a) Turn ratio $= \frac{N_{HV}}{N_{LV}} = \frac{5}{2} = 2\frac{1}{2}:1$

(b) Voltage ratio $= \frac{E_P}{E_S} = \frac{100}{250} = 1:2\frac{1}{2}$

(c) Current ratio $= \frac{I_P}{I_S} = \frac{10}{4} = 2\frac{1}{2}:1$

(d) Turn ratio $= \frac{N_{HV}}{N_{LV}} = \frac{9}{4} = 2\frac{1}{4}:1$

(e) Voltage ratio $= \frac{E_P}{E_S} = \frac{200}{89} = 2\frac{1}{4}:1$

(f) Current ratio $= \frac{I_P}{I_S} = \frac{100}{225} = 1:2\frac{1}{4}$

(g) Turn ratio $= \frac{N_{HV}}{N_{LS}} = \frac{5}{5} = 1:1$

(h) Voltage ratio $= \frac{E_P}{E_S} = \frac{50}{50} = 1:1$

(i) Current ratio $= \frac{I_P}{I_S} = \frac{60}{60} = 1:1$

FIG. 558.—Illustrating examples of ratios of transformation.

873. The power, current, voltage, and power-factor relations in transformer primary and secondary windings may be readily derived from the fact that the power input to any device must be always equal to its power output, plus the power losses in the device. Now properly designed transformers have very small losses—they are very efficient. Hence, for practical purposes, it can be assumed that the power input to the primary of a transformer equals the power output of its secondary. That is,

$$P_P = P_S \text{ (watts)} \tag{318}$$

Now, since the power of any alternating-current circuit is $P = E \times I \times p.f.$, it follows that $P_P = E_P \times I_P \times p.f._P$ Also, $P_S = E_S \times I_S \times p.f._S$ Hence, substituting these values in the above (318) equation,

$$E_P \times I_P \times p.f._P = E_S \times I_S \times p.f._S \tag{319}$$

or,

$$P_P = E_S \times I_S \times p.f._S \tag{320}$$

and

$$P_S = E_P \times I_P \times p.f._P \tag{321}$$

It can be shown (see note following) that $p.f._P = p.f._S$. Hence

$$E_P \times I_P = E_S \times I_S \tag{322}$$

That is, primary current $\times$ primary voltage = secondary current $\times$ secondary voltage. The working formulas become

$$E_P = \frac{E_S \times I_S}{I_P} \text{ (volts)} \tag{323}$$

$$I_P = \frac{E_S \times I_S}{E_P} \text{ (amp.)} \tag{324}$$

$$E_S = \frac{E_P \times I_P}{I_S} \text{ (volts)} \tag{325}$$

$$I_S = \frac{E_P \times I_P}{E_S} \text{ (amp.)} \tag{326}$$

Wherein the symbols have the same meanings above given except that I_P = current in primary, in amperes, effective
I_S = current in secondary, in amperes, effective.
P_P = power input to primary, in watts.
P_S = power output of secondary, in watts.

Example.—The input to a transformer is 8 kw. The secondary e.m.f. is 220 volts, and the power factor is 80 per cent. What is the secondary current? *Solution.*—From equation (320) above, $P_P = E_S \times I_S \times p.f._S$, hence, $I_S = P_P \div (E_S \times p.f._S)$. Then, substituting in this equation, $I_S = 8{,}000 \div (220 \times 0.80) = 45.5$ amp. Hence, the current in the secondary in this transformer is 45.5 amp.

Example.—The primary voltage of a certain transformer is 2,200. The primary current is 5 amp., and the secondary current is 100 amp. What is the secondary voltage? *Solution.*—From equation (325) above, $E_S = E_P \times I_P \div I_S = 2{,}200 \times 5 \div 100 = 110$ volts.

NOTE.—The proof that the power factor of the primary circuit equals the power factor of the secondary circuit is this: The magnetizing effect of the primary ampere turns may (if the primary magnetizing current, which is relatively very small, be disregarded) be taken as equal to that of secondary ampere turns. That is, $I_P \times N_P = I_S \times N_S$ or

$$\frac{N_P}{N_S} = \frac{I_S}{I_P} \tag{327}$$

But from equation (310), $N_P \div N_S = E_P \div E_S$, that is

$$\frac{N_P}{N_S} = \frac{E_P}{E_S} \tag{328}$$

hence

$$\frac{I_S}{I_P} = \frac{E_P}{E_S} \tag{329}$$

or

$$I_S \times E_S = I_P \times E_P \tag{330}$$

Then, dividing equation (319) above by (330), thus:

$$\frac{E_p \times I_p \times p.f._p}{I_P \times E_P} = \frac{E_s \times I_s \times p.f._s}{I_S \times E_S} \tag{331}$$

the result is $$p.f._P = p.f._S \text{ (power factor)} \tag{332}$$

This means that the power factor of the primary circuit of a transformer equals the power factor of the secondary circuit—if the magnetization current and losses in the transformer be neglected. These can be neglected in practice with little error.

874. The Currents in the Primary and Secondary Circuits of a Transformer Vary Inversely as the Voltages of the Windings to Which They Are Connected and Inversely as the Number of Turns in the Winding.—This statement logically follows from the previous discussion; in fact it is merely a statement in words of the meanings of equations (322) and (327).

875. The counter e.m.f. of a transformer may be computed (see formula (231), Art. 779) from the equation

$$E_{PC} = \frac{4.44 \times f \times \phi \times N}{100{,}000{,}000} = \frac{4.44 \times f \times A \times B_M \times N}{100{,}000{,}000} \tag{333}$$

Wherein E_{PC} = counter e.m.f. of transformer winding in volts.
f = the frequency of the circuit in cycles per sec.
ϕ = the total flux developed in the core.
N = the number of turns in the primary winding.
A = the area of the core in sq. in.
B_M = the maximum flux density in lines per square inch, that is, the flux density at the instant when the exciting current is a maximum.

By using one or the other of the above formulas, and assuming that the counter e.m.f. E_{PC} = the impressed e.m.f. E_P, which it does for all practical purposes, the principal dimensions of a transformer may be determined. The above formulas are very important inasmuch as they are used repeatedly in connection, not alone with transformers, but also with induction and synchronous motors and other alternating-current apparatus.

Example.—How many turns should be used in the primary winding of a transformer upon which an e.m.f. of 140 volts is to be impressed at a frequency of 60 cycles? The core upon which the winding is to be served has a sectional area of 1⅝ sq. in., and a maximum permissible flux density of 35,000 lines is assumed. *Solution.*—Substitute in equation (333) which has been solved for N

$$N = \frac{100{,}000{,}000 \times E_{PC}}{4.44 \times f \times A \times B_M} = \frac{100{,}000{,}000 \times 140}{4.44 \times 60 \times 1.625 \times 35{,}000} = 924$$

Hence the primary winding of this transformer should have 924 turns.

NOTE.—In designing a transformer the flux density is taken at a value which experience shows will involve iron losses that are as small as is consistent. Usually B_M is taken as 30,000 to 40,000 lines per sq. in. of core area. The number of primary turns is selected upon the basis of what experience has shown desirable and may be computed from an empirical formula.[1] Then by substituting these assumed values from B_M and N in the formula, the area, A, of the core required can be computed. To design transformers most effectively, that is, so that they will have a maximum efficiency with a minimum expenditure of material, is a specialized art and requires much experience and skill for its successful accomplishment.

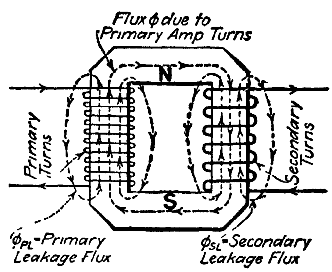


FIG. 559.—Illustrating magnetic leakage in a voltage transformer.

876. Magnetic leakage actually occurs in a transformer, although it has been disregarded in the foregoing discussion because in a well-designed transformer its effects are negligible from a practical standpoint. When there is no load on the secondary of a transformer (Fig. 559) no current flows in the secondary winding, and there is then nothing to oppose the creation of a flux in the core. Hence, at no load, there is very little leakage. When, however, a load is connected across the secondary circuit and a current flows in the secondary winding, a counter flux ϕ_{SL} (Fig. 559) is created by its secondary current. This counter flux opposes the flux produced by the primary (Lenz's law) as suggested by the arrows in Fig. 559. There is then a tendency toward the creation of magnetic poles (N and S, as shown at one particular instant) at the top and bottom of the core. Under these conditions, a proportion of the flux produced by the primary winding is prevented from following a circuit

[1] See Pender, "American Handbook for Electrical Engineers."

around through the core. The result is that this primary leakage flux ϕ_{PL} does not cut the secondary turns and the secondary is reduced accordingly. Obviously the primary leakage flux will increase with the load on the secondary, which tends to cause a decrease in the secondary voltage as the load increases.

877. Magnetic leakage may be largely minimized by so disposing the primary and secondary windings that all or most

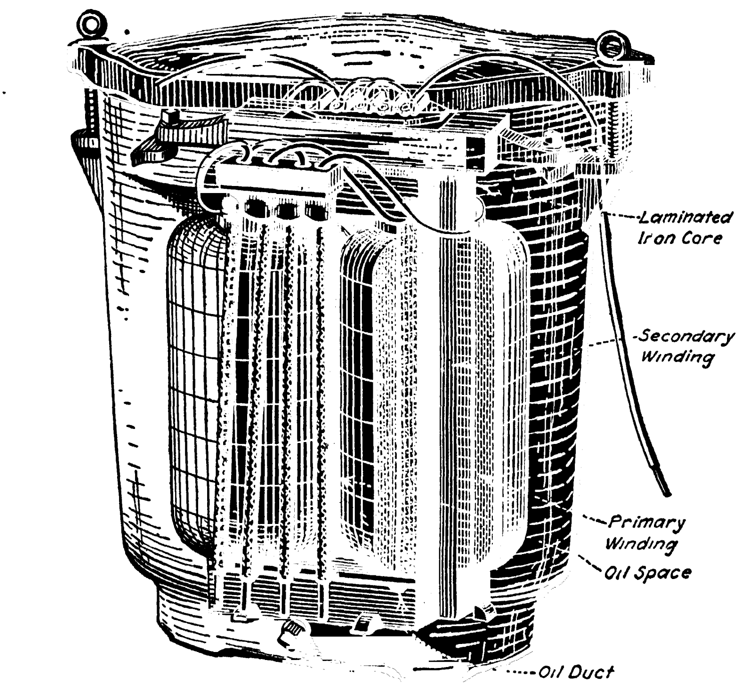


FIG. 560.—Phantom view of a typical distribution transformer assembled.

of the flux which is produced by the primary must cut the secondary turns. In practice this is accomplished by winding the primary and secondary coils over one another as suggested in Figs. 560 and 561, or by winding them in sections and interleaving.

878. Core-type and shell-type transformers are indicated diagrammatically in Fig. 562. Those of the core type are characterized by a long average length of magnetic circuit and a short average length of winding. Those of the shell type have a short average length of magnetic circuit and a long average

length of winding. The result of this condition is that for a given output and performance a core-type transformer will have

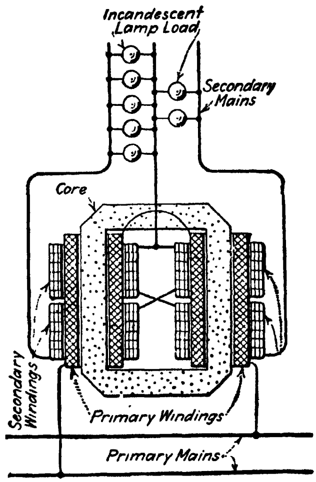
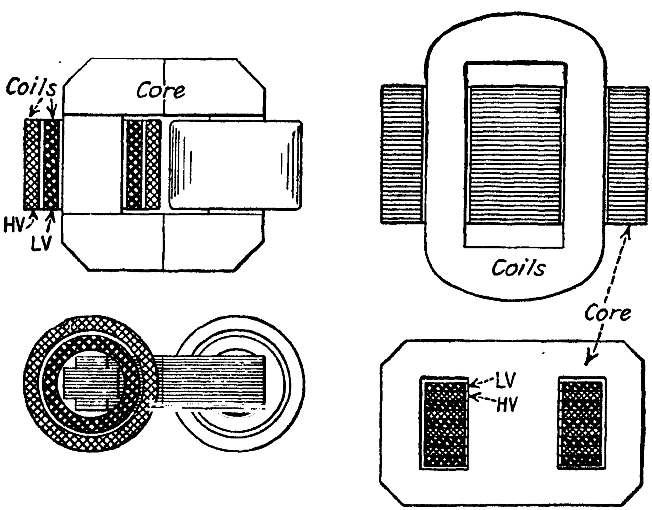


FIG. 561.—Illustrating typical arrangement of transformer primary and secondary coils. (Transformer secondary is feeding a three-wire system.)



I–Core type. II–Shell type.

FIG. 562.—Illustrating core-type and shell-type transformer construction.

a smaller area of core and a larger number of turns than will a corresponding one of the shell type. Either type may be so

designed as to be economical, but as a rule the core-type construction is more economical for high-voltage transformers than is the shell type.

879. The losses in the cores of transformers are of two kinds: (1) hysteresis losses (2) eddy-current losses. These taken together are called the iron loss, since they both occur in the iron core of the device. The hysteresis loss is due to the fact that the flux in a core is constantly alternating and power is required to effect reversals of polarity in the iron. The eddy-current losses are really $I^2 \times R$ losses in the iron, due to the currents induced in the material by the alternating flux. To minimize the intensities of these eddy currents and consequently the loss due to them, the cores are laminated. Since the alternating flux, ϕ, in the core produces both of the iron losses, and since this flux is practically constant at all loads, it follows that the iron losses are practically constant at all loads, that is, they occur whether or not the transformer is loaded.

880. To determine the iron losses in a transformer, the power input to the primary, at normal voltage and frequency, with the secondary open, can be measured with a wattmeter. The reading will be the power loss in watts inasmuch as the $I^2 \times R$ copper loss in the primary winding itself will be negligible with only the exciting current flowing in it.

881. The copper loss in a transformer equals the sum of the $I^2 \times R$ losses which occur in the windings. Hence, the copper losses may be readily computed for any load by multiplying the square of the current for that load in the primary or the secondary, respectively, by the resistance of the primary or secondary, and then adding together the two values thus obtained. That is,

$$\text{Copper loss in watts} = (I_p^2 \times R_p) + (I^2 \times R_s) \qquad (334)$$

882. The efficiencies of transformers are very high, usually being greater than 95 per cent, and with transformers of large capacity greater than 98 per cent. The efficiency of a transformer may be expressed as a formula, thus:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{copper loss} + \text{iron loss}} \qquad (335)$$

Example.—The output of a certain transformer is 50 kw. Its iron loss is 320 watts and its copper loss is 520 watts. What is its efficiency? *Solution.*—Substitute in equation (335)

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{copper loss} + \text{iron loss}}$$
$$= \frac{50{,}000}{50{,}000 + 520 + 320} = \frac{50{,}000}{50{,}840} = 98.3 \text{ per cent}$$

883. The three-phase transformer is ordinarily arranged substantially as shown in Fig. 563. The windings for all three of the phases are placed on the same core, which usually results in a material saving over the cost of three single-phase transformers of equivalent capacity. Each of the three components operates as if the others were not present. The windings may be connected to the external circuit either in delta or in Y.

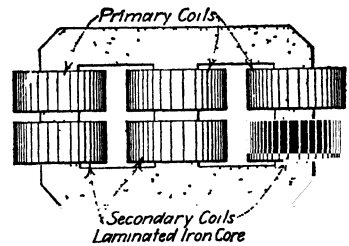


FIG. 563.—Diagram illustrating construction of a three-phase transformer.

884. The cooling of transformers is necessary to dissipate the heat developed due to the losses, above referred to, which occur within them. Special provisions are not ordinarily necessary for the cooling of very small transformers, say those under 1 kva. capacity, because the relatively large surfaces which they expose to the surrounding atmosphere dissipate the heat with sufficient rapidity to prevent the windings of the transformer from attaining a temperature which would be injurious. Larger transformers are usually placed within steel or cast-iron tanks (Fig 564), filled with oil. The oil transfers the heat produced within the core and windings to the iron case, which is usually corrugated, for capacities from 50 to 500 kva., or, in larger sizes from 500 to 2,000 kva., provided with radiating pipes, to increase their effective radiating surfaces. Not only does the oil provide a medium for the dissipation of heat, but it also provides additional insulation between parts of the transformer winding which operate at different potentials. The exposed surface of the case in many instances can be made ample and thus effectively dissipate all the heat produced. In very large transformers, with capacities greater than 2,000 kva., cooling coils through which water is circulated are submerged in the oil within the tank. Sometimes where real estate is expensive, and hence floor space must be economized, air-blast transformers are used for pressures less than 30,000 volts. These are cooled with a current of air, circulated past the core and windings by a blower.

885. The current transformer is different from the shunt or voltage transformer only in the method of its application. With the current transformer (Figs. 556,II and 565), the impedance

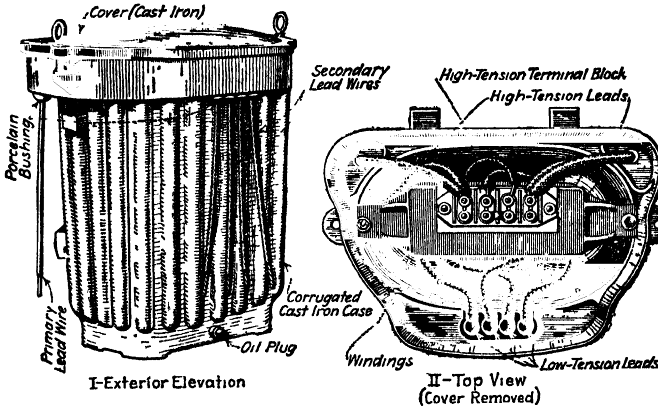


FIG 564 —Illustrating typical distribution transformer construction.

of the secondary circuit is ordinarily constant; hence any change in load that occurs must be due to a simultaneous change of primary current and voltage. In designing a series transformer, little attention is concentrated on the ratio of the primary power input to the secondary power output. However, there should be a definite ratio between the primary amperes and the secondary amperes.

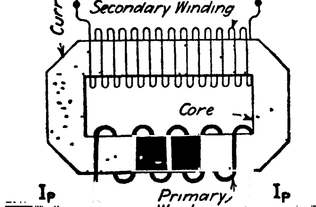


FIG. 565 —Current transformer serving an ammeter

886. The important applications of current transformers are for electrical instrument circuits (Fig. 565) and for series street lighting. Their function when used with instrument circuits is to produce through the instrument a current I_s (Fig. 565) proportional to, but of much less intensity than, the main current, I_p, which is being metered. Furthermore, where a current transformer is used, the high voltage of the main circuit is not impressed on the instrument. In practice, all instrument current transformers are usually so designed that when full-load current

flows in the primary winding, a current of 5 amp. will flow in the secondary. The instruments which are used with them are so calibrated that their pointers will show a full-scale deflection with 5 amp. flowing through the instrument. *Caution.—Never open the secondary.*

Example.—A 2,400-amp. current transformer has a ratio of 480 to 1. That is, with a current of 2,400 amp. in its primary circuit, there will flow in the secondary circuit: 2,400 amp. ÷ 480 = 5 amp. The ammeter used with this transformer would have its scale so calibrated that when a current of 5 amp. flowed through it the pointer would indicate 2,400 amp.

887. Current transformers for series-incandescent-lighting circuits are applied as suggested in Fig. 556,II, for providing a current through the lamps of greater or lesser intensity than the constant current flowing in the main circuit.

888. Distribution transformers, so-called, are those for capacities up to possibly 200 kva. and primary pressures up to 17,500

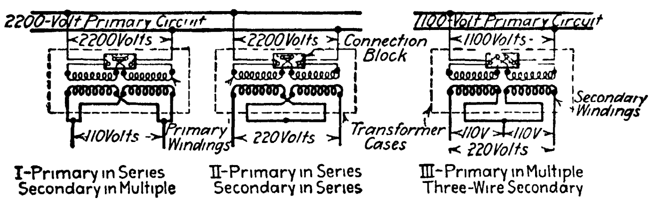


FIG. 566.—Showing standard arrangement of the primary and secondary windings of distribution transformers.

volts, which are used for the distribution of energy at secondary low voltages—500 or lower—from high-voltage mains. Figures 560 and 564 indicate typical construction for transformers of this class. They are usually arranged (Fig. 566) with two primary and two secondary coils all on the same core. With this arrangement, a given transformer may, for example, with its primary windings in parallel, be used on a 1,100-volt primary circuit (Fig. 566,III) and with the coils in series on a 2,200-volt primary circuit (I and II). Furthermore, the secondary coils may be connected either in series or in parallel, providing a secondary e.m.f. of 220 or of 110 volts, or a three-wire e.m.f. of 110 to 220 volts (see Fig. 566).

889. Transformer connections[1] may be made in an almost endless number of combinations, but those shown in Fig. 566

[1] See Terrell Croft, "American Electricians' Handbook," McGraw-Hill Book Company, Inc.

are the ones most frequently employed for single-phase distribution transformers. In Fig. 567 are shown four methods of connecting single-phase transformers to three-phase mains, so that three-phase secondary e.m.fs. of different intensities may be obtained.

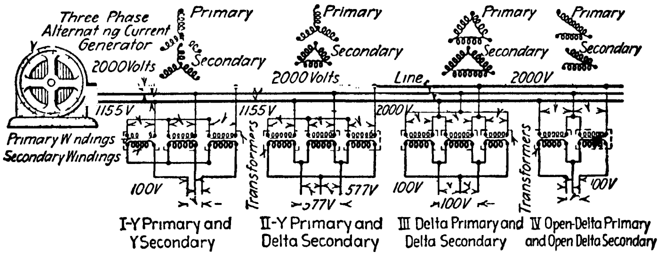


FIG. 567.—Connections of single-phase transformers on a three-phase line.

890. The autotransformer is shown diagrammatically in Fig. 568. If a winding, AD, on an iron core be connected across an alternating-current supply main of an e.m.f. E_1, any lower e.m.f., E_2 or E_3, may be obtained from the coil by tapping it as suggested. The e.m.f. E_2 will then be proportional to the

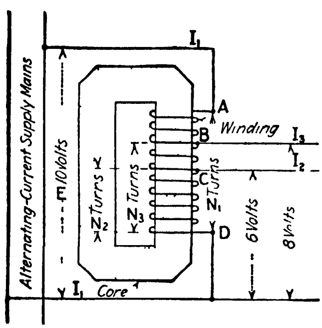


FIG. 568.—The autotransformer

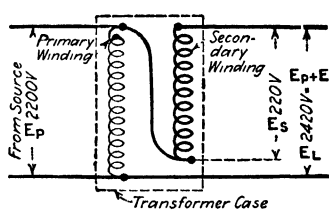


FIG 569.—The booster transformer.

number of turns between C and D. The e.m.f. E_3 will be proportional to the number of turns between D and B. It follows, then that

$$E_1 \text{ is to } E_2 \text{ as } N_1 \text{ is to } N_2 \text{ and as } I_2 \text{ is to } I_1 \qquad (336)$$

Also

$$E_1 \text{ is to } E_3 \text{ as } N_1 \text{ is to } N_3 \text{ and as } I_3 \text{ is to } I_1 \qquad (337)$$

The autotransformer is very efficient and is economical where the ratio transformation is not great. Autotransformers may be used either to step up or step down the voltage.

Example.—The autotransformer winding AD in Fig. 568 has 10 turns, and an e.m f. of 10 volts impressed across it. If 6 of the turns, CD, are tapped, what would be the voltage across CD, and if a current I_2 of 20 amp. flows in the secondary, what will be the current in the primary circuit, I_1? *Solution.*—$E_2:E_1 .N_2 N_1$; therefore $E_2 = N_2 \times E_1 \div N_1 = 6 \times 10 \div 10 = 6.0$ volts. That is, the pressure across CD would be 6.0 volts. Also $I_1 \cdot I_2::N_2:N_1$. Then $I_1 = N_2 \times I_2 \div N_1 = 6 \times 20 \div 10 = 12$ amp. That is, the current in the primary circuit will be 12 amp.

891. A booster transformer is diagramed in Fig. 569. If a transformer be arranged in a circuit with its primary winding

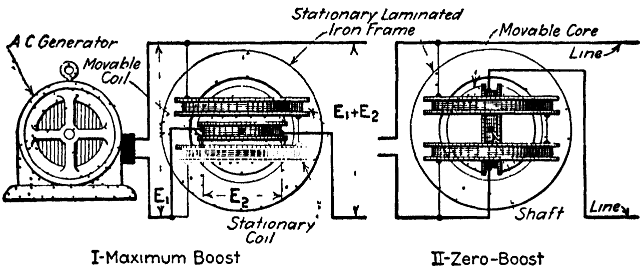


FIG 570—The induction regulator and its action.

connected across a constant voltage of the circuit E_P and its secondary winding in series with the circuit, the voltage impressed on the load will be increased by the amount of the e.m.f. E_S induced in the secondary winding. That is, the voltage impressed on the load $E_L = E_P + E_S$.

Example.—If a transformer having a primary winding designed for 2,200 volts and a secondary winding designed to deliver 220 volts be connected in a 2,200-volt alternating-current circuit as a booster as shown in Fig. 569, what would be the e m f impressed on the load? *Solution*—The secondary winding will develop 220 volts This added to the voltage impressed on the line = (2,200 + 220) = 2,420 volts, which will be the pressure on the load.

892. The induction regulator (Fig. 570) is really a booster transformer, so arranged that the e.m.f. induced in its secondary winding may be varied at will within the range for which it is designed. The device is shown diagrammatically in Fig. 570, which indicates top views. The stationary windings are arranged

on a hollow cylindrical laminated iron core. The movable winding is mounted on a laminated iron cylinder within the stationary windings. With the movable coil in the position shown at Fig. 570,I, the voltage induced in the secondary, E_2, is a maximum, because then practically all of the flux induced by the stationary coils passes through the movable coil. If, however, the movable coil be turned to position II, practically none of the flux produced by the stationary coils cuts the movable coil. Then the voltage induced by the movable coils is zero. Hence, with this device, E_2, the voltage added to the impressed voltage E_1, may be made any desired value between zero and a

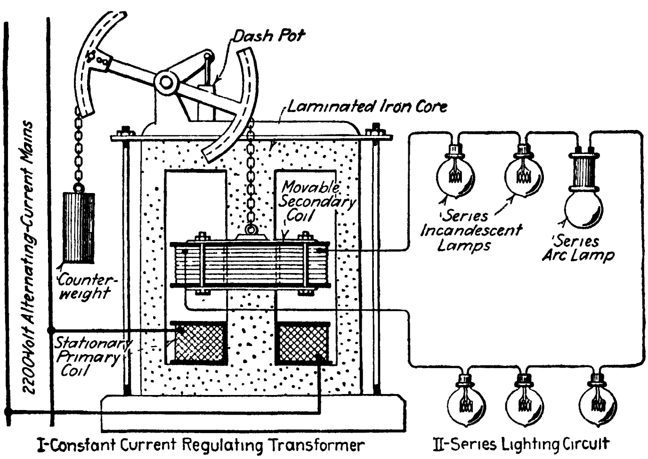


FIG. 571.—Constant-current regulating transformer.

maximum. Automatic induction regulators can be obtained wherein the movable core is turned by a small motor with a relay. The arrangement is so adjusted that when the line current increases, the boosting effect of the regulator is increased and when the line current decreases, its boosting effect is decreased. Thus, with a constant or varying e.m.f. impressed on the regulator, the e.m.f. impressed on the load can be maintained constant, even though the load varies through a wide range.

893. A constant-current regulating transformer is a device for maintaining a current of constant intensity—6.6 amp. for example—in a secondary circuit. Constant-current transformers are utilized widely for series incandescent lighting circuits. Figure 571 indicates diagrammatically the construction.

Explanation.—The primary coil is stationary and has impressed on it the constant voltage of the supply circuit The secondary coil is free to move and is counterbalanced A dashpot prevents sudden movement and jumping. With the secondary circuit open the coils lie close together, and practically all of the flux generated by the primary coil cuts the turns of the secondary coil Now, if the secondary circuit be closed, current will flow through it and through the secondary coil. There will then be a repulsive action (Lenz's law, Art 470), between the two coils, and the secondary will be forced away from the primary coil. As it is forced away the magnetic leakage between the coils increases Fewer turns cut the secondary coil and it induces a smaller e m f It will move away from the primary until it is at such a distance therefrom that its voltage is just sufficient to force the current, which the regulating transformer is designed to maintain,

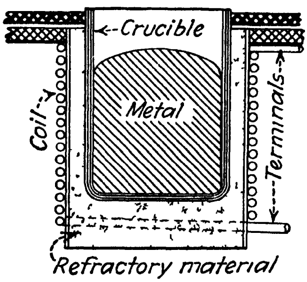


FIG 572 — Illustrating the principle of the induction furnace

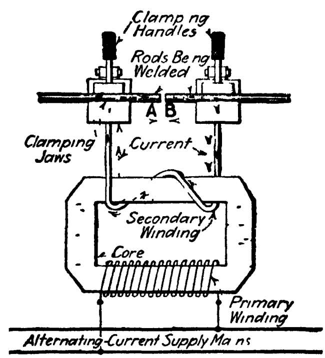


FIG 573 —Principle of the electric welder

through the secondary circuit If lamps are cut out of the secondary circuit, the secondary current tends to increase so that the coils are forced farther apart to another position at which the voltage induced in the secondary will be just sufficient to impel the normal secondary current. If lamps are connected into the secondary circuit, it tends to decrease the secondary current, thus permitting the coils to come closer together until they assume a new position in which the flux that cuts them will just be sufficient to induce a voltage that will force the normal current through the circuit.

894. The principle of the induction furnace is shown in Fig 572. In its simplest analysis the furnace consists of a crucible of refractory material containing the metal charge, while around the crucible is wound a helical, current-conducting coil of copper The copper coil acts as the primary winding of a transformer, and the charge itself forms the secondary. When high-frequency current flows through this copper coil, or primary winding, all

of the space inside of the coil is subjected to a rapidly alternating electromagnetic field, and any electrical conductor, such as the metallic charge, that is placed within this field will have currents induced in it. These induced currents cause a rapid and effective heating followed by melting of the charge in the crucible.

895. The electric welder operates on the principle diagramed in Fig. 573. The metal pieces to be welded together, *A* and *B*, are so clamped in jaws that they form a part of the path of the secondary circuit. When the two pieces are forced together with pressure in the directions indicated by the arrows, intense heat is developed at their junction. This fuses the metal to the welding temperature.

SECTION 53

THREE-WIRE DISTRIBUTION AND SYSTEMS

896. The Three-wire System Is Used Because It Saves Copper (see Figs. 574 and 577).—Incandescent lamps are now standardized for 110 volts; and although lamps for higher voltages may be obtained, they are much more expensive. A circuit carrying any considerable load and operating at the low pressure of 110 volts would require very large conductors to maintain the $I \times R$ or line voltage and the $I^2 \times R$ power loss in the line within reasonable limits. With the three-wire system, a low voltage, say 110, is impressed on the receivers—incandescent

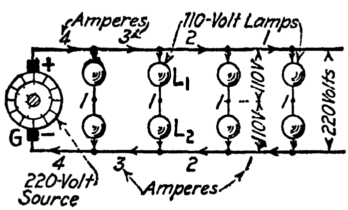


Fig. 574.—110-volt lamps arranged in multiple series across 220 volts.

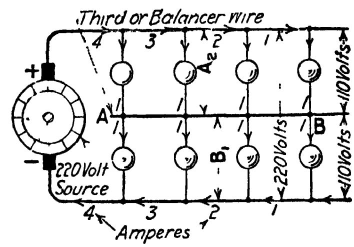


Fig. 575—Balancing wire *AB* arranged between lamps.

lamps—whereas one twice as great, say 220, is used for transmission. Since the weight of the metal in conductors for a given power loss varies inversely as the square of the voltage, it is evident that a considerable saving in conductor material is possible with the three-wire system. In this country the three-wire system is of greatest importance as applied to 110- to 220-volt lighting systems.

897. The principle of the three-wire system is illustrated in Figs. 574 to 577. Incandescent lamps for 110 volts could be connected two in series across 220 volts, as shown in Fig. 574, and although each lamp would operate at 110 volts, the energy of the group would be transmitted at 220 volts, and the outside conductors could, with equal loss, be but one-fourth the size that

would be necessary if the energy were transmitted at 110 volts. This arrangement (Fig. 574), although it would operate, is not commercially feasible because each lamp (of each pair of lamps in series) must be of the same size, and if one lamp, L_1, for example, goes out, its partner, L_2, is also extinguished. These disadvantages might be partially corrected by installing a third or balancer wire as at Fig. 575. Then one lamp might be turned off, and the others would burn. Also, a single lamp might be added to either side of the system, between the third wire and either of the outside wires. But unless the total resistance of all of the lamps connected to one side circuit, A_2, was practically equal to that of all of the lamps connected to the other side circuit, B_1, the voltage across one side circuit would be

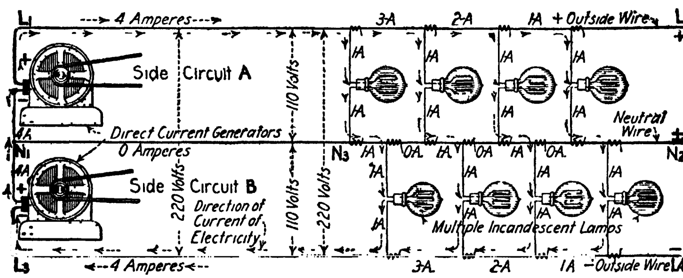


FIG. 576.—Three-wire direct-current circuit.

higher than that across the other. On the side having impressed on it the higher voltage the lamps would burn brightly, and on the side of the lower voltage they would burn dimly. Obviously, it is not feasible in practice so to arrange or balance the side circuits that they will have the same resistance. Hence, some other method must be used in practicable three-wire systems whereby the electricity will be transmitted at 220 volts, and the pressure across the lamps will be 110 volts.

898. Commercial three-wire systems consist (Figs. 576 and 577) of (*a*) two outer conductors (L_1–L_2 and L_3–L_4, Fig. 576) having—for lighting installations—a pressure of 220 volts impressed across them; and (*b*) a neutral wire (N_1–N_2) so connected to sources of voltage that the pressure between it and either of the outside wires is 110 volts. In Fig. 576, direct-current generators are the sources of voltage. The neutral wire joins at the point, N_1, where the generators are connected

together. When the system is perfectly balanced, the neutral wire carries no current, and the system is then, in effect, a 220-volt system. Perfect balance seldom obtains in practice. When the balance is not perfect, the neutral wire conveys a current equal to the difference between the current taken by one side circuit and that taken by the other side circuit. Note from Fig. 577 that the current in different parts of the neutral wire may be of different intensities and that it is not necessarily in the same direction in all parts of the neutral wire. Each incandescent lamp in Fig. 577 is assumed to take 1 amp., and the small figures indicate the currents in different parts of the circuit.

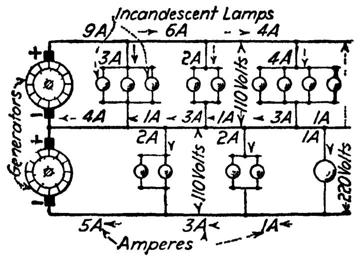


FIG. 577 —Showing the distribution of current in a three-wire system.

NOTE.—A three-wire circuit may, in a sense, be considered as a combination of two two-wire circuits Thus, the two circuits of Fig. 578 would transmit the electrical energy to the eight incandescent lamps which they serve, as satisfactorily as does the equivalent three-wire circuit of Fig 576. However, the Fig. 576 arrangement would, as will be shown, require much

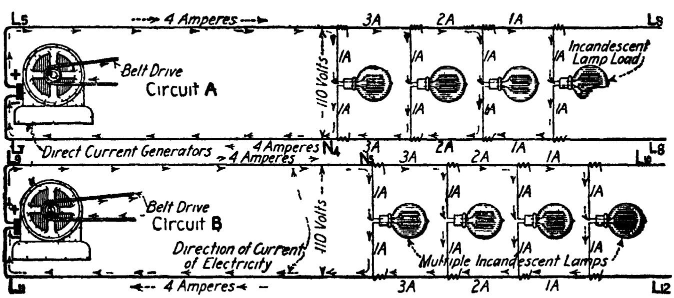


FIG 578.—Two two-wire direct-current circuits.

less copper in the line conductors for a given watts power loss and a given percentage voltage drop than would that of Fig. 578. The small numerals indicate the current intensities, in amperes, in the different parts of the circuits in the two illustrations.

899. A three-wire circuit with the neutral the same size as the outers requires only three-eighths the copper required for an equivalent two-wire circuit providing the same percentage

volts line drop and the same watts power loss. The reason for this may be derived from a consideration of the three-wire circuit of Fig. 576 and the equivalent two two-wire circuits of Fig. 578. It is evident since there is a current of 4 amp. to the *left* in section L_7–N_4 (of Fig. 578) and a current of 4 amp. to the *right* in section L_9–N_5 that there will be no current in section N_1–N_3 of Fig. 576. This follows because the three-wire circuit is balanced. With the arrangement of Fig. 576, one less wire is required than with that of Fig. 578. Now, the current in the outer line wires is 4 amp., both in the three-wire circuit of Fig. 576 and in the equivalent two two-wire circuits of Fig. 578.

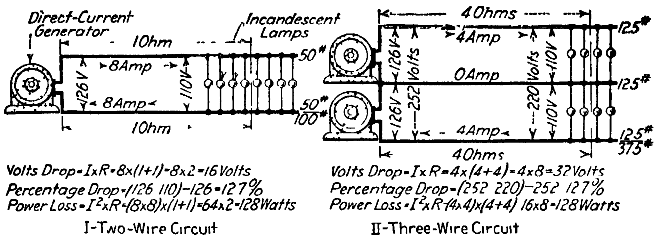


FIG. 579.—Showing a comparison of volts line drop and watts power loss in equivalent two-wire and three-wire circuits.

However, in Fig. 576 the 4-amp. line current traverses only half the length of line conductor that the 4-amp. currents in Fig. 578 traverse, because no—or practically no—current flows in the neutral wire. Hence the line wires in Fig. 576 may be one-half the size of those in Fig. 578, for the same percentage voltage drop. Therefore, since three-fourths the number of wires is required for Fig. 576 as for Fig. 578, and since each of these wires may be one-half the size of that necessary in Fig. 578, the amount of copper in Fig. 576 is $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ that necessary in Fig. 578 for the same percentage drop—it being assumed that the neutral is made the same size as the outer wires. The neutral may frequently be made smaller than the outer wires, as outlined later.

Example.—Consider the 110-volt two-wire circuit of Fig. 579,I, and the equivalent three-wire circuit of II. The transmission distance is the same for each of the two circuits. The load—eight incandescent lamps each taking 110 watts—is the same on I as on II. Referring to the two-wire

circuit of I, each of the line wires is assumed to have a resistance of 1 ohm. Since each lamp takes a current of 1 amp., the line current is 8 amp. Hence, as indicated, the line drop is 16 volts, the percentage line drop is 12.7 per cent, and the line power loss is 128 watts. If it be assumed that each line wire weighs 50 lb., the total weight of the two line wires will be 100 lb.

If a 110- to 220-volt three-wire circuit is arranged to transmit the same power with the same loss, it will have the characteristics diagramed in Fig. 579,II. Since it is in effect a 220-volt circuit, the current will be half that of the circuit of I. Since power loss in a conductor = I^2R, if the current is halved, a conductor of four times the resistance will carry it (the current) with the same power loss and with the same percentage voltage loss. Thus, for the same line power loss and the same percentage voltage drop, as in I, the outer wires of II will each have a resistance of 4 ohms.

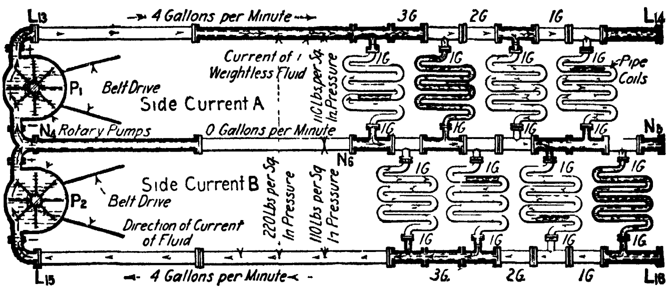


FIG. 580.—Hydraulic analogy to a three-wire direct-current circuit.

There is no drop or loss in the neutral wire since—the load being balanced—it carries no current. Then, as indicated, the line drop is 32 volts but the percentage volts drop is 12.7 per cent—the same as in I. The power loss is 128 watts, the same as in I. Each outer wire of II, since it has four times the resistance of an outer of I, will have a weight of $\frac{1}{4} \times 50 = 12.5$ lb. If the neutral is made the same size as the outers, the total weight will be $3 \times 12.5 = 37.5$ lb. Now 37.5 lb. is three-eighths of 100 lb.; hence the three-wire circuit of II transmits the same power the same distance with the same power loss and same percentage voltage drop as does the two-wire circuit of I—but with three-eighths the weight of conductor.

900. If the neutral wire is made half the size of the outers, only 31.3 per cent of the copper is required that would be necessary for a two-wire system transmitting the same power the same distance with the same power-line loss and percentage voltage drop. If the neutral is made one-third the size of the outers, the relative weight of copper is then 29.2 per cent.

901. A hydraulic analogy of a three-wire circuit is shown in Fig. 580. The two rotary pumps, connected in series, are analo-

gous to two direct-current generators connected in series for three-wire service. The pipe lines L_{13}, L_{14} and L_{15}, L_{16} correspond to the outer wires of a three-wire circuit. The pipe line N_4–N_5 corresponds to the neutral wire in a three-wire circuit. The current of weightless incompressible fluid in the pipe circuit—analogous to a current of electricity—will be impelled as shown by the arrows. The circulation is due to the hydraulic pressure (voltage) developed by the pumps. No fluid flows in the neutral between N_4 and N_6 because, with the circuit balanced as shown, there is no difference in hydraulic pressure between these two points. The small numerals with the letter "G" following them, indicate the gallons per minute—analogous to amperes—flowing in the different parts of the circuit.

902. Size of Neutral in Actual Installations.—In out-of-door distribution systems the neutral is often made one-half the size of the outer wires. For interior wiring, the neutral is usually made the same size as the outers, where the outers are of No. 6 A.W.G. or smaller wire. Where the outers are larger than No. 6, the neutral is usually selected so as to have about two-thirds the sectional area of one of the outers (see the author's "American Electricians' Handbook").

903. The amount of unbalance that may come on a three-wire circuit depends on local conditions. In ordinary three-wire lighting systems the unbalanced load seldom exceeds 10 per cent of the total load. Probably 5 per cent is a fair average for a well-laid-out system. Balancer sets for interior three-wire systems are frequently specified of sufficient capacity to take care of a 10 per cent unbalance. Sometimes the unbalance on a poorly laid out system may be 20, 30 per cent, or even more.

904. The methods of obtaining three-wire voltages will now be discussed. That suggested in Fig. 576, while excellent for certain conditions, is not economical for small installations because the operation of two generators is necessary. Its first cost is more and its efficiency is less than with an arrangement requiring the use of but one generator. The advantage of a two-generator installation (Fig. 576) is that any percentage of unbalance up to the capacity of one generator can be handled by it. The important methods of obtaining three-wire voltages are listed in the following table.

905. Arrangements for Deriving Three-wire Voltages.—

Direct current	1. A 220-volt generator in combination with a motor-generator balancer set, Fig. 581. 2. A three-wire or Dobrowolsky generator, Fig. 582.
Alternating current	1. With a transformer, Fig. 583,I. 2. With an autotransformer or balance coil, Fig. 583,II.

906. If the neutral wire opens in a three-wire circuit, the lamps (Fig. 584) on the side, *A*, having the most lamps connected

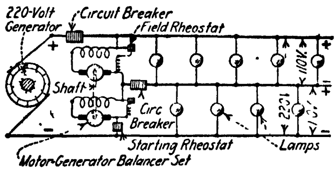


FIG. 581.—A 220-volt generator and a motor-generator balancer.

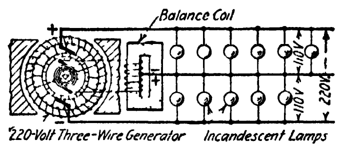


FIG. 582.—Three-wire or Dobrowolsky generator.

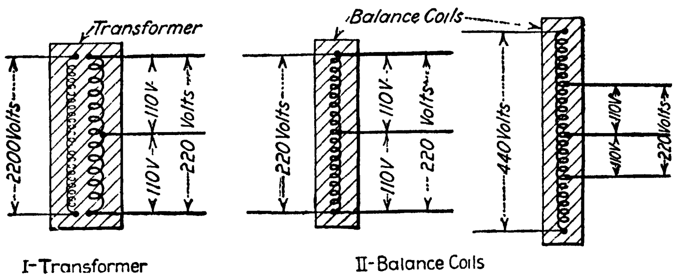


FIG. 583.—Methods of obtaining three-wire voltages with transformers and balance coils.

to it, will burn dimly or at under voltage, whereas the lamps connected to the side having the least number of lamps, *B*, will burn brighter than normal or at overvoltage. Any device, a motor, *C*, for instance, connected to the two outside wires will operate normally.

907. If one side of a lamp on a three-wire system becomes grounded, G_1, and there is another ground, G_2, on the opposite side of the system (Fig. 585) 220 volts will be impressed across the lamp, and it will be burned out.

908. If one of the generators of a three-wire system becomes reversed. as at G_1 (Fig. 586), the lamps connected between both

of the outside wires and the neutral will receive normal voltage, but the polarity will be reversed. Any receiver connected across the outside wires, M, for example, will have no difference of potential across it, and hence no current will be forced through it—it will not operate.

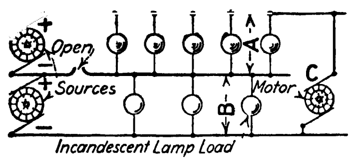


FIG. 584.—Open neutral in a three-wire circuit

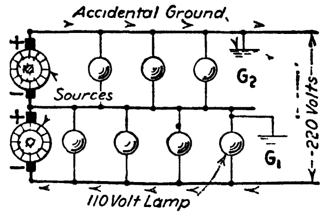


FIG. 585.—Ground on three-wire system outer wire.

909. If one of the outside wires of a three-wire system opens, O (Fig. 587), a device, H, connected across the two outside wires on the distant side of the break will receive some current at 110 volts through the lamps, L_1, L_2, and L_3, connected to the broken side between the device and the break. These lamps will burn dimly when the device is connected, but the device will not receive enough current to operate it properly.

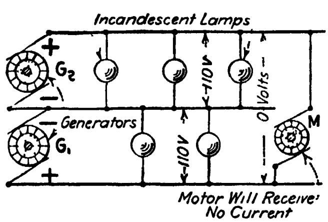


FIG. 586.—Reversal of generators.

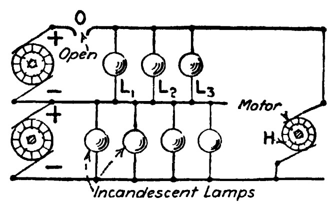


FIG. 587.—Open in the outer of a three-wire system

910. The general arrangement of three-wire feeders and mains may be substantially the same as that for two-wire systems. The important thing is so to balance the circuits as to maintain the currents in the two outside wires approximately equal. The lamps and other devices should be divided between the two side circuits of the system so that the loads will be, as nearly as possible, balanced when the circuit is loaded to its full capacity or when loaded to any fraction of it. All three wires—two outers and the neutral—should be routed to every location

where any considerable amount of power is required; to every building in an out-of-door distribution and to every distribution center in distributions within buildings. Where many lamps are to be lighted simultaneously the group should be controlled by three-pole switches to prevent unbalancing. Not only must loads on three-wire systems be balanced on the two side circuits, but the loads on the side circuits must also be distributed along both the circuits in approximately the same manner. If they are not, there may be considerable extra drop. This local unbalancing is one of the principal causes of the excessive voltage variation that may sometimes occur on three-wire systems.

SECTION 54

ELECTRONIC TUBES

911. An electronic tube consists of a vessel, either highly evacuated or gas filled, containing metal electrodes which may be connected to external electric circuits. The usefulness of an electronic tube depends on the actions of electrons within it (Art. 4). Primarily its function is to control electric current or to emit certain radiations.

It has a very great number of practical applications and does many things which are impossible without its use.

912. Types of Electronic Tubes.—There are two general classes of such tubes: (1) the cold-cathode tube and (2) the hot-cathode or thermionic tube. In the first the operation does not depend on the heating of a part of the tube. In the second, one electrode of the tube is heated in order to make it emit electrons freely. Nearly all of the electronic tubes at present in practical use are of the hot-cathode type. The photoelectric cell, which constitutes a third class, depends upon the fact that electrons are emitted from one of its electrodes when light strikes that electrode.

There is another way in which vacuum tubes may be classified into two general kinds. Tubes of the first kind have a high vacuum, and gas is not intentionally left or inserted in them. Tubes of the second kind are known as gaseous tubes, and in them a small amount of a certain gas is purposely enclosed.

Tubes of these various classes and kinds will be discussed in the following paragraphs, though the discussion will not always follow the order of division as mentioned. There is still another type of tube which depends upon electron action. This type is the tube in which an arc is maintained, such as the neon lamp and the mercury arc lamp. These tubes will not be discussed here. They are covered in the author's "Practical Illumination" and "Electrical Machinery," respectively.

913. The action of all electronic tubes depends on the fact that electrons are emitted more or less freely from substances under certain conditions. These electrons travel inside the

tube from one terminal to another. If the terminals are connected to each other through an electrical circuit outside the tube, an electric current will flow in that circuit. This current may be used to perform useful functions.

A vacuum or very low gas pressure is used because the electronic action is much greater under low pressure than under ordinary atmospheric pressure.

The tubes are of glass or metal. Air is pumped out of them and they are sealed, or the low pressure may be maintained by continual pumping during operation of the tube. In gaseous tubes a small amount of certain gases is inserted before sealing.

The metal terminals inside the tube are mounted apart and insulated from each other. Each has a wire or other connector which passes through a sealed hole in the wall of the tube so that electrical connections may be made outside.

914. The thermionic vacuum tube makes use of the fact that when metal is heated, electrons tend to escape from its surface somewhat as particles of vapor escape from a liquid during evaporation. The electrons which act in this way are called "free" electrons. In number they amount to only a small fraction of all the electrons in a substance. They have temporarily escaped from the control of the positive nucleus of the atoms to which they belong (Art. 16). They move readily between the atoms until they again become attached to some atomic system. Free electrons are present in all substances. It is estimated that copper contains about 10^{19} (10,000,000,000,000,000,000) free electrons per cubic centimeter, and air contains from 1,000 to 5,000 per cubic centimeter.

915. The free electrons circulate easily within a substance and they are continually moving, some at a greater speed than others However, at the surface they are prevented from leaving the substance by a very large force. This force is similar to the cohesive force which is present in the surface of all liquids. To penetrate the surface an electron must have a high speed. The required speed is different for each kind of substance. For tungsten it is 1.26×10^8 cm., or about 800 miles per sec. To escape from the metal thorium the speed must be 1.09×10^8 cm., or about 700 miles per sec. Thus it is seen that electrons escape more easily from thorium, and for this reason thorium is used in the filaments of some vacuum tubes.

916. When the temperature is raised the motion of the free electrons in the metal becomes more rapid. The velocity of some of them may become so great that they will penetrate and leave the surface of the substance and pass out of its control. Others may start away from the surface but without sufficient velocity to carry them out of its control. They will be drawn back again. In air or other gas at ordinary pressures, the electrons which do escape soon collide with the comparatively heavy molecules in the air and their energy is quickly dissipated. However, in a vacuum, the escaped electrons move without obstruction, and thermionic action is greatly increased.

917. The Two-electrode Tube.—The simplest form of thermionic tube has two electrodes. One of these is arranged so that it may be heated. Usually this is a length of wire called a filament. Connections pass from each end of the filament to terminals on the exterior of the tube. In order to heat the filament, electric current is passed through it from a battery or other source. This heats the filament just as current heats the filament in an incandescent lamp (but not to so high a temperature). The heating of the filament is the only purpose of this electric current. If the heating could be accomplished in another way, the result would be the same. This electrode which emits electrons is called the cathode.

The second electrode, termed the anode, is usually in the form of a thin metal sheet surrounding the hot electrode and located at a distance from it. When electronic tubes were first made, this electrode was a flat plate. The word "plate" is still generally used to designate the electrode, though it may have cylindrical or other form.

In operating the tube, the filament is kept heated by its current. Electrons disengage themselves from the filament and fly outward. Many of them strike the cold anode. Now, if an electric circuit is run outside the tube from the plate to the filament, electric current will flow in the circuit. Each of the electrons passing to the anode brings a negative charge. As the stream of electrons continually deposits its charges, an equivalent current flows in the external circuit from the anode to the filament.

918. Space Charge.—One factor which tends to prevent electrons from leaving a heated surface is the space charge. This may be explained by referring again to the heated filament

in a vacuum. As already explained, the electrons which have a high enough speed will pierce the surface and start away from the filament. As soon as an electron leaves the filament, however, the filament has a surplus of positive charge. The electron has taken away one negative charge which the filament formerly had when it was neutral with all charges balanced. This leaves one unneutralized positive charge on the filament. This positive charge tends to attract the negatively charged electron. Now when another electron leaves the surface, it is likewise attracted backward by the positive charge on the filament. In addition, this electron is repelled toward the filament by the electron which has preceded it. The electrons, both being negative, repel each other. As more electrons leave the filament they also are pulled back by the charge on the filament and pushed back by the charge of the other electrons which are outside the filament. Some of the electrons are forced back into the filament, and a stable condition is reached in which the surface of the filament is surrounded by a cloud of electrons and the number of electrons leaving the filament just equals the number returning into it. If the temperature of the filament is raised, this equilibrium is destroyed and the number of electrons around the plate increases until another equilibrium is reached.

This phenomenon of a charge due to separate electrons in a space is known as a "space charge." The space charge due to electrons is normally a negative charge. It is possible also to have a positive space charge. This is caused by positive ions in a space. These ions are atoms which have lost one or more electrons and which therefore contain a surplus of positive charge (Art. 42).

919. If a battery is connected in series with the circuit as shown in Fig. 588, the flow of current will be increased. In this case the negative side of the battery is connected to the filament and the positive side to the plate. The action then is as follows. Suppose an electron escapes from the surface of the filament. This electron has a negative charge of electricity. As the filament is charged negatively by the battery, it will repel the electron. At the same time the plate will attract the electron because it is charged positively by the battery. Therefore the electron will have a greater speed than if the battery were not connected to the plate. Also, a greater number of

electrons will escape from control of the filament. When the battery was not connected, some of these electrons did not have quite enough speed to escape control of the filament. With the battery connected, the speed of these electrons is increased sufficiently for them to pass entirely away from the filament. In this way the current is increased by connecting a battery in the circuit. For this reason a battery or other source of electricity is always used in practical applications of vacuum tubes.

If the voltage of the battery is increased, more electrons will pass from filament to plate and the current will increase. However, there is a limit to increasing the current in this way. When the voltage of the battery has been raised to a certain value, practically all of the electrons which penetrate the surface of the filament are propelled across to the plate, none returning to the filament. Raising the voltage beyond this point gives very little increase in current.

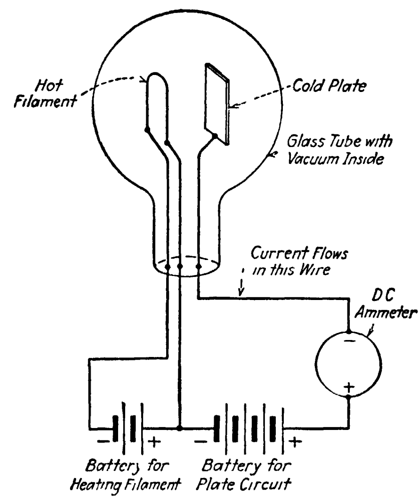


FIG. 588.—Two-electrode thermionic vacuum tube connected to batteries.

However, if the temperature of the filament is raised, the current will then increase. At higher temperature more electrons will penetrate through the surface of the filament. Then as the voltage is increased, the current will increase until another point is reached where all the electrons escaping at this temperature are passing to the plate. Such a point as this is called the tem-

perature saturation point. For each *different temperature of* the filament there is a different saturation current.

920. Tubes Pass Current in One Direction Only.—Now suppose the connections of the battery are reversed, with the positive side of the battery connected to the filament and the negative side to the plate. Under this condition no current would flow. Any electrons which penetrate the surface of the filament are again attracted toward it by its positive charge from the battery. Likewise the electrons are repelled by the negative charge on the plate.

Thus it is seen that the tube will allow current to flow in one direction but not in the other. This property makes such a tube

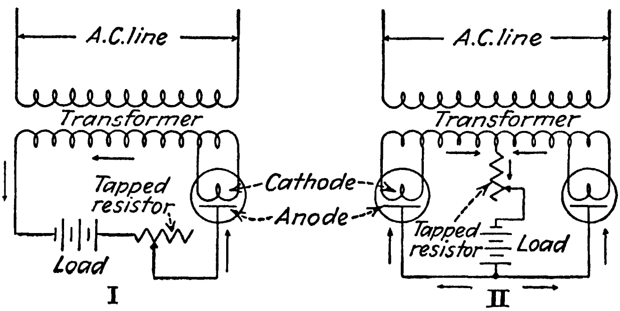


FIG. 589.—Two-electrode thermionic tubes used as rectifiers. I, half-wave rectification; II, full-wave rectification.

suitable for use as a rectifier for changing alternating current into direct current and as a radio detector.

921. Thermionic Rectifier Tubes.—When the two-element tube is used as a rectifier it is connected to a circuit as shown in Fig. 589. Alternating current is supplied to this circuit from an alternating-current line or from a transformer. The alternating voltage is positive first in one direction and then in the other direction. This is similar to connecting the battery first in one direction and then in the reversed direction as explained in Art. 920. The tube allows current to flow when the voltage is in the direction which makes the plate positive. However, when the voltage is reversed no current flows. The result is that current flows in the circuit in one direction only and it flows only when the voltage is positive on the plate. When the voltage is negative on the plate no current is flowing.

The current through the circuit is a pulsating one, varying from zero to a maximum value, and always flowing in one direction.

922. Steadying the Direct Current.—*This fluctuating direct current is not satisfactory for some purposes. Therefore an arrangement is often employed which will give a steadier current.* This is sometimes accomplished by connecting reactance coils in the circuit and condensers across the circuit as shown in Fig. 590. When the voltage in the direct-current portion of the circuit is high, electricity flows into the condenser and is tempo-

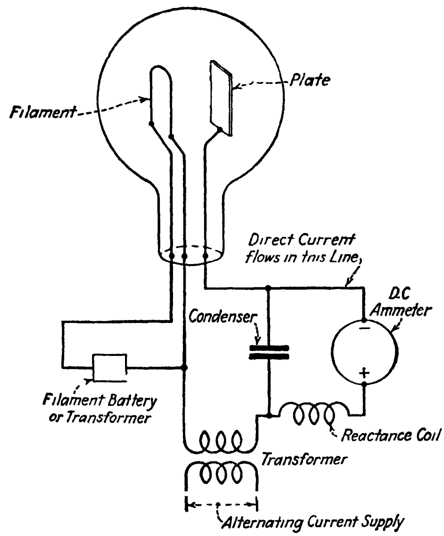


Fig. 590.—Thermionic rectifier with reactance coil and condenser connected to smooth out fluctuations in direct-current output.

rarily stored there. When the supply voltage becomes zero the electricity in the condenser is discharged back into the circuit. This electricity cannot pass through the tube because the tube will not pass current in the reverse direction. The electricity is consequently discharged through the circuit where it is wanted. This discharge takes place while the current from the tube is zero. Thus, by help of the condenser the current in the load circuit never becomes zero. It is a continuous direct current, fluctuating more or less but never reaching zero if the condenser is large.

The reactance coil still further smooths out the fluctuations. A reactance coil consists of a large number of turns of insulated

wire wound around an iron core. Such a coil in a circuit always tends to prevent changes in the current flowing in the circuit. Thus this coil assists the condenser in smoothing out the current. In the same way any inductive load in the circuit will assist in smoothing the current. If very steady current is needed, more complicated arrangements of condensers and reactance coils may be employed.

923. Use of Two Rectifiers.—Two rectifiers are usually employed to produce a smoother direct current than that obtained from a single tube. This is illustrated in Fig. 589,II. During the interval while no current is flowing from the first rectifier, the second rectifier is delivering current. Thus one rectifier is always operative and the load current is fairly steady.

The arrangements suggested in Fig. 589 are for operation on single-phase current. By using three-phase current, the direct-current pulsations will overlap, with the result of further smoothing out the pulsating current. This smoothing out effect is still further increased by the use of six- or twelve-phase connections, as are now employed in large rectifier installations for industrial and railway applications.

Similar results may be obtained by the use of the two-plate rectifier which, as its name indicates, has two plates and one filament in one tube.

924. Uses of Thermionic Rectifiers.—Thermionic rectifiers are largely used in radio work, testing, and experiment. They are particularly valuable where high voltages and low currents are desired. For radio they are made with voltages as low as 200 or 300 volts and for currents of 0.1 amp. to a few amperes. For testing purposes voltages of 50,000 volts have been achieved with low currents. If high-voltage long-distance direct-current power transmission ever becomes practicable, it is probable that rectifiers of this kind will be used to change alternating current to the direct current which will be transmitted. Rectifiers of large capacity for direct-current voltages as high as three thousand volts are at present employed for the operation of railroads.

Two-element tubes serve satisfactorily as radio detectors and formerly were used for that purpose. They have largely been superseded now by tubes with three or more elements. These tubes are more sensitive and have the ability to amplify radio

signals as well as to detect them. The two-electrode tube cannot be used as an amplifier.

925. Three-electrode Tubes.—The addition of a third electrode to a two-element vacuum tube multiplies its worth many times. The three-electrode tube has made possible the development of radio and modern methods of telephony and telegraphy. It also has many other industrial and scientific uses. This type of tube is called a triode.

The third element is known as the "grid." It was given this name because in its original form it resembled a gridiron.

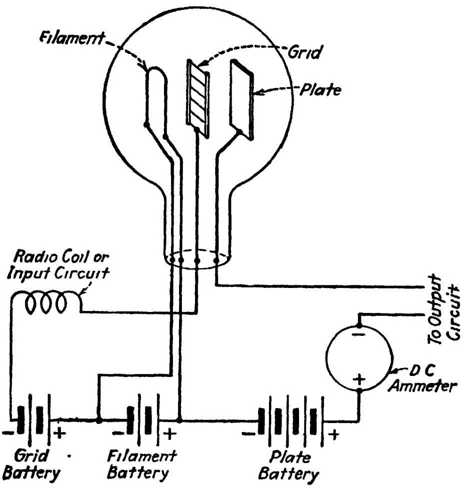


FIG. 591.—Three-electrode thermionic tube.

It was formed of parallel wires in the same plane separated from each other and attached by their ends to two heavier wires. The whole grid was in the shape of a rectangle. A grid of this form is still used in some tubes. In others the grid is cylindrical, sometimes formed by a cylindrical coil of wire with the turns spaced apart. The use of the term "grid" has, however, continued regardless of the shape which it may take.

The grid is located in the space between the filament and the plate as shown in Fig. 591. It has a connection terminal outside the tube If the grid is not connected outside the tube, the tube will act practically the same as a two-electrode tube.

To understand the operation of a three-electrode tube, suppose the filament is heated and the plate and filament are connected

to a battery as in Fig. 591, with the plate positive and the filament negative. Then electrons will flow from the filament to the plate through the spaces in the grid. Some of the electrons may strike the wires of the grid, but most will pass through.

926. Action of the Grid.—Now suppose the grid is made more negative than the filament. This is done by connecting it to the negative terminal of a battery, while the filament is connected to the positive side of this battery. When the grid becomes negative some of the electrons leaving the filament are repelled back to the filament by the negative charge on the grid. Other electrons may have a high enough speed to continue through the grid to the plate. If so, some current will flow in the external circuit, but the circuit will be less than before because it has been reduced by the negative charge on the grid.

If the grid is made still more negative, a point will be reached where practically no current will flow. The negative charge on the grid will be so strong that it will repel all of the electrons back to the filament and none will flow to the plate.

If the grid is made less negative, the current will increase. Thus it is seen that by making the grid more or less negative the current in the plate circuit will be decreased or increased respectively. In other words, by varying the voltage of the grid it is possible to vary the current in the plate circuit. The very important and useful feature of this action is that thereby we can get amplification. By varying the grid voltage by a small amount, the plate current can be varied by a large amount. Furthermore, the tube may be so constructed that the variation of the plate current is proportional to the variation in grid voltage throughout a considerable range. If we double the variation of grid voltage we double the variation in plate current. This permits accurate reproduction of the changes of grid voltage in amplified form. In a ratio tube this gives amplification without distortion. Figure 592 is a curve for a certain tube showing how the plate current changes as the grid voltage is changed. The voltage at the filament is zero and the voltage on the plate is kept constant at 90 volts. It is seen that when the grid voltage is zero, the plate current is 0.006 amp. or 6 milliamp. When the grid voltage is minus 1 volt, the plate current is 4 milliamp. When the grid voltage is minus 2 volts, the plate current is 2 milliamp.

This explains the action of the tube as an amplifier. In practice the voltage to be amplified is impressed from the filament to the grid. The grid, for instance, may be connected to one end of a coil of wire in a radio set as in Fig. 591. This coil is arranged so that the radio signals pass through it. The voltage across the coil varies as the radio signals vary. The other end of the coil is connected to a grid battery of the proper voltage, connected as shown in Fig. 591. Then as the radio signals vary, the voltage on the grid varies, and this causes the plate current to vary. The plate circuit is connected through coils or otherwise to the next tube in the radio set. This tube then receives, in amplified form, a duplicate of the signal impressed on the first

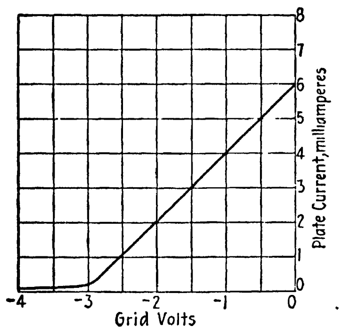


FIG. 592.—Curve showing relation between grid voltage and plate current in a three-electrode vacuum tube.

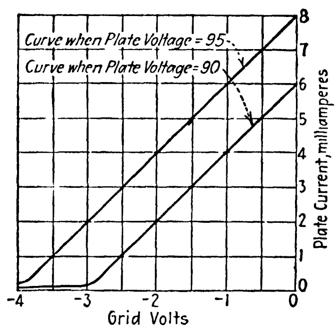


FIG. 593.—Curve showing relation between grid voltage and plate current for two different values of plate voltage, 90 and 95 volts.

tube. A sufficient number of tubes is thus connected one after another to give the desired amplification. This same principle is used in other applications of electronic tubes.

If the voltage is raised on the plate of a three-electrode tube the plate current will increase just as explained for the two-electrode tube. Figure 593 shows two curves, one for a plate voltage of 90 and the other for a plate voltage of 95. It will be seen that for any value of grid voltage, the plate current is greater with 95 volts plate voltage than it is for 90 volts.

927. Amplification Factor.—It is seen from the curves that on the 90-volt curve the plate current is 6 milliamp. when the grid voltage is zero. On the 95-volt curve, the plate current is the same, 6 milliamp., when the grid voltage is minus 1. In

other words, for the same plate current, the plate voltage has been changed from 90 to 95, a change of 5 volts, and the grid voltage has been changed from 0 to minus 1, a change of 1 volt. The ratio of the change is 5 to 1. This ratio is called the "amplification factor" of the tube, and the amplification factor of this particular tube equals 5. This quantity is in a way a measure of the amplification which may be obtained from the tube, though in a practical circuit the amplification obtained is always less than the amplification factor. The value of this factor depends upon the construction of the tube. In commercial radio tubes the theoretical value ranges from 3 to 50 or more for three-electrode tubes and to 1,500 or more for screen-grid tubes (Art. 936).

The amplification factor may be increased by increasing the distance from the plate to the grid. It also depends on the size and spacing of the grid wires. Larger wire and closer spacing give higher amplification factor. The symbol commonly employed for amplification factor is the Greek letter μ (mu).

928. Plate Resistance.—The amplification, which can be secured from a tube as well as the current, depends on the plate resistance. This means the resistance which the tube offers to alternating current in flowing from the filament to the plate. This resistance increases as the grid voltage becomes more negative. This is why the current decreases when the grid becomes more negative, as already explained in Art. 926. The plate resistance may be estimated from curves as shown in Fig. 593. Suppose it is desired to get the plate resistance at minus 1 volt on the grid. Taking the two curves shown, the plate current is 6 milliamp. for the upper curve and 4 milliamp. for the lower curve. Subtracting, the difference equals 2 milliamp. The plate voltage for the upper curve is 95, and that for the lower curve is 90. The difference is 5 volts. Then, taking this 5 volts and dividing by the 2 milliamp., we get $5 \div 2/1{,}000 = 2{,}500$ ohms. This is the plate resistance at a grid voltage of minus 1. In actual tubes the plate resistance varies with different plate voltages to some extent.

929. Mutual conductance is another term applied to the characteristics of an electronic tube. It tells how much the plate current changes per volt change in grid voltage. The plate voltage is supposed to remain steady during this increase. For

instance, in the curve of Fig. 592 suppose the grid voltage changes from minus 2 to 0, a change of 2 volts. At the same time the plate current increases from 2 to 6 milliamp., a change of 4 milliamp. Then, dividing 4 milliamp. by 2 volts, we get 0.004 amp. ÷ 2 = 0.002 amp. per volt. In other words, the plate current changes 0.002 amp. per volt change in grid voltage. A value of this kind is expressed in mhos or micromhos. (The word "mho" is "ohm" spelled backward.) As the values of this constant are small for electronic tubes, the quantities are usually expressed in micromhos, which are one-millionth of a mho. Therefore, the mutual conductance of the tube exemplified by Fig. 592 would be 0.002 × 1,000,000 = 2,000 micromhos. It will be noted that the value may be obtained by dividing the amplification factor by the plate resistance.

930. Changes in filament temperature in a three-electrode tube give results similar to such changes in a two-electrode tube (Art. 919). At a certain plate and grid voltage, if the temperature is not high enough, an insufficient number of electrons will flow so that the plate current will be limited and the tube will not act as a true amplifier.

931. Grid Current.—In the foregoing paragraphs the grid has been considered as having a negative voltage or zero with respect to the filament. Under this condition no current will flow in the grid circuit. However, if the grid is connected to a battery so as to make the grid positive with respect to the filament, a current will flow in the grid circuit. When the grid is positive, some of the electrons coming from the filament will be attracted to the grid. They will strike the grid, and current will flow through the external grid circuit to the filament. In this case the grid will act as another plate. When grid current is flowing, the tube does not act as a true amplifier. Therefore, on amplifier tubes it is necessary to keep the grid always negative with respect to the filament.

932. Three-electrode Tube as Detector.—In some radio detector circuits, the grid is connected through a resistance to a positive voltage. A small condenser is connected across the ends of the resistor. A tube connected in this way is called a "grid-resistance" detector.

In a grid-bias detector, a different arrangement is used. In this case the grid is connected to a circuit so that when no signal

is being received the grid has a certain negative voltage. For the tube illustrated by Fig. 592, this grid-bias voltage would be made minus 3 volts. At minus 3 volts the curve reaches the point where the plate voltage becomes practically zero. If the voltage is changed in one direction, say it is made minus 2 volts, the plate current increases to 2 milliamp. However, if the voltage is made more negative than minus 3 volts, say it is made minus 4 volts, there is practically no change in plate current; it remains at practically zero. Change in grid voltage on one side of the minus 3 point causes a change in plate current; but a change in grid voltage on the other side of the minus 3 point causes no change of plate current. This uneven action makes the tube useful as a detector.

933. Filament Material.—The filament must be of a material which gives off electrons freely when heated but will not deteriorate quickly. Tungsten wire was formerly used. It was found that if a small amount of thorium oxide was mixed with the tungsten the electron emission was greatly increased. Such a filament is called thoriated tungsten. Since tungsten is expensive it has been largely replaced by certain nickel alloys coated with oxides which emit electrons freely. Oxides of barium and strontium are widely used.

934. Alternating current may be used to heat the filament of a tube under certain conditions. If alternating current is used the plate current will fluctuate in value as the alternations of current occur. For some purposes fluctuations of current may not be objectionable, but in a radio receiver or sending set large fluctuations will cause distortion. The distortion will be small if the voltage across the filament is small compared with the grid-bias voltage. For instance, there are very good radio tubes which operate with a filament voltage of 2½ volts alternating current and a grid-bias voltage of minus 30 volts. Another cause of fluctuations when using alternating current is that the temperature of the filament fluctuates as the current repeatedly changes from its maximum to zero. To overcome this change in temperature a relatively thick filament is used. In this the temperature remains more steady than it does in a fine wire.

In another type of tube which operates satisfactorily on alternating current, the filament itself is not used for emitting electrons. It is surrounded closely by a tube or sleeve but insulated

from it. The surface of the sleeve is coated with a substance such as strontium or barium oxide which readily emits electrons when heated. This sleeve is the cathode. It is connected to a terminal outside the tube. When the filament is heated by the alternating current, the sleeve becomes hot enough to emit electrons just as the filament does in the direct-current tubes. Current then passes to the plate and through the outside circuit back to the cathode. Thus, the circuit is completed just as in a direct-current tube.

935. Uses of Three-electrode Tube.—The three-electrode tube has a wide variety of uses. In radio, telephony, telegraphy, and television it serves both receiving and sending stations. It acts as amplifier, detector, and generator of oscillations.

It is used in voltmeters to measure very small voltages or voltage changes. The small voltage is impressed on the grid and is recorded in a meter in the plate circuit. Sometimes more than one tube is used in order to amplify the voltage to such a value as can be read on a standard meter.

Its uses in industry are many. Wherever a small voltage or current must be used to give an indication or to control an electrical circuit, the three-electrode tube can be used to advantage. Its amplifying ability permits a small voltage to control large currents or voltages.

936. Screen-grid Tube.—The addition of a fourth electrode to the three-electrode tube makes practicable a very high amplification factor. In the three-electrode tube the amount of amplification attainable is limited because the filament and the plate act inside the tube like the plates of a condenser (Art. 795). These plates are small and the permittance of this condenser is very small. However, at the very high frequencies used in radio, even a small condenser offers little resistance to the passage of current. When the tube is operating, the voltage of the plate is continually varying. This varying voltage is impressed on the grid through this small condenser action. This interaction, if large, prevents proper functioning of the tube.

In constructing a tube, the spacing, etc., must be such as to minimize this interaction. Unfortunately, a design which does this will not give very high amplification.

However, addition of the screen grid reduces the undesired reaction and at the same time permits a construction which gives a high amplification factor.

The screen grid consists of a mesh or other form of fine wire very similar to the control grid. It is placed between the plate and the control grid. It is connected to the filament through a large condenser outside the tube. Since the resistance of a large condenser is small at high frequencies, the filament and the screen grid are always at the same voltage as far as high-frequency voltages are concerned. The filament is always at zero voltage, and therefore the screen grid is at zero voltage as far as high frequencies are concerned. Therefore, the screen grid is a screen of unvarying voltage between the plate and the grid, and voltage variations on the plate do not affect the grid.

With this protection, it is possible to construct a tube with a high amplification factor, which will operate satisfactorily.

It was stated above that the screen grid is at the same high-frequency voltage as the filament. This is true, but it is not at the same direct-current voltage. The filament is connected to zero voltage on the power supply, while the screen grid is connected to a positive voltage which is less than the plate voltage, the exact value depending on the tube and the circuit employed. For instance, one tube operates with a screen-grid voltage of plus 100 and a plate voltage of plus 250.

The screen-grid tube may be used for purposes which require distortionless high amplification of voltage without high currents.

937. The Magnetron.[1]—If a tube consisting of an axial filament and a concentric cylindrical plate is placed near a magnetic field, so that the lines of force are perpendicular to the direction of flow of the electrons from filament to plate, the electrons will describe curved paths in planes perpendicular to the lines of force. The curvature of these paths increases with the field strength until it becomes so great that the electrons cannot reach the plate but return to the filament. The current through the tube can thus be controlled entirely by the magnetic field. This tube is used in conjunction with an axial magnetic field. Electrons flowing from the filament to the plate under the combined influence of the electric and the magnetic fields give the tube a volt-ampere characteristic which has a negative slope over a definite range. Such a negative resistance can be used to generate oscillations.

938. The Pentode.—The pentode, or five-electrode tube, is similar to the screen-grid tube with another grid added. This

[1] General Electric Company's trade name.

arrangement is used to give a high amplification factor together with high current (low plate resistance). The fifth electrode is between the screen grid and the plate. It is connected inside the tube to the cathode which has zero voltage. In this position it prevents what is called secondary emission of electrons from the plate. When the electrons from the filament strike the plate, they dislodge some of the electrons from the plate material. In a screen-grid tube some of these electrons have enough velocity to travel to the screen grid. The screen grid, being positive, attracts these electrons and they constitute a current which flows from the screen grid externally to the filament. This current, however, is wasted, as it is lost from the plate current where it would be useful. In the pentode, the fifth electrode is inserted next to the plate. It is quite negative with respect to the plate. Therefore, after the secondary electrons leave the plate they tend to be again attracted to it rather than pass the more negative fifth electrode which repels them. Thus, they do not subtract from the plate current. In the pentode it is possible to impress a high voltage on the screen grid as there is no danger that it will steal away electrons from the plate. This increases the emission from the filament and gives the electrons a high speed so that they travel readily to the plate.

939. Other Multi-electrode Tubes.—There are numerous other types of tubes having more than three electrodes. In one type there are two independent control grids. Thus, two signals may be impressed on the tube at one time. The output will be a combination of the two signals. An ordinary screen-grid tube may be used in this manner.

940. Gas-filled Tubes.—If a small amount of certain gases is present in a tube the current is generally larger than the current in a high vacuum. Advantage is taken of this fact in the construction of several kinds of tubes. The gas must be one which is not chemically changed by electronic action. Argon, neon, and helium are used at a fairly low pressure. The increase in current is caused by ionization of the gas. When electrons are emitted from the cathode some of them collide with the atoms of the gas. When an electron strikes an atom it knocks other electrons out of the atom. These electrons travel on toward the plate. The remaining part of the atom has now lost some

negative electrons, and therefore it has a positive charge. It is now called an "ion" or an "ionized" atom. It is attracted toward the cathode, which is negative. The positive ions thus formed create a positive charge in space, and this neutralizes part of the negative space charge created by electrons leaving the filament. As explained in Art. 918, this negative space charge tends to prevent electrons from leaving the filament. Therefore, when the positive ions neutralize this negative space charge, they make it easier for electrons to pass to the plate. As a result of these actions more electrons pass over and the plate current of the tube increases.

This ionization causes a dull light or glow to be emitted from the gas. For this reason gaseous tubes are sometimes called glow-discharge tubes.

941. Types of Gas-filled Tubes.—The gas-filled tube may have a hot cathode or a cold cathode, depending upon its purpose. Rectifiers, photoelectric cells, grid-glow tubes, and voltage-limiting tubes are among the types employing gases. Radio detectors and X-ray tubes containing gas have been made, but they have largely been replaced by high-vacuum tubes. Tubes designed to give amplification without distortion must not contain gas.

942. Gaseous Thermionic Rectifier.—A widespread use of a two-electrode gaseous thermionic tube is as a rectifier for charging storage batteries and for similar purposes. Such rectifiers give large currents at low voltages. In a popular rectifier of this type argon gas is used at a pressure equal to 5 cm. of mercury when the tube is cold. The filament or cathode is of tungsten wire and the anode is of graphite.

Rectifiers of this type are not suitable for higher voltages. The operation of a rectifier depends on its ability to resist flow of current while the alternating voltage is positive at the filament. With a high voltage this type of tube does not offer much resistance to flow of current in the reverse direction, that is, when the filament is positive. Therefore, the rectifier action is not satisfactory under high voltages.

943. Cold-cathode Gaseous Rectifier.—It is possible to obtain electronic action and ionization without a heated cathode. Cold-cathode tubes have a relatively high voltage drop and a comparatively small current-carrying capacity.

944. Glow-discharge Tubes.—A type of gaseous tube is known as the glow-discharge or the grid-glow tube. The tubes have a wide range of application due to their sensitivity, high speed of operation, durability as rapid-duty contactors, and general adaptability for automatic operations. The grid-glow tube resembles the ordinary three-electrode vacuum tube in that it contains an anode, a cathode, and a grid, and in operation functions as a grid-controlled rectifier. In the vacuum tube, however, the current is carried between the principal electrodes entirely by negative electrons emitted from the filament. The grid-glow tubes, on the other hand, are filled with an inert gas, neon, helium, or mercury vapor at a few millimeters pressure. The conduction of current is in the form of a glow or arc discharge through this gas. Passage of current through glows or arcs does not consist of a flow of electrons alone. The current is carried jointly by electrons moving toward the anode and by positive ions moving toward the cathode. Their relative distributions vary continuously over the length of the discharge, electrons being most abundant at the anode and positive ions at the cathode. At some intermediate point they are present in approximately equal numbers. The positive ions render a valuable service in neutralizing the negative space charge, such as is present in the operation of vacuum tubes where it accumulates in proportion to the current and causes a high tube resistance. The voltage drop of the hot cathode gaseous discharge tube is of a low essentially constant value, only sufficient to ionize the gas content, so that the efficiency is considerably higher than that of the vacuum type. A tube of the same size can therefore control considerably more power. In a glow discharge or an arc, the grid will always receive current excepting at one critical value of potential, this being the particular value which causes electrons and positive ions to arrive at the grid in equal numbers.

The function of the grid in this type of tube is merely to "trigger off" the tube. When the grid is given the proper potential in relation to the anode-cathode potential, the tube breaks down and current is conducted in the form of a glow, or arc, discharge. Once this is started, the tube voltage drop reaches a low constant value throughout the period of conduction, and the grid is no longer effective in controlling the discharge.

Hence the tube possesses a lock-in characteristic when used with direct-current sources. The anode potential must then be interrupted to stop the discharge. The tubes are generally used with alternating-current sources where the discharge is periodically extinguished, and the grid control restored, by reversal of the anode-cathode voltage. Thus grid control is never lost for more than one-half cycle. The breakdown or ionization time varies from 1 to 50 microseconds and the deionization time (the time for restoration of grid control after the tube has been conducting) is not in excess of one-thousandth second.

945. The various grid-glow tubes may be classified with respect to the type of cathode as. (1) the cold-cathode type; (2) the hot-cathode or thermionic type; and (3) the mercury-pool cathode type.

In the operation of tubes of the first class, electrons are emitted from the cathode due to the positive ion bombardment, breakdown of the tube being possible because of a small ever-present ionization of the gas. A higher tube voltage drop and consequent higher power loss is involved in the use of this type of tube, hence it is used mainly for the control of small amounts of power in sensitive relay applications.

In the tubes of the two other classes electrons are emitted thermionically from the cathode and therefore have a low anode-cathode voltage drop. Although some cathode power for electron emission is required constantly during the use of tubes of these classes, one or the other is used wherever appreciable amounts of power are to be handled.

An important difference existing between tubes of class 2 and class 3 is that the maximum current passed by the hot-cathode tube must never exceed the electron-emitting power of the cathode because, whenever this is exceeded, the tube drop rises and the resulting positive ion bombardment of the cathode rapidly destroys the coating, while in the case of the mercury-pool tube, the emission increases in proportion to the current drawn, and the only injury which may be worked by high currents is that due to excessive heating of the tube parts. This type thus has advantages in applications where the current rises to very high values momentarily.

946. The applications of grid-glow tubes of the various types range all the way from ultrasensitive uses to the control of large

spot welders. Among some of the applications of these tubes may be mentioned: production-line control, oil-burner-safety control, cable testing, voltage regulation, time-delay relays, temperature control, the stroboglow, theater-light dimming, motor-speed control, and control of welding machines.

947. The Inverter.—Description has been given of the use of the electronic tube for the conversion of alternating into direct current by the principle of rectification. A three-element gas tube may also be used for converting direct into alternating current, in which form the tube is known as an inverter. In inverters some form of energy storage is necessary to commutate the flow of current from one winding to another and also to aid

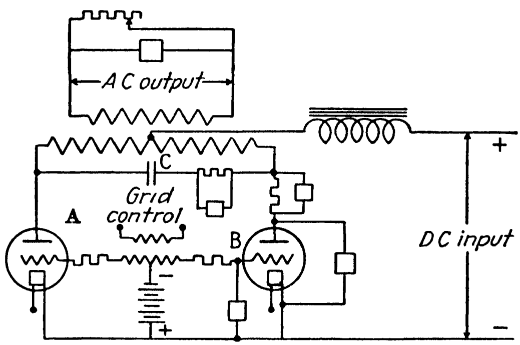


FIG. 594.—Simple single-phase inverter circuit for converting direct current into alternating current.[1]

in deionizing the tube so as to establish grid control. Numerous methods for the connections of the tubes for this purpose are employed, but the principle is illustrated by the simple single-phase inverter circuit shown in Fig. 594.

In operation, the grid of tube *A*, for example, is made sufficiently positive to establish conduction, and the grid of tube *B* is held sufficiently negative to maintain control. Current then flows from the direct-current source through the inductance and one-half of the transformer winding, completing the circuit through tube *A*. Commutating condenser *C* becomes charged negatively at tube *B* and positively at tube *A*, to a potential the value of which is nearly twice that of the direct-current line. At the end of the cycle the polarity of the grids is reversed and

[1] General Electric diagram.

tube B is made conducting. Current that previously had been flowing through tube A is diverted to the condenser, and as conduction ceases the anode of tube A is driven negative, the grid thereby gaining control. Meanwhile, current flow is transferred to tube B and its half of the transformer winding; commutation of the tube currents is then complete. The condenser is charged again but with opposite polarity for repeating the switching operations. On the output side of the transformer the tube voltage waves combine to produce an alternating voltage that by proper circuit constants can be made practically sinusoidal.

One of the outstanding applications of the inverter is that of converting direct-current power into alternating-current power for the operation of electrical apparatus such as induction furnaces, induction motors, radio receivers, etc., in localities where direct current only is available from the supply mains.

948. Photoelectric Cells.—The photoelectric cell is a type of electronic tube which has many useful applications. It is used in television, in measuring intensity of light, in matching colors, and in various applications where the movement of a body is employed for controlling an electrical circuit. This electrical circuit may be used for counting, as, for instance, the number of persons or vehicles passing a certain point. Or it may be used for performing other operations, such as turning lights on and off and starting or stopping machinery.

949. Photoelectric Effect.—The photoelectric cell depends for its action on the fact that when light falls on certain substances, they emit electrons. The action is similar to that which results when substances are heated, as in the thermionic tube. A number of substances have been found to give this photoelectric effect to a marked degree. The most prolific sources of electrons are the alkali metals, so-called because their salts are alkalies. These metals include potassium, sodium, caesium, and lithium. In many of the present commercial types of photoelectric cells the light sensitive element is caesium oxide deposited on the cathode.

950. Types of Photoelectric Cells.—Some of these cells are made with a high vacuum while others contain small amounts of certain gases at low pressure, such as helium, neon, and argon.

In the commonest form the sensitive metal electrode, cathode, is large and partly surrounds the anode (Fig. 595). The cathode may be a plate in the form of a half cylinder, or it may be a coating on the inside surface of the tube. In any case a translucent opening must be left for entrance of the light. The anode may be a ring or grid which will not obstruct much light. It is placed in the center of the tube.

In the vacuum type, the total current is made up of the electrons emitted from the sensitive surface. In the gas-filled type, greater effective response is obtained due to gas amplification and secondary emission. The ratio of the tube's response at the operating voltage to its response at the voltage where the gas amplification begins, for a given illumination, is known as the gas ratio.

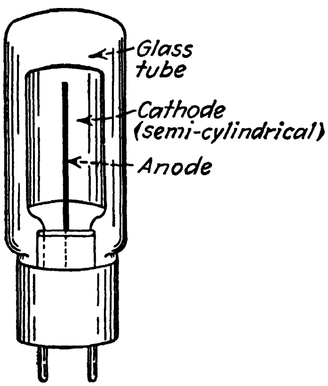


FIG. 595—A typical photoelectric tube.[2]

Photoelectric cells are of three general types as follows:[1]

951. Photoemissive cells, in which a beam of light causes a surface to emit electrons. In a manner similar to that in which in the thermionic tube heat energy is converted into electrical energy, so in the photoelectric tube, light energy is converted into electrical energy. The electrons are attracted toward a plate maintained at a positive potential by an external battery. The current flowing is of low value, and one or more stages of amplification by thermionic tubes are therefore usually employed to increase this current to a value sufficient to perform the desired functions of the devices with which the tubes are used.

952. Photoconductive cells, in which the resistance of a material to the flow of current is materially altered when it is illuminated by a beam of light. Selenium is the best-known example of this type of light-sensitive material.

[1] This classification is taken from Keith Henney, "Electron Tubes in Industry," McGraw-Hill Book Company, Inc.

[2] Sketch from Austin V. Eastman, "Fundamentals of Vacuum Tubes," McGraw-Hill Book Company, Inc.

953. Photovoltaic cells, in which the passage of electrons from one metallic surface to another is accelerated by illuminating the surface with light. These tubes act as rectifiers, the electrons passing more readily from one plate or surface to another than they do in the opposite direction. The voltage developed by these surfaces is independent of the area illuminated; the output varies with the area.

954. Uses of Photoelectric Cells.—As already stated, such cells have a multitude of uses. Television would be impossible without them. They are used in transmitting pictures by wire or radio. In one form of sound moving picture the variations in light received through an edge of the picture film are received by a photoelectric cell, the current of which is amplified to produce sounds and music by means of loud speakers. They may be used to measure the light from lamps or other sources. They are even employed to measure the light from stars. They are useful in matching and sorting objects of varying color, as, for instance, in sorting cigars, separating the light-colored ones from the dark. They are installed at airports and in factories to turn on lamps when the daylight has decreased to a certain intensity. Likewise, in the morning they turn out the lights. They are used as smoke detectors which give an indication of the smoke density, turn on blowers for removing the smoke, give an alarm, or start a sprinkler. When used for such a purpose, a photoelectric cell is mounted opposite a lamp which is always lighted. When there is smoke or vapor in the air between the lamp and the cell, the light reaching the cell is diminished. This causes the current in the cell to decrease. At a certain density of smoke the current is of the correct value to operate a relay, and this connects other electric circuits which perform the desired function.

Another wide field of use depends on the blocking of the light by an object moving between the lamp and the cell. When the light is blocked the current drops and relays act to perform various duties. Thus, objects may be counted as they pass a certain point. Vehicles passing by a cell may cause traffic lights to operate. The cell is used in a mill where steel bars must be cut to a certain length. As soon as the end of a long strip of steel reaches a certain point, it intercepts the light shining

on a photoelectric cell. At that instant, by means of a relay, a shearing machine is operated and a piece of the desired length is thus cut off. There are many other applications too numerous to mention here.

955. The Cathode-ray Tube.—Cathode rays are the streams of electrons which flow from the negative electrode, or cathode, of electron devices. The flow of electrons from the cathode has been explained under two-electrode thermionic tubes. In the cathode-ray tube the construction is such that these rays in considerable quantity may pass out of the tube in a concentrated stream.

The cathode rays travel in straight lines, but they may be deflected by magnetic or electrostatic fields. They are stopped by heavy objects but pass through thin ones. They affect

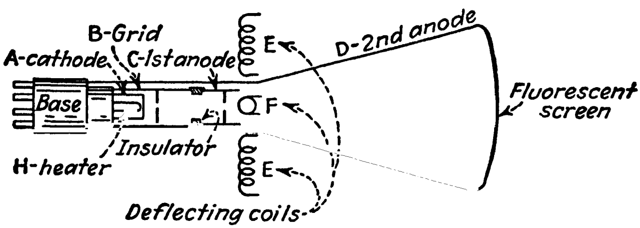


FIG. 596.—Illustrating the principle of the cathode-ray tube.

photographic plates and cause certain substances to "fluoresce" or give off a glowing light.

The action of the cathode-ray tube is illustrated in Fig. 596 which is a diagrammatical section of a typical tube.[1] Variation in details will of course be found in different types of tubes to accommodate them to special purposes, but this outline shows the general principle upon which the tubes operate. In this drawing, A is the cathode the emitting surface of which is heated by the heater H. B is the grid for controlling the intensity of the beam. This is in the form of a metal sleeve having a cylindrical disk immediately in front of the cathode. A small aperture in the center of this disk permits the passage of those electrons which are traveling in the right direction and the small size of the opening concentrates the electrons in a fairly narrow beam. The potential of the grid is made negative in respect to the

[1] The description of the operation of the cathode-ray tube is taken largely from A. V. Eastman's "Fundamentals of Vacuum Tubes," McGraw-Hill Book Company, Inc.

cathode, and the amount of this negative potential determines the number of electrons in the beam, therefore its intensity.

C is the first anode, which is made moderately positive with respect to the cathode, the exact potential depending upon the size of the tube. It is very similar to the grid in construction, having one or more small perforated disks in the path of the electron stream. The positive potential of this anode draws electrons through the grid aperture into the beam, and the small opening in its disk, known as the masking aperture, cuts off some of the peripheral electrons, much as the stop in a camera cuts off some of the light. Beyond the second aperture the beam enters the field of the second anode *D*, which is maintained at a potential of from 300 volts in small tubes to several thousand volts in large ones. The field set up by this anode, besides providing further acceleration, gives a radial velocity to the electrons of the beam, acting toward the axis, such that, if the axial velocity is correct, all electrons will concentrate on a single spot on the fluorescent screen at the end of the tube.

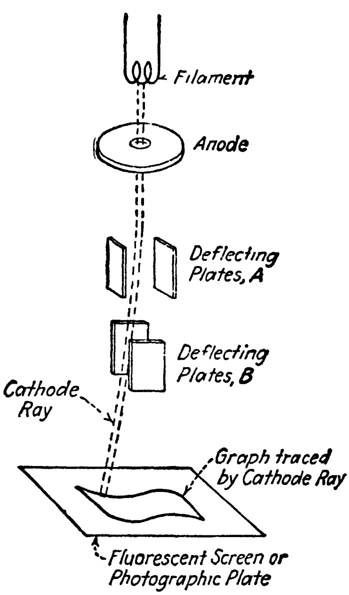


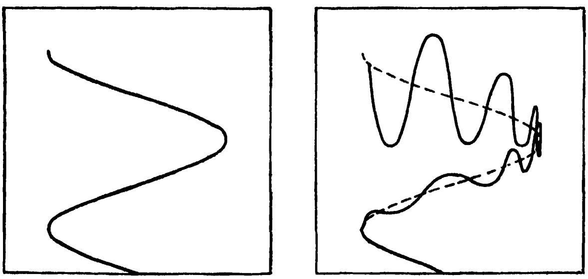
FIG. 597.—Diagram of cathode-ray oscillograph.

Deflecting coils are shown at *E* and *F*, coils *E* deflecting the beam in a horizontal direction while coils *F* act in a vertical plane. Passage of the current to be investigated through one pair of coils and a sweep or timing current through the other, will produce an image of the unknown current on the screen. The cathode-ray tube is essentially a visual device rather than a photographic one. However, the intensity of the spot on the screen is sufficient to photograph when the phenomenon under investigation is a repeating one. Transient phenomena are generally too fast to record satisfactorily.

956. The Cathode-ray Oscillograph.—The properties of cathode rays make them useful in a high-speed type of oscillograph.

The oscillograph is an instrument for giving a visual or photographic record of high-speed changes of voltage or current. This record is in the form of a curve. In the cathode-ray oscillograph the curve is traced by a fine cathode ray which moves in accordance with the changes of the recorded voltage.

In its simplest form the cathode-ray oscillograph is shown diagrammatically in Fig. 597. Electrons from the filament pass through a small hole in the anode and emerge as a fine ray. This ray passes between two metal plates, *A*. If one of these plates, say the left-hand one in Fig. 597, is connected to the positive side of a battery, and the right-hand plate to the negative side, the charges on the plates will force the ray to swing over toward the left. Suppose that, instead of the battery, the voltage



I II

FIG. 598.—Curves traced by du Four oscillograph. I shows the base line traced before the unknown voltage is applied. II shows the record when the unknown voltage has been applied.

to be recorded is connected across these plates. If the voltage varies, the ray will swing across in step with the voltage. The ray would then trace a straight line from side to side on the photographic plate or fluorescent screen at the bottom of the tube. In order to get a curve, the ray must at the same time be swung from back to front. This is accomplished by use of another set of deflecting plates, *B*. If a steadily increasing positive voltage is impressed on the front plate, the ray will pass steadily from back to front. These two motions combined will trace a curve on the screen. This curve will show just how the voltage on plates *A* is changing in value.

The oscillograph may have a fluorescent screen on which the ray will trace a visible curve; or a photographic plate may be used to get a permanent record.

A form of cathode-ray oscillograph, known as the du Four oscillograph, has advantages for recording very fast fluctuations which do not repeat themselves regularly. Such fluctuations if recorded by the tube previously described would give a curve in which the waves would be too close together to be easily seen and analyzed. In order to spread the record, a third motion is added to the travel of the ray. This is accomplished by placing two coils at the sides of the ray. Changing current passes through these coils and swings the ray from side to side in the same way as changing voltage on the plates. This gives a third motion. The action may be explained by reference to Fig. 598. The curve at I is that which is traced by the two known voltages

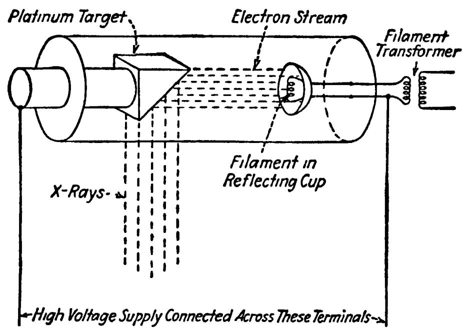


FIG. 599.—Diagram of X-ray tube.

impressed on the coils and one set of plates. When the unknown voltage is impressed on the other set of plates, the curve is as shown in II. In this curve the base line is the long curved dotted line. The record of the unknown voltage passes back and forth across this base line.

957. X-ray Tubes.—X-rays are radiations which can penetrate substances to a greater degree than any other known rays except some of the radiations from radium and other radioactive materials. The degree of penetration depends on the density of the substances. They affect photographic plates and make certain materials fluorescent. Therefore, they can be used for making photographs or for visual inspection. They are used for medical examination, for medical treatment, for inspection of metals, for scientific purposes, etc.

A simple form of X-ray is shown diagrammatically in Fig. 599. A vacuum tube encloses a cathode like the filament shown,

and an anode which is called a "target." A source of high voltage is connected across the two electrodes. The electrons pass from the filament to the target. When the target is struck by the electrons, it emits X-rays which pass on out of the tube.

A high vacuum is employed in most tubes. The use of gas-filled tubes, which were employed in some of the earlier types, has now become practically obsolete.

The cathode is generally a heated filament, tungsten being used and sometimes tantalum. A metal focusing cup is placed back of the filament to direct the electrons toward the target. When the electrons strike the target, they are largely absorbed and most of their energy is given up as heat, but about 0.1 to 0.2 per cent comes out as X-rays.

The target must absorb and radiate heat. This is sometimes mounted on a metal piece extending outside the tube and having fins for radiating heat. It may be arranged to be cooled by running water. The face of the target is generally inclined to the electron stream by 45 to 75 deg. Sometimes it is conical in shape so as to concentrate the rays.

The entire tube is often immersed in oil to keep it cool and prolong its life.

On account of their active penetrating power, too long exposure to X-rays will cause injury to the human body. Therefore, protective screens of lead are placed about the X-ray apparatus to guard persons who are nearby. Lead is a very dense metal and the rays do not penetrate it to any considerable distance.

APPENDIX

Sines and Cosines of Angles

ϕ°	$\sin\phi$	$\sin^3\phi$	$\cos\phi$	$\cos^2\phi$	$\cos^3\phi$	ϕ°	$\sin\phi$	$\sin^3\phi$	$\cos\phi$	$\cos^2\phi$	$\cos^3\phi$
0	0 0000	0 0000	1 000	1 000	1 000	43	0 682	0 317	0 731	0 535	0 391
1	0 0175	0 0000	1 000	1 000	1 000	44	0 695	0 335	0 719	0 517	0 372
2	0 0349	0 0000	0 999	0 998	0 998	45	0 707	0 354	0 707	0 500	0 354
3	0 0523	0 0001	0 999	0 997	0 996	46	0 719	0 372	0 695	0 483	0 335
4	0 0698	0 0003	0 998	0 995	0 993	47	0 731	0 391	0 682	0 465	0 317
5	**0 0872**	**0 0007**	**0 996**	**0 992**	**0 989**	**48**	**0 743**	**0 410**	**0 669**	**0 448**	**0 300**
6	0 105	0 0011	0 995	0 989	0 984	49	0 755	0 430	0 656	0 430	0 282
7	0 122	0 0018	0 993	0 985	0 978	50	0 766	0 450	0 643	0 413	0 266
8	0 139	0 0027	0 990	0 981	0 971	51	0 777	0 469	0 629	0 396	0 249
9	0 156	0 0038	0 988	0 976	0 964	52	0 788	0 489	0 616	0 379	0 233
10	**0 174**	**0 0052**	**0 985**	**0 970**	**0 955**	**53**	**0 799**	**0 509**	**0 602**	**0 362**	**0 218**
11	0 191	0 0069	0 982	0 964	0 946	54	0 809	0 530	0 588	0 345	0 203
12	0 208	0 0090	0 978	0 957	0 936	55	0 819	0 550	0 574	0 329	0 189
13	0 225	0 0114	0 974	0 949	0 925	56	0 829	0 570	0 559	0 313	0 175
14	0 242	0 0142	0 970	0 941	0 913	57	0 839	0 590	0 545	0 297	0 162
15	**0 259**	**0 0173**	**0 966**	**0 933**	**0 901**	**58**	**0 848**	**0 610**	**0 530**	**0 281**	**0 149**
16	0 276	0 0209	0 961	0 924	0 888	59	0 857	0 630	0 515	0 265	0 137
17	0 292	0 0250	0 956	0 915	0 875	60	0 866	0 650	0 500	0 250	0 125
18	0 309	0 0295	0 951	0 905	0 860	61	0 875	0 669	0 485	0 235	0 114
19	0 326	0 0345	0 946	0 894	0 845	62	0 883	0 688	0 470	0 220	0 103
20	**0 342**	**0 0400**	**0 940**	**0 883**	**0 830**	**63**	**0 891**	**0 707**	**0 454**	**0 206**	**0 0936**
21	0 358	0 0460	0 934	0 872	0 814	64	0 899	0 726	0 438	0 192	0 0842
22	0 375	0 0526	0 927	0 860	0 797	65	0 906	0 744	0 423	0 179	0 0755
23	0 391	0 0596	0 921	0 847	0 780	66	0 914	0 762	0 407	0 165	0 0673
24	0 407	0 0673	0 914	0 835	0 762	67	0 921	0 780	0 391	0 153	0 0597
25	**0 423**	**0 0755**	**0 906**	**0 821**	**0 744**	**68**	**0 927**	**0 797**	**0 375**	**0 140**	**0 0526**
26	0 438	0 0843	0 899	0 808	0 726	69	0 934	0 814	0 358	0 128	0 0460
27	0 454	0 0936	0 891	0 794	0 707	70	0 940	0 830	0 342	0 117	0 0400
28	0 470	0 104	0 883	0 780	0 688	71	0 946	0 845	0 326	0 106	0 0345
29	0 485	0 114	0 875	0 765	0 669	72	0 951	0 860	0 309	0 0955	0 0295
30	**0 500**	**0 125**	**0 866**	**0 750**	**0 650**	**73**	**0 956**	**0 875**	**0 292**	**0 0855**	**0 0250**
31	0 515	0 137	0 857	0 735	0 630	74	0 961	0 888	0 276	0 0760	0 0209
32	0 530	0 149	0 848	0 719	0 610	75	0 966	0 901	0 259	0 0670	0 0173
33	0 545	0 162	0 839	0 703	0 590	76	0 970	0 914	0 242	0 0585	0 0142
34	0 559	0 175	0 829	0 687	0 570	77	0 974	0 925	0 225	0 0506	0 0114
35	**0 574**	**0 189**	**0 819**	**0 671**	**0 550**	**78**	**0 978**	**0 936**	**0 208**	**0 0432**	**0 00899**
36	0 588	0 203	0 809	0 655	0 530	79	0 982	0 946	0 191	0 0364	0 00695
37	0 602	0 218	0 799	0 638	0 509	80	0 985	0 955	0 174	0 0302	0 00524
38	0 616	0 233	0 788	0 621	0 489	81	0 988	0 964	0 156	0 0245	0 00376
39	0 629	0 249	0 777	0 604	0 469	82	0 990	0 971	0 139	0 0194	0 00270
40	**0 643**	**0 266**	**0 766**	**0 587**	**0 450**	**83**	**0 993**	**0 978**	**0 122**	**0 0149**	**0 00181**
41	0 656	0 282	0 755	0 570	0 430	84	0 995	0 984	0 105	0 0109	0 00114
42	0 669	0 300	0 743	0 552	0 410	85	0 996	0 989	0 0872	0 0076	0 00060

INDEX

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